Bounded Region of Solitary Wave Solutions Along with Higher Order Amplitudes and Slow Mode Phase Velocity in Non-isothermal Plasma

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Abstract. Ion-acoustic solitary waves are investigated in non-isothermal two-temperature electron plasma in presence of positron. The compressive solitary wave solutions of first (ϕ_1), second (ϕ_2) and third (ϕ_3)order in non-isothermal electron plasma are calculated here by a perturbative new approach known as 'tanh – method' as well as the condition for the existence of a potential well with warm positive ion, warm negative ion including their streams and warm positron. The bounded region of the compressive solitary wave solution is carefully discussed in this paper from which periodic and aperiodic solutions are analysed. The well-defined boundaries in this paper highlight the bifurcation of all the waves which appear due to the interaction of trapped electrons in plasma acoustic waves. Two kinds of phase velocities are also investigated in this paper. In our study, we compare the first (ϕ_{01}), second (ϕ_{02}) and third (ϕ_{03}) order amplitudes between two temperature and single temperature non-isothermal electron plasmas under the variation of different temperatures of positive and negative ions. Also we find out slow ion-acoustic phase velocity (V_s) and profiles of Sagdeev potential function $\psi(\phi)$ of the solitary waves for some plasma parameters. All these are represented graphically by the respective figs. 1 – 18.

Highlights:
(i) Phase plane trajectories under different plasma parameter variations are discussed clearly in a four component plasma.
(ii) Comparison of amplitudes and slow mode phase velocity are another attracting matter.
(iii) Profiles of Sagdeev potential function are drawn for determination of soliton nature.

Keywords: Sagdeev Pseudopotential, non-isothermal electron, tanh-method, two-temperature electrons, positron, amplitude, phase velocity, spiky and explosive solitons.

I. INTRODUCTION

Ion-acoustic solitary waves for two-temperature and single temperature isothermal plasma containing positive ion, negative ion and positron have been studied theoretically by a large number of scientists [1-7]. Some of the renowned physicists [8-13] showed the behaviour and nature of solitary waves by experimental and theoretical observations in the plasmas. On the basis of the experimental results of Nakamura et al [14] and Ludwig et al [15], the existence of ion-acoustic solitary waves in a multicomponent plasma with negative ions was investigated properly. The Boltzmann relation for electron density (n_e) did not ensure the account of the trapped electrons which expects a strong interaction with solitary wave during its evolution in plasmas. Since for ion-acoustic wave, the trapped particles are those whose velocities are closed to the ion-acoustic speed and the number of trapped ions is generally ignored because of its negligible account in the dynamical system. On the other hand, since the shift in the electron distribution is very small, the number of trapped electrons is very large and thereby the effect of the trapped electrons, whose distribution may not be Maxwellian, should also not be taken into account under consideration of the ion-acoustic dynamics. Schamel [16-17] first studied the ion-acoustic solitons for non-isothermal plasma and made an attempt to deal such trapping of electrons causing the appearance of different electrons viz. cold and warm. Based on the reductive perturbation technique Schamel [16,17] then derived the K-dV equation in non-isothermal plasma to study the small amplitude solitary wave in the form of soliton profile. By using the same reductive perturbation method, Tagare [18] also showed a non-linear ion-acoustic solitary wave in a collisionless plasma consisting of negative ions, positive ions and warm non-isothermal electrons by the modified Korteweg-de-Vries (K-dV) equation. After that, Kalita and Bujarbarua [19] extended the works of many previous authors [20-21] for higher order contribution to ion-acoustic solitary waves. In presence of negative ions, solitary waves have an interesting characteristics .The study of ion-acoustic solitary waves in a multicomponent plasma with negative ions investigated by Das [22] had a considerable impact in laboratory plasmas. It is important to note in this context that non-isothermal electrons can be observed in many astrophysical and space environments. Gill et al [23] studied the effects of non-isothermal electrons on ion-acoustic solitary waves with positive and negative ions and showed only the compressive solitons. The plasma with this negative ions exhibits compressive and rarefactive solitons in isothermal case whereas in non-isothermal plasma it is completely different. It has been shown that in case of a small percentage of non-isothermality, the solitary wave equation exhibits both kinds of solitons in various plasmas with respect to non-isothermality. Tagare and Reddy [24] studied the higher order ion-acoustic solitary waves in presence of negative ions together with positive ions and non-isothermal electrons. Actually the plasma behaves non-isothermally in presence of resonant electrons and interesting results are found in multicomponent plasma consisting of non-isothermal electrons. In case of two-ion plasma with single temperature isothermal electron, Bhattacharyya et al [25] found bounded solution of solitary waves with interior and exterior regions of the separatrices correspond to periodic and aperiodic solutions respectively. But in the case of two-temperature non-isothermal electron plasma many physicists [26-28] assumed Schamel's plasma model and investigated the effects of non-isothermality for formation of solitons. Majumdar et al [29] studied the first, second and third order contributions to ion-acoustic solitons along with their amplitudes, widths and Mach number in a plasma with two types of cold positive ions and two-temperature non-isothermal electrons by the renowned Bogoliubov – Mitropolsky method. Chattopadhyay et al [30] studied first and second order compressive solitary wave solutions in non-isothermal single temperature electron for cold positive and cold negative ions with second order W – type soliton shape in nature. Following Chattopadhyay et al [31], the present author took the warm positive and warm negative ions with streams finding critical positron density (χ_c), critical negative ion temperature (σ_{jc}), critical positive ion temperature (σ_{ic}) and critical velocity (V_c) of two modes. In this paper the first(ϕ_1) , second(ϕ_2) and third(ϕ_3) order compressive solitary wave solutions are obtained by a perturbative new approach known as the' tanh – method' [32] with first(ϕ_{01}) , second $(\phi_{02})$and third (ϕ_{03})order amplitudes in two-temperature non-isothermal electron plasma . The third order(ϕ_3) solutions are known as the spiky and explosive solitary wave solutions obtained by Das et al [33] and this theoretical solutions are related to satellite observations made in interplanetary space plasmas. Considering two-temperature non-isothermal electron plasma with warm positron, warm positive and negative ions with streams Chattopadhyay [34] also investigated theoretically the profiles of Sagdeev potential function ψ(ϕ) against ϕ for fully non-linear situation and small amplitude double layer profiles from Sagdeev potential function ψ(ϕ) against ϕ, mentioning the amplitudes of the solitary waves under the variation of different plasma parameters.

The plan of the paper is arranged in the following manner:

In Section II, the Sagdeev potential function $[\psi(\phi)]$ for non-isothermal two temperature electron plasma is calculated from the basic set of equations and critical values of the positron density (χ_c), negative ion temperature (σ_{jc}) and phase velocity of slow (V_S) and fast (V_F) mode are obtained from the condition for the existence of a solitary wave solutions. Slow mode phase velocity is also discussed in detail and shown graphically. In this section, the boundary region of the compressive solitary wave solution is critically analysed so that periodic and aperiodic solutions are obtained and later on the condition for compressive solitary wave solution is discussed carefully. The separatrices of the boundary region of compressive solitary wave solutions are shown graphically under the variation of different plasma parameters. The nature of Sagdeev potential curve $[\psi(\phi)]$ gives us the characteristics such as amplitude and depth of the solitary waves potential, shown graphically by that curve under the variation of some plasma parameters. Section III contains the first, second and third order compressive solitary wave solutions along with the first, second and third order amplitudes for the non-isothermal two temperature electron plasma. The same first, second and third order amplitudes for non-isothermal two and single temperature electron plasmas are also compared later by graphical approach. The entire discussion is done very carefully in Section IV with the variation of different plasma parameters. Concluding remarks are given in Section V.

II. FORMULATION

For studying solitary wave solution in non-isothermal plasma we consider a plasma consisting of warm positive ions, warm negative ions with streams and warm positron. The governing normalized basic equations for non-linear behaviour of ion-acoustic solitary waves along x-axis in a collisionless, unmagnetised warm plasma for positive and negative ions are given below in the following form:

For positive ions

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial p_i}{\partial t} + u_i \frac{\partial p_i}{\partial x} + 3\, p_i \frac{\partial u_i}{\partial x} = 0 \tag{3}$$

For negative ions

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j u_j) = 0 \tag{4}$$

$$\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} + \frac{\sigma_j}{Q n_j}\frac{\partial p_j}{\partial x} = \frac{Z}{Q}\frac{\partial \phi}{\partial x} \tag{5}$$

$$\frac{\partial p_j}{\partial t} + u_j \frac{\partial p_j}{\partial x} + 3\, p_j \frac{\partial u_j}{\partial x} = 0 \qquad (6)$$

All these equations are closed with the normalised Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i + Z\, n_j - n_p \qquad (7)$$

where the normalized positron density (n_p) is $n_p = \chi\, e^{-\sigma_p \phi}$ (8)

In this case, the plasma parameters n_i , u_i , σ_i , p_i are known as the concentration , velocity , temperature and pressure of positive ions while n_j , u_j , σ_j , p_j are the same for negative ions. Also Q, Z, n_e ,ϕ , n_p, t, x, σ_p and χ are respectively known as the ratio of negative to positive ion masses , charge of ions , concentration of non-isothermal electrons, electrostatic potential, concentration of positron , time, distance, temperature ratio of electron and positron and density of positron at $\varphi = 0$.
Here $\sigma_i = \frac{T_i}{T_e}$, $\sigma_j = \frac{T_j}{T_e}$,$\sigma_p = \frac{T_e}{T_p}$; T_i , T_j ,T_e and T_p are the temperatures of positive ion, negative ion, electron and positron. Q = $\frac{m_j}{m_i}$, m_j and m_i are masses of negative and positive ions. The charge neutrality condition is

$$\chi + n_{i0} = 1 + Zn_{j0}$$

In absence of positron (i.e. $\chi = 0$) equation (8a) supports Ref. [30]. The above equations (1) to (8) are normalized as the usual conventional way.

In order to get the solitary wave solution we assume the Galelian transformation $\eta = x - Vt$, where V is the velocity of the solitary wave .The boundary conditions are

$n_i \to n_{io}, n_j \to n_{jo}$,$u_i \to u_{io}$, $u_j \to u_{jo}$, $p_i \to 1$, $p_j \to 1$, $n_e \to 1$, $n_p \to \chi$ and $\phi \to 0$ at $|x| \to \infty$

Following Chattopadhyay [35] and using the above boundary conditions we get finally from equations (1) to (8)

$$n_i = \frac{1}{2}\sqrt{\frac{n_{io}^3}{3\sigma_i}}\left[\sqrt{\left(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi} - \sqrt{\left(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi}\right] \qquad (9)$$

$$n_j = \frac{1}{2}\sqrt{\frac{Qn_{jo}^3}{3\sigma_j}}\left[\sqrt{\left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}} - \sqrt{\left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}}\right] \qquad (10)$$

and

$$\frac{d^2\phi}{d\eta^2} = n_e - \frac{1}{2}\sqrt{\frac{n_{io}^3}{3\sigma_i}}\left[\sqrt{\left(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi} - \sqrt{\left(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi}\right]$$

$$+\frac{Z}{2}\sqrt{\frac{Qn_{jo}^3}{3\sigma_j}}\left[\sqrt{\left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}} - \sqrt{\left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}}\right] - \chi e^{-\sigma_p \phi} \qquad (11)$$

For real values of concentration of ions, the following restrictions on the electrostatic potential ϕ is

$$-\frac{Q}{2Z}\left(V-u_{jo}-\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2<\phi<\frac{1}{2}\left(V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 \qquad (12)$$

In this paper, the speciality of the plasma model taken here is with free electrons of temperature T_{ef} along with some electrons of temperature T_{et}. Some fraction of these kind of electrons other than free electrons are moving in the potential well and losing energy continuously and as a result of which, the electrons bounce back and forth within the potential well and ultimately get trapped in the potential well within the plasma. The trapped electrons can change the features of plasma acoustic waves experimentally and this was supported by many theoretical observations. The distribution of trapped electrons in normalized form is given by the form of electron density (n_e), following Schamel [16,17], as

$$n_e(\phi)=\int_{-\propto}^{\propto} f_e\,(x,v)\mathrm{dv}$$

$$=\exp(\phi)\mathrm{erfc}\left(\sqrt{\phi}\right)+\frac{1}{\sqrt{\beta_1}}\begin{bmatrix} exp(\beta_1\phi)erf\left[\sqrt{|\beta_1\phi|}\right] & for\ \beta_1>0 \\ \frac{2}{\sqrt{\pi}}\,exp\left[-\left\{\sqrt{|(-\beta_1\phi)|}\right\}^2\int_0^{\sqrt{|(-\beta_1\phi)|}} exp(X^2)\,dX\right] & for\ \beta_1<0 \end{bmatrix}$$

(12a)

Where $\mathrm{erf(x)}=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{dt}$ and $\mathrm{erfc(x)} = 1 - \mathrm{erf(x)}$

In this case, f_e(x,v) is the electron distribution function and $\beta_1=\frac{T_{el,f}}{T_{eh,t}}$ is the temperature ratio of free $\left(T_{el,f}\right)$ and trapped $\left(T_{eh,t}\right)$ electrons in low and high temperatures.

We are now interested to discuss the bounded region of solitary wave solutions, Sagdeev potential profiles along with the first , second and third order solitary wave solutions, their respective amplitudes and slow ion-acoustic phase velocity (V_s) in two-temperature non-isothermal electron plasma.

Two-Temperature Non-isothermal Electron Plasma

The normalized electron density (n_e) for two-temperature non-isothermal electron plasma is obtained by Taylor series expansion from (12a) under the condition φ < 1 as

$$n_e = n_{el}+n_{eh}$$

$$=\mu\left[1+\left(\frac{\phi}{\mu+\nu\beta_1}\right)-\frac{4}{3}b_l\left(\frac{\phi}{\mu+\nu\beta_1}\right)^{\frac{3}{2}}+\frac{1}{2}\left(\frac{\phi}{\mu+\nu\beta_1}\right)^2-\frac{8}{15}b_l^{(1)}\left(\frac{\phi}{\mu+\nu\beta_1}\right)^{\frac{5}{2}}+\frac{1}{6}\left(\frac{\phi}{\mu+\nu\beta_1}\right)^3+\cdots\right]$$

$$+\nu\left[1+\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)-\frac{4}{3}b_h\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)^{\frac{3}{2}}+\frac{1}{2}\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)^2-\frac{8}{15}b_h^{(1)}\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)^{\frac{5}{2}}+\frac{1}{6}\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)^3+\cdots\right]$$

$$=1+\phi-\frac{4}{3}\frac{\left(\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{3}{2}}}\phi^{\frac{3}{2}}+\frac{1}{2}\frac{\left(\mu+\nu\beta_1^2\right)}{\left(\mu+\nu\beta_1\right)^2}\phi^2-\frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{5}{2}}}\phi^{\frac{5}{2}}+\frac{1}{6}\frac{(\mu+\nu\beta_1^3)}{\left(\mu+\nu\beta_1\right)^3}\phi^3-\cdots$$

$b_l=\frac{1-\beta_l}{\sqrt{\pi}}$, $b_h=\frac{1-\beta_h}{\sqrt{\pi}}$, $b_l^{(1)}=\frac{1-\beta_l^2}{\sqrt{\pi}}$, $b_h^{(1)}=\frac{1-\beta_h^2}{\sqrt{\pi}}$, $\beta_1=\frac{T_{el,f}}{T_{eh,t}}$, $\beta_l=\frac{T_{el,f}}{T_{el,t}}$, $\beta_h=\frac{T_{eh,f}}{T_{eh,t}}$, μ+ν = 1

Where μ and ν are respectively the unperturbed number density of low temperature and high temperature electrons , β_l is the temperature ratio of free and trapped electrons in low temperature, β_h is the temperature ratio of free and trapped electrons in high temperature and β_1 is the temperature ratio of free and trapped electrons in low and high temperatures. when $b_l \rightarrow b_1$ (i.e. $\beta_l \rightarrow \beta$), $\mu \rightarrow 1$, $\nu \rightarrow 0$, $b_l^{(1)} \rightarrow$, b_2 (i.e. $\beta_l \rightarrow \beta$) , $b_h \rightarrow 0$ (i.e. $\beta_h \rightarrow 1$), $b_h^{(1)} \rightarrow 0$ then the expression for non-isothermal two-temperature electron plasma concentration (n_e) reduces to the result of non-isothermal single temperature electron plasma concentration.

Again from equations (11) after putting the value of n_e we get

$$\frac{d^2\phi}{d\eta^2} = 1+\phi-\frac{4}{3}\frac{\left(\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{3}{2}}}\phi^{\frac{3}{2}}+\frac{1}{2}\frac{\left(\mu+\nu\beta_1^2\right)}{\left(\mu+\nu\beta_1\right)^2}\phi^2-\frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{5}{2}}}\phi^{\frac{5}{2}}+\frac{1}{6}\frac{(\mu+\nu\beta_1^3)}{\left(\mu+\nu\beta_1\right)^3}\phi^3-\cdots$$

$$-\frac{1}{2}\sqrt{\frac{n_{io}^3}{3\sigma_i}}\left[\sqrt{\left(V-u_{io}+\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2-2\phi}-\sqrt{\left(V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2-2\phi}\right]$$

$$+\frac{Z}{2}\sqrt{\frac{Qn_{jo}^3}{3\sigma_j}}\left[\sqrt{\left(V-u_{jo}+\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2+\frac{2Z\phi}{Q}}-\sqrt{\left(V-u_{jo}-\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2+\frac{2Z\phi}{Q}}\right]-\chi e^{-\sigma_p\phi} \tag{13}$$

Sagdeev Potential Equation and Related Discussion

Equation (13) can be written in the following form:

$$\frac{d^2\phi}{d\eta^2} = -\frac{\partial\psi}{\partial\phi} \tag{14a}$$

Integrating and using proper boundary condition we get from equation (14a),

$$\text{Or, } \frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 + \psi(\varphi) = 0 \tag{14b}$$

$$\Rightarrow \frac{d\phi}{d\eta} = \pm\sqrt{-2\psi(\phi)} \tag{14c}$$

$$\Rightarrow \int \frac{d\phi}{\sqrt{-2\psi(\phi)}} = \pm\,\eta$$

Here ψ(φ) is the Sagdeev pseudopotential function, $\frac{d\phi}{d\eta}$ is termed as the velocity of the pseudoparticle in the potential well and equation (14b) is known as Sagdeev potential (Energy) equation .

The Sagdeev potential ψ(ϕ) for two - temperature non-isothermal electron plasma is

$$\psi(\varphi) = \left[-\varphi-\frac{1}{2}\phi^2+\frac{8}{15}\frac{\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}}{(\mu+\nu\beta_1)^{\frac{3}{2}}}\phi^{\frac{5}{2}}-\frac{1}{6}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2}\phi^3+\frac{16}{105}\frac{\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}}{(\mu+\nu\beta_1)^{\frac{5}{2}}}\phi^{\frac{7}{2}}\right.$$

$$\left.-\frac{1}{24}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3}\phi^4+\ldots\ldots\right]$$

$$+\frac{1}{6}\sqrt{\frac{n_{io}^3}{3\sigma_i}}\left[\begin{array}{c}\left\{\left(V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2-2\phi\right\}^{\frac{3}{2}}-\left\{\left(V-u_{io}+\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2-2\phi\right\}^{\frac{3}{2}}\\+\left(V-u_{io}+\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3-\left(V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3\end{array}\right]$$

$$+\frac{1}{6}\sqrt{\frac{Q^3n_{io}^3}{3\sigma_j}}\left[\begin{array}{c}\left\{\left(V-u_{jo}-\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2+\frac{2Z\phi}{Q}\right\}^{\frac{3}{2}}-\left\{\left(V-u_{jo}+\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2+\frac{2Z\phi}{Q}\right\}^{\frac{3}{2}}\\+\left(V-u_{jo}+\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^3-\left(V-u_{jo}-\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^3\end{array}\right]+\frac{\chi}{\sigma_p}(1-e^{-\sigma_p\phi}) \quad (15)$$

The function ψ(φ) in equation (15) two-temperature non-isothermal electron plasma with positive ion, negative ion and positron may be easily reducible to two-temperature isothermal electron plasma if $b_1 = b_2 = 0$ (i.e. for β = 1). For μ = 1, ν = 0 equation (15) is reduced to the result of single temperature non-isothermal electron plasma.

Also from (15) for μ = 1, ν = 0 and χ = 0 with $\sigma_i = 0$, $\sigma_j = 0$, equation (15) supports Ref.[30].

The conditions for the existence of solitary wave solutions are obtained easily from the properties of motion of the pseudoparticle in the potential ψ. If the pseudoparticle has a positive initial velocity $\left(\frac{d\phi}{d\eta} > 0\right)$ a second condition follows from the fact that $\boldsymbol{\psi}(\phi)$ and n_i are complex for $\phi > \phi_0 = \frac{1}{2}\left(V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2$. Physically, complex n_i and $\boldsymbol{\psi}(\phi)$ are not allowed. In order that the particle moving into the region ϕ > 0 be reflected before reaching the region of complex $\boldsymbol{\psi}(\phi)$, it is necessary that

$$\psi\left[\phi = \phi_0 = \frac{1}{2}\left(V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2\right] > 0.$$

In this type of solution (ϕ > 0), compressive solitary waves are found and there will be a chance of potential hump with a real ion density.

On the other hand for non-isothermal plasma, the electrostatic potential ϕ must be positive (ϕ > 0) otherwise ψ(ϕ) will be complex and complex ψ(ϕ) will not be allowed.

The solitary wave solution for non-isothermal plasma will be obtained or available in the region $0 < \phi < \phi_0 = \frac{1}{2}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2$ where ψ(ϕ) is always real. No solitary wave solution will exist for $\phi > \phi_0 = \frac{1}{2}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2$ and ϕ < 0. Thus two kinds of solution regions are constituted in this problem – one is solitary wave region where $0 < \phi < \phi_0 = \frac{1}{2}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2$ and the other is no solution region where $\phi > \phi_0 = \frac{1}{2}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2$ and ϕ < 0.

Now from equation (15) after using $\psi\left[\phi = \phi_0 = \frac{1}{2}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2\right] > 0$ we get finally

$$\frac{n_{i0}^{\frac{3}{2}}}{\sqrt{3\sigma_i}}\left[8\left\{(V-u_{i0})\sqrt{\frac{3\sigma_i}{n_{io}}}\right\}^{\frac{3}{2}}+\left(V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3-\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3\right]$$

$$+\frac{(Qn_{j0})^{\frac{3}{2}}}{\sqrt{3\sigma_j}}\left[\begin{array}{c}\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^2+\frac{Z}{Q}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2\right\}^{\frac{3}{2}}-\\ \left\{\left(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^2+\frac{Z}{Q}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2\right\}^{\frac{3}{2}}+\left(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^3-\\ \left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^3\end{array}\right]$$

$$+\frac{\chi}{\sigma_p}\left[1-e^{-\frac{1}{2}\sigma_p\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2}\right]$$

$$>3\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2.$$

$$\left[\begin{array}{c}1+\frac{1}{4}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2-\frac{2\sqrt{2}}{15}\frac{\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}}{(\mu+\nu\beta_1)^{\frac{3}{2}}}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3\\ +\frac{1}{24}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^4-\frac{2\sqrt{2}}{105}\frac{\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}}{(\mu+\nu\beta_1)^{\frac{5}{2}}}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^5\\ +\frac{1}{192}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3}\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^6\end{array}\right]$$

The Sagdeev potential function ψ(ϕ) is much more important for formation of solitary waves. From equation (14c), it is evident that $\frac{d\phi}{d\eta}$ is regarded as the infinitesimal velocity of a pseudoparticle at position ϕ and time η in the potential well for solitary wave motion in (ϕ,η) plane. Also $\frac{d\phi}{d\eta}$ is symmetric about ϕ axis when $\frac{d\phi}{d\eta}$ is plotted against ϕ. The curve so formed by $\frac{d\phi}{d\eta}$ against ϕ represents a bounded region of solitary wave solutions where the restriction on ϕ by the inequation (12) is satisfied. For this reason, the characteristics of the graph $\frac{d\phi}{d\eta}$ against ϕ is inevitable, all valid solutions lie within that specified boundary. Under different plasma parameters, we must try to draw the graph of $\frac{d\phi}{d\eta}$ against ϕ for its nature and behaviour. The bounded solution is now classified into two categories namely periodic and aperiodic solutions. In this context, it is easy to mention that the interior regions of separatrices correspond to periodic solution while the exterior regions of the separatrices correspond to aperiodic solutions. Again it is easy to express the time (η) in terms of the position ϕ in the (ϕ,η) plane.

In a two-ion plasma with single temperature isothermal electron, Bhattacharyya et al [25] took heavier (h) and lighter (ι) ions with equal ion temperature (σ_h = σ_l = 0.01 and σ_h = σ_l = 0) for allowed region of soliton formation while plotting $\frac{d\phi}{d\eta}$ against ϕ [25]. But in our case we take two different temperatures of positive and negative ions ($\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$) with positive and negative ion drifts (u_{i0} , u_{j0}) and warm positron maintaining the validity of expansion i.e. convergence is assured for the specified range of the electrostatic potential (ϕ) for two-temperature non-isothermal electron.

The Sagdeev pseudopotential function $\psi(\phi)$ must satisfy the following conditions for solitary wave solution:

i) $\psi(\phi) = 0 = \frac{\partial\psi}{\partial\phi}$ for all V at $\phi = 0$

ii) $\frac{\partial^2\psi}{\partial\phi^2} < 0$ at $\phi = 0$

iii) $\psi(\phi) = 0$ for some $\phi = \phi_m$, ϕ_m is some max. value of ϕ

iv) $\psi(\phi) < 0$ in $0 < |\phi| < |\phi_m|$ (16)

Following chattopadhyay [4] we get the condition for the existence of a solitary wave solution in presence of positron as

$$\frac{n_{i0}}{(V-u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}} + \frac{Z^2 n_{j0}}{Q(V-u_{j0})^2 - \frac{3\sigma_j}{n_{j0}}} < 1 + \chi\sigma_p \qquad (17)$$

In absence of positron, this inequality (17) supports Ref.[30] for warm ion plasma and also supports Ref.[30] for cold ion plasma (i.e. σ_α =0).
From inequality (17), we get the following linear dispersion relation:

$$\frac{n_{i0}}{(V-u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}} + \frac{Z^2 n_{j0}}{Q(V-u_{j0})^2 - \frac{3\sigma_j}{n_{j0}}} = 1 + \chi\sigma_p \qquad (18)$$

When streams are absent i.e. $u_{i0} = 0$ and $u_{j0} = 0$ then the above equation (18) reduces to the form:

$$\frac{n_{i0}}{V^2 - \frac{3\sigma_i}{n_{i0}}} + \frac{Z^2 n_{j0}}{QV^2 - \frac{3\sigma_j}{n_{j0}}} = 1 + \chi\sigma_p \qquad (19)$$

Again when negative ions and streams are absent (i.e. $n_{j0} = 0$ and $u_{i0} = 0 = u_{j0}$) then we can write from (18)

$$V = \pm\sqrt{\frac{1-\chi}{1+\chi\sigma_p} + \frac{3\sigma_i}{1-\chi}} \qquad (20)$$

Therefore in absence of streams ($u_{i0} = 0$ and $u_{j0} = 0$) and negative ions ($n_{j0} = 0$), equation (18) gives two values of V. Practically the negative value of V does not exist and thus meaningless. The positive value of V in this case is known as the phase velocity of normal ion-acoustic mode.
From inequation (18), the critical positron density (χ_c) is

$$\chi_c = \frac{1}{\sigma_p}\left[\frac{n_{io}}{(V-u_{io})^2 - \frac{3\sigma_i}{n_{io}}} + \frac{Z^2 n_{jo}}{Q(V-u_{jo})^2 - \frac{3\sigma_j}{n_{jo}}} - 1\right] \qquad (21)$$

Now from equation (21), we can say that the ratio (σ_p) of electron (T_e) and positron (T_p) temperature will be greater than 1 when $T_e > T_p$ where χ_c is decreasing and that ratio σ_p will be less than 1 when $T_e < T_p$ where χ_c is increasing.

The critical negative ion temperature (σ_{jc}) from the inequation (18) is obtained as

$$\sigma_{jc} = \frac{n_{j0}}{3}\left[Q(V-u_{j0})^2 - Z^2 n_{j0}\left\{\left(1+\chi\sigma_p\right) - \frac{\left(1+Zn_{j0}-\chi\right)^2}{(1+Zn_{j0}-\chi)(V-u_{i0})^2 - 3\sigma_i}\right\}^{-1}\right] \qquad (22)$$

In absence of negative ion (i.e. n_{j0}= 0, σ_j = 0) we have from the inequation (17)

$$\frac{(1-\chi)}{(V-u_{io})^2 - \frac{3\sigma_i}{1-\chi}} < 1 + \chi\sigma_p$$

The critical velocity (V_c) is obtained from the above inequality in presence of positron as

$$V_c = u_{io} \pm \sqrt{\frac{1-\chi}{1+\chi\sigma_p} + \frac{3\sigma_i}{1-\chi}}$$

The two values of the above critical velocities (V_c) are known as the phase velocity of slow and fast mode.

Thus in presence of positive ion streams ($u_{i0} \neq 0$) the phase velocity of fast ion-acoustic mode (V_F) is

$$V_F = u_{io} + \sqrt{\frac{1-\chi}{1+\chi\sigma_p} + \frac{3\sigma_i}{1-\chi}}$$

And the phase velocity of slow ion-acoustic mode (V_S) in presence of positive ion streams ($u_{i0} \neq 0$) is

$$V_S = u_{io} - \sqrt{\frac{1-\chi}{1+\chi\sigma_p} + \frac{3\sigma_i}{1-\chi}}$$

It is also found very clearly that $V_F > V_S$.

The condition for the existence of slow ion-acoustic mode in absence of negative ion $(n_{j0} = 0)$ is

$$\sigma_i < \frac{1}{3}(1-\chi)\left[u_{i0}^2 - \frac{1-\chi}{1+\chi\sigma_p}\right]$$

In absence of positron, the above relation reduces to the form

$$\sigma_i < \frac{1}{3}(u_{i0}^2 - 1)$$

Thus it is evident from above that $u_{i0}^2 > 1$ for real values of σ_i . Now if we take $u_{i0}^2 = 1.001 > 1$ then $\sigma_i < 0.000333$. Under this condition, slow ion-acoustic phase velocity (V_s) appears. Therefore we conclude that slow ion-acoustic mode exists for lower values of σ_i but for higher values of σ_i , slow mode disappear or does not exist whereas fast mode exists for those two cases.

Moreover the critical positive ion temperature (σ_{ic}) in absence of negative ion $(n_{j0} = 0)$ is found as

$$\sigma_{ic} = \frac{n_{i0}}{3}\left[(V-u_{i0})^2 - \frac{n_{i0}}{1+\chi\sigma_p}\right]$$

In absence of negative ions, the stream velocity (u_{i0}) has a great significance and influence on the above two modes of the phase velocities V_F and V_S. The slow ion-acoustic phase velocity (V_S) is meaningless for u_{i0} = 0 whereas fast ion-acoustic phase velocity (V_F) is meaningful and is regarded as the normal ion-acoustic phase velocity. Again in presence of stream velocity($u_{i0} \neq 0$) V_F always exists whereas V_S does not exist always but under certain restriction on stream velocity of positive ions(u_{i0}), the slow ion-acoustic mode V_S will be positive and acceptable.

III. SOLITARY WAVE SOLUTIONS AND CORRESPONDING AMPLITUDES

In order to find the nature and number of solitary wave solutions, we are now going to discuss the different orders and kinds of the differential equation related to solitary wave solutions .The perturbative new approach "tanh – method" is used presently for finding the solitary wave solutions of first, second and third order in non-isothermal plasma along with their respective amplitudes for cold and warm ions in two-temperature electron plasma. From equation (13) after simplifying some steps we get finally

$$\frac{d^2\phi}{d\eta^2} = A_1\,\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 - A_4\phi^{\frac{5}{2}} + A_5\,\phi^3 - \ldots\ldots\ldots \quad (23)$$

Where for two-temperature non-isothermal electron plasma

$$A_1 = \left[1 - n_{i0}\left\{(V-u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}\right\}^{-1} - Z^2 n_{j0}\left\{Q(V-u_{j0})^2 - \frac{3\sigma_j}{n_{j0}}\right\}^{-1} + \chi\sigma_p\right]$$

$$A_2 = \frac{4}{3}\frac{\left(\mu b_l + \nu b_h \beta_1^{\frac{3}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{3}{2}}}$$

$$A_3 = \frac{1}{2}\left[\begin{array}{l}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-3} - \left(V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-3}\right\} \\ + \frac{Z^3 n_{j0}^{\frac{3}{2}}}{2Q\sqrt{3\sigma_{jQ}}}\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-3} - \left(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-3}\right\} - \chi\sigma_p^2\end{array}\right]$$

$$A_4 = \frac{8}{15}\frac{\left(\mu b_l^{(1)} + \nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{5}{2}}}$$

$$A_5 = \frac{1}{2}\left[\begin{array}{l}\frac{1}{3}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-5} - \left(V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-5}\right\} \\ + \frac{Z^4 n_{j0}^{\frac{3}{2}}}{2Q^2\sqrt{3\sigma_{jQ}}}\left\{\left(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-5} - \left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-5}\right\} + \frac{\chi\sigma_p^3}{3}\end{array}\right] \quad (24)$$

For cold positive and negative ions with streams, positron and two-temperature non-isothermal electron the modified expressions for A_1 , A_2 , A_3 , A_4 and A_5 are given below from (24).

$$A_1 = \left[1 - \frac{n_{i0}}{(V-u_{i0})^2} - \frac{Z^2 n_{j0}}{Q(V-u_{j0})^2} + \chi\sigma_p\right]$$

$$A_2 = \frac{4}{3}\frac{\left(\mu b_l + \nu b_h \beta_1^{\frac{3}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{3}{2}}}$$

$$A_3 = \frac{1}{2}\left[\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2} - \frac{3n_{i0}}{(V-u_{i0})^4} - \frac{3Z^3 n_{j0}}{Q^2(V-u_{j0})^4} - \chi\sigma_p^2\right]$$

$$A_4 = \frac{8}{15}\frac{\left(\mu b_l^{(1)} + \nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{5}{2}}}$$

$$A_5 = \frac{1}{2}\left[\frac{1}{3}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3} - \frac{5n_{i0}}{(V-u_{i0})^6} - \frac{5Z^4 n_{j0}}{Q^3(V-u_{j0})^6} + \frac{\chi\sigma_p^3}{3}\right] \quad (25)$$

We are now discussing a special method namely "tanh-method" for solving the differential equation (23). To solve the differential equation (23) upto certain powers of ϕ, we use the well-known "tanh-method". For this purpose, we substitute z = tanh(η) and w(z) = ϕ(η) in equation (23) from where we get a Fuchsian-like non-linear ordinary differential equation and after that a Frobeneous series solution is found there. Finally returning to the original variable, solitary wave solutions of first, second and third orders with their respective amplitudes are found out. By this way we are going to solve the differential equation (23) upto certertain powers of ϕ respectively. Since we are only interested to find the first, second and third order amplitudes of the respective solitary wave solution that's why we are not trying to show the actual procedure for solution of the differential equation (23). So we write the solutions of the differential equations straight forward.

Taking terms upto $\phi^{\frac{3}{2}}$ from equation (23) we get

$$\frac{d^2\phi}{d\eta^2} = A_1\,\phi - A_2\phi^{\frac{3}{2}} \quad (26)$$

The first order solitary wave solution (ϕ_1) in non-isothermal plasma [1,33] is

$$\phi_1 = \left(\frac{5A_1}{4A_2}\right)^2 Sech^4\left(\sqrt{\frac{A_1}{16}}\,\eta\right) \quad (27)$$

For non-isothermal plasma the first order amplitude (ϕ_{01}) of the solitary wave solution (27) is

$$\phi_{01} = \left(\frac{5A_1}{4A_2}\right)^2 \quad (28)$$

Similarly taking terms upto ϕ^2 from equation (23) we get

$$\frac{d^2\phi}{d\eta^2} = A_1\,\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 \quad (29)$$

The second order solitary wave solution (ϕ_2) in non-isothermal plasma [1,33] is

$$\phi_2 = \left[\frac{3A_2}{5A_3} \pm \sqrt{\frac{3}{2A_3}\left(\frac{2}{3}\frac{9A_2^2}{25A_3} - A_1\right)}\,Sech\left(\frac{1}{2}\sqrt{A_1 - \frac{9A_2^2}{25A_3}}\,\eta\right)\right]^2 \quad (30)$$

The second order amplitude (ϕ_{02}) of the solitary wave solution (30) is

$$\phi_{02} = \left[\frac{3A_2}{5A_3} \pm \sqrt{\left\{\left(\frac{3A_2}{5A_3}\right)^2 - \frac{3}{2}\left(\frac{A_1}{A_3}\right)\right\}}\right]^2 \quad (31)$$

Again from equation (23) taking terms upto $\phi^{\frac{5}{2}}$ we get

$$\frac{d^2\phi}{d\eta^2} = A_1\,\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 - A_4\phi^{\frac{5}{2}} \quad (32)$$

The third order [28] solitary wave solution (ϕ_3) [1,33] of the above equation in non-isothermal plasma is

$$\phi_3 = \left(\frac{7A_3}{18A_4}\right)^2 Sech^4 \left[\begin{array}{c} \left(\frac{7A_3}{7A_3 - 18\,A_4\sqrt{\phi_3}}\right)^{\frac{1}{2}} \pm \frac{7A_3}{108\,A_4}\sqrt{\frac{A_3}{2}}(\eta - \eta_0) \\ + sech^{-1}\left(\frac{18A_4}{7A_3}\sqrt{\phi_m}\right)^{\frac{1}{2}} - \left(\frac{7A_3}{7A_3 - 18\,A_4\sqrt{\phi_m}}\right)^{\frac{1}{2}} \end{array} \right] \quad (33)$$

Where ϕ_m is the optimal amplitude of the acoustic mode.

The above solution ϕ_3 is an implicit function of η and is known as the profile of a spiky solitary wave solution defined in the region $\sqrt{\phi_3} > 0$.

In a similar way for the region where $\sqrt{\phi_3} < 0$ we get the profile of an explosive (ϕ_3) solitary wave [33] solution in the implicit form

$$\phi_3 = \left(\frac{7A_3}{18A_4}\right)^2 Cosech^4 \left[\begin{array}{c} \left(\frac{7A_3}{7A_3 - 18\,A_4\sqrt{\phi_3}}\right)^{\frac{1}{2}} \pm \frac{7A_3}{108\,A_4}\sqrt{\frac{A_3}{2}}(\eta - \eta_0) \\ + cosech^{-1}\left(\frac{18A_4}{7A_3}\sqrt{\phi_m}\right)^{\frac{1}{2}} - \left(\frac{7A_3}{7A_3 - 18\,A_4\sqrt{\phi_m}}\right)^{\frac{1}{2}} \end{array} \right] \quad (34)$$

Where ϕ_m is the optimal amplitude of the acoustic mode

The third order amplitude (ϕ_{03}) of the spiky and explosive solitary wave solution (33) or (34) is

$$\phi_{03} = \left[\left\{\frac{1}{2}(a_1 + b_1)\right\}^{\frac{1}{3}} + \left\{\frac{1}{2}(a_1 - b_1)\right\}^{\frac{1}{3}} + \frac{7A_3}{18A_4}\right]^2 \quad (35)$$

Where $a_1 = \left[\frac{1}{4}\left(\frac{7A_3}{9A_4}\right)^3 - \frac{49}{90}\frac{A_2 A_3}{A_4^2} + \frac{7A_1}{4A_4}\right]$

And $b_1^2 = \left|\left[a_1^2 + \frac{4}{27}\left\{\left(\frac{7A_2}{5A_4}\right) - \frac{1}{3}\left(\frac{7A_3}{6A_4}\right)^2\right\}^3\right]\right|$

The first, second and third order solitary wave solutions are obtained by a perturbative new approach known as "tanh-method". Out of these three solutions, only third order is not observed by Freja Scientific Satellite observations though first and second order solitary wave solutions are observed. It is thus a new achievement.

IV. DISCUSSION

In our present study for the higher order solitary wave solution in two temperature non-isothermal electron plasma with warm positive ions, warm negative ions and warm positron by the new approach "tanh-method", the important and interesting phenomena is the determination of the boundary regions of compressive solitary wave solutions under the variation of different plasma parameters, the corresponding respective amplitudes and their comparisons among three orders. The slow mode phase velocity (V_s) is also discussed here for different plasma parameters and the profiles of Sagdeev potentials are graphically discussed for the variation of the concentration of negative ions, mass ratios of negative to positive ions, positron concentration and temperature of positive and negative ions. Again for highlighting the physical insight of this problem, it is mentioned in this context that the non-isothermal electron distributions can be observed in many Astrophysical and Space environments, dusty (complex) plasmas and in Fluid dynamics. In this present work, the bounded region represents the compressive solitary wave solution and the condition for its appearance is satisfied whereas outside boundary, the condition for solitary wave solution is not satisfied. Interior region of the separator known as periodic solutions while for exterior region of the separator known as aperiodic solutions are observed in this paper. The interior region of the separatrix gives a physically acceptable ion-acoustic solitary wave observed in space plasma.

At first we are discussing the boundary regions of the compressive solitary wave solutions and these are represented graphically by the respective figs.1 – 6.

In Fig. 1, the bounded compressive solitary wave solutions with the variation of the concentration of two-temperature electron (μ,ν) are shown very distinctly for the two-temperature non-isothermal electron plasma. The curves so formed by $\frac{d\phi}{d\eta}$ against ϕ for three different values of the concentration of two-temperature electrons μ and ν are represented by the separatrices S_1(μ = 0.15, ν = 0.85), S_2 (μ = 0.25, ν = 0.75) and S_3 (μ = 1, ν = 0) respectively. The interior and exterior regions of the separatrices S_1 , $S_2$and S_3 are known as periodic and aperiodic solutions. In this case, we are discussing only the bounded compressive periodic solitary wave solutions lying within the range -0.552097< ϕ < 0.291009 for our used parameters in this figure and all solutions within the three boundaries S_1 , S_2 and S_3 are valid solutions. Again when (μ, ν) changes from μ = 0.15, ν = 0.85 to μ = 1, ν = 0, the boundary region gradually increases with higher values of the areas of compressive periodic solutions.

Figure 2 shows the phase plane trajectories of compressive solitary waves for four different values of the temperature of positive (σ_i) and negative (σ_j) ions with a particular value of the concentration of two-temperature non-isothermal electron (μ,ν) and positron temperature (σ_p). The separatrices formed in this figure are represented by S_1 ($\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$) , S_4 ($\sigma_i = \frac{1}{25}$, $\sigma_j = \frac{1}{30}$) , S_5 ($\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{30}$) and S_6 (σ_i = 0 , σ_j = 0) respectively. Each separatrix shown in this figure represents a bounded localized structure with stable periodic solution. For S_1 and S_5 separatrices, the bounded solutions are valid for such values of ϕ which lies in the range - 0.552097 < ϕ < 0.291009 and the solution within that closed area are allowed and acceptable. The solution within the separatrix S_4 is valid for – 0.703044 < ϕ < 0.266980. Also for cold positive (σ_i=0) and cold negative (σ_j=0) ion, the solution region bounded by the separatrix S_6 is valid and allowed for – 3.38 < ϕ < 0.605 and in this case the bounded area under S_6 separatrix is the largest while the area under S_4 separatrix is the least. It is also clear from the profiles of this figure that the region under finite ion temperature is reduced compared to cold ions which does not restrict the allowed region for soliton formation.

In Fig. 3, the bounded periodic compressive solitary wave structure with the variation of negative ion concentration (n_{j0}) is shown. The separatrices S_1,S_7 and S_8 are respectively marked for n_{j0} = 0.05, 0.09 and 0. It is evident from this figure that as negative ion concentration (n_{j0}) increases from 0 to 0.09, the bounded regions are gradually decreasing in area which reflects the important influence of negative ion.

For S_7 separatrix ϕ lies in -1.044446 < ϕ < 0.296689 and for S_8 separatrix ϕ lies in 0 ≤ ϕ < 0.291009 , but the area under S_8 separatrix is greatest whereas area under S_7 separatrix is least.

Figure 4 shows the bounded compressive solitary wave trajectories under the variation of the mass ratios of negative to positive ions (Q) [Q = 4 (He^+, O^-) plasma, Q = 8.875 (He^+, Cl^-) plasma, Q = 35.5 (H^+,Cl^-) plasma]. As Q increases from 4 to 35.5, the bounded regions are larger in area and all solutions within that area are satisfied and stable. The separatrices thus formed for Q = 4, 8.875 and 35.5 are represented by S_1, S_9 and S_{10} respectively where ϕ lies in -0.552097 < ϕ < 0.291009 for S_1 separatrix, for S_9 separatrix ϕ lies in – 2.699623 < ϕ < 0.291009 and for S_{10} separatrix ϕ lies in - 19.197994 < ϕ < 0.291009. In this figure, the area generated by S_{10} separatrix is largest whereas the area under S_1 separatrix is least.

In Fig. 5, the bounded localized structure for the stable periodic solution is shown for the three different values of the concentration of positron (χ). The separatrix S_1 is for χ = 0.17, separatrix S_{11} is for χ = 0 and separatrix S_{12} is for χ = 0.12. Out of these three separatrices S_1, S_{11} and S_{12} , only separatrix S_{11} is the lowest bounded area where ϕ lies in - 0.552097 < ϕ < 0.313151. For separatrix S_{12} , ϕ lies in the range - 0.552097 < ϕ < 0.302060 and for S_1 separatrix , ϕ lies in the range - 0.552097 < ϕ < 0.291009 where all periodic solutions within the above mentioned separatrices are satisfied, valid and stable.

Figure 6 shows the phase plane trajectories of compressive solitary waves for three different values of the temperature of positron (σ_p) with a particular value of the temperature of positive (σ_i) and negative (σ_j) ions and concentration of two-temperature non-isothermal electron (μ,ν). The separatrices so formed are represented by S_1 (σ_p = 0.41), S_{13} (σ_p = 0.30) and S_{14} (σ_p = 0.55) respectively and they show a bounded localized structure S_1 , S_{13} and S_{14} with stable periodic solution. For S_1, S_{13} and S_{14} separatrices, the bounded solutions are valid for such values of ϕ which lies in the range - 0.552097 < ϕ < 0.291009 and the solution within that closed area are stable, but for certain value of σ_p = 0.09, 0.70, the so formed trajectory is unbounded and it's solution is unstable.

Secondly we are now investigating the comparisons of first (ϕ_{01}) , second (ϕ_{02}) and third (ϕ_{03}) order amplitudes of the ion-acoustic solitary waves in presence of non-isothermal electrons together with drifting warm positive ion, warm negative ion and warm positron for two and single temperature non-isothermal electron plasma under the variation of different temperatures of positive (σ_i) and negative (σ_j) ions. These are represented graphically by the figs. 7 - 10.

In Fig. 7, first (ϕ_{01}), second (ϕ_{02}) and third (ϕ_{03}) order amplitudes of the corresponding solitary wave solutions verses negative ion concentration (n_{j0}) are shown very clearly between two-temperature and single temperature non-isothermal electron plasma for the ion temperature σ_i = 0, σ_j = 0 when μ = 0.15, ν = 0.85 and μ = 1, ν = 0. These are shown respectively by the figures l_1 , l_2 , l_3 and k_1, k_2, k_3. It is also found that for any particular values of negative ion concentration (n_{j0}) and also for increasing values of negative ion concentration (n_{j0}), the amplitudes are normally decreasing in nature for two-temperature [l_3>l_2>l_1] and single temperature [k_3>k_2>k_1] non-isothermal electron plasma in presence of positron. Moreover among these three types of amplitudes only $\phi_{01}$are generally larger but ϕ_{02} and ϕ_{03} are normally less values for single temperature non-isothermal electron than those of two-temperature non-isothermal electron plasma [k_1>l_1, k_2< l_2 , k_3< l_3] in presence of positron.

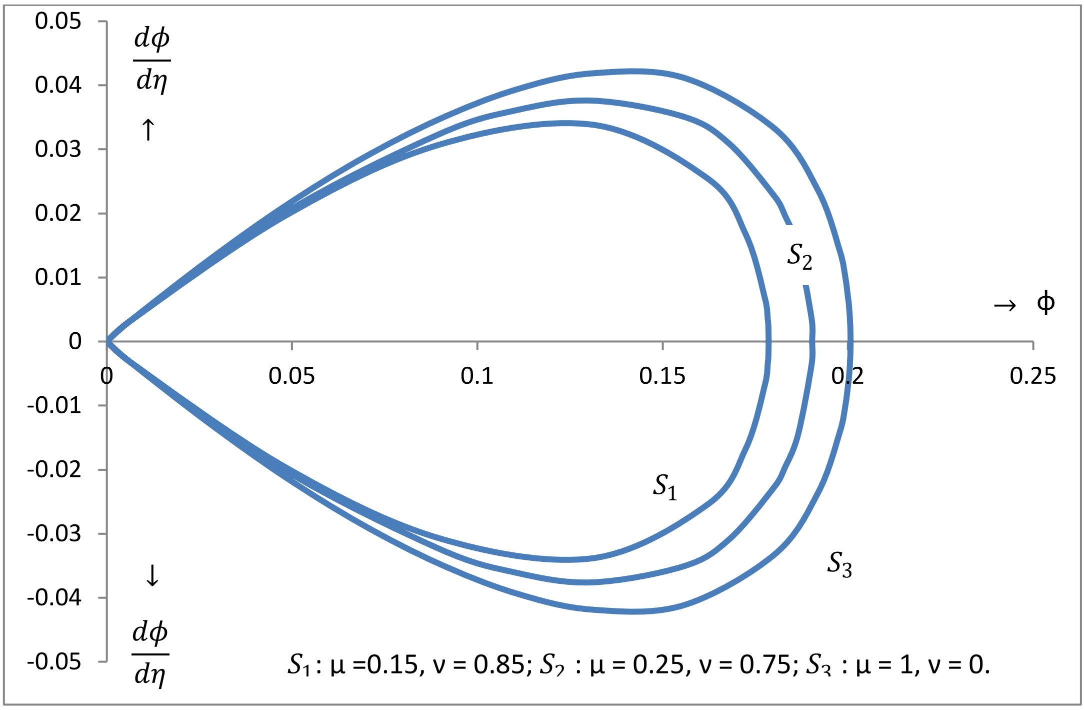
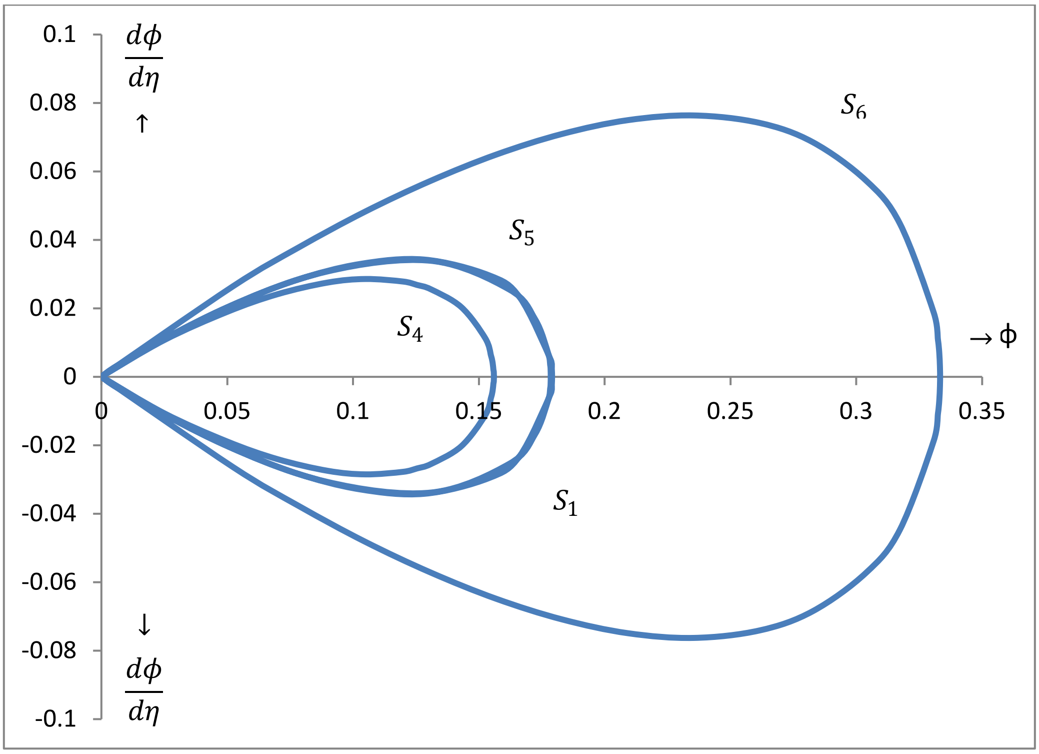


FIGURE 1. Bounded solitary wave solutions with the variation of the concentration of two-temperature electron (μ, ν) for V = 1.6, $u_{i0} = 0.5$, $u_{j0} = 0.3$, Q = 4, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, $n_{j0} = 0.05$, $n_{i0} = 0.88$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.25$, $b_h^{(1)} = 0.51$, Z = 1.



S_1 : $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$; S_4 : $\sigma_i = \frac{1}{25}$, $\sigma_j = \frac{1}{30}$; S_5 : $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{30}$; S_6 : $\sigma_i = 0$, $\sigma_j = 0$

FIGURE 2. Bounded solitary wave solutions with the variation of the temperature of positive(σ_i) and negative(σ_j) ions for V = 1.6, $u_{i0} = 0.5$, $u_{j0} = 0.3$, Q = 4, $n_{j0} = 0.05$, $n_{i0} = 0.88$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.25$, $b_h^{(1)} = 0.51$, Z = 1, μ = 0.15, ν = 0.85.

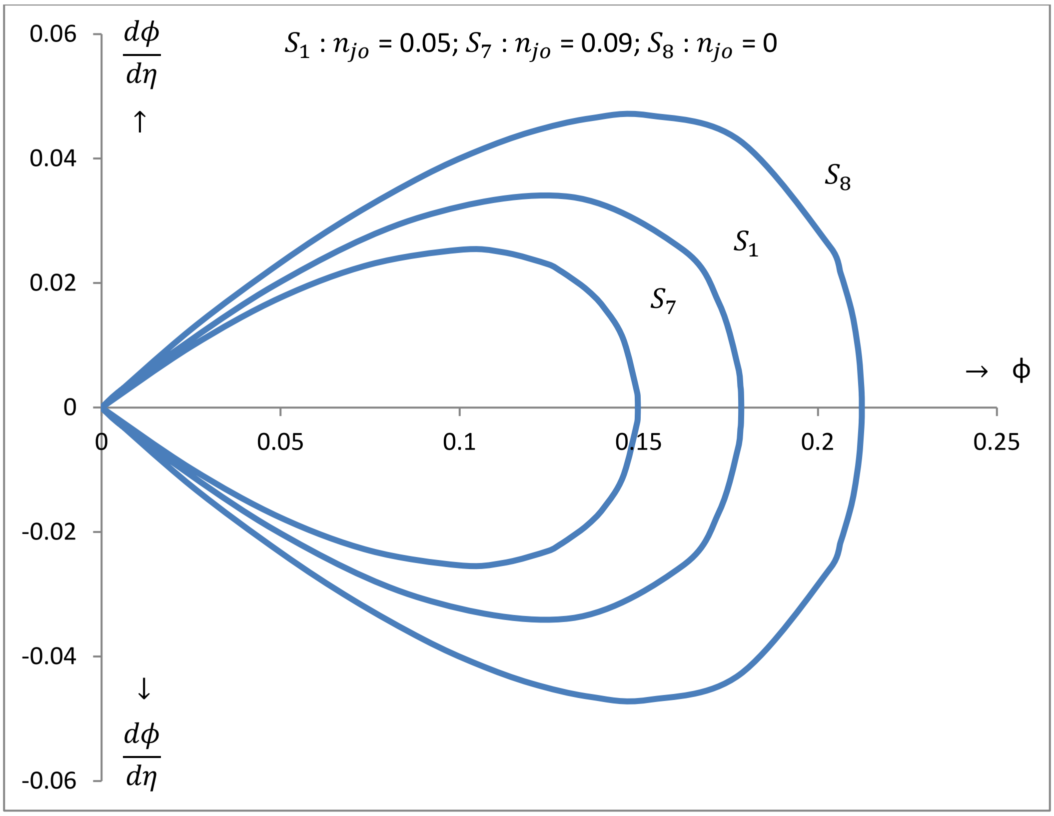


FIGURE 3. Bounded region of solitary wave solution with the variation of the concentration of negative ion (n_{jo}) for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, Q = 4, χ = 0.17, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, Z = 1, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, μ = 0.15, ν = 0.85.

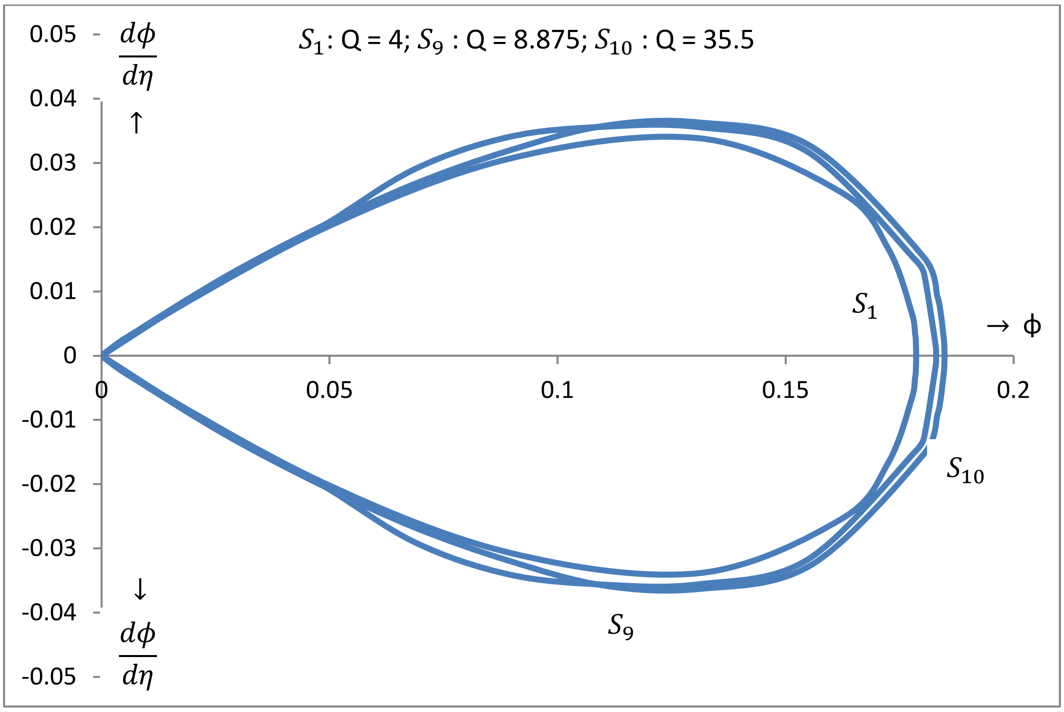


FIGURE 4. Bounded region of solitary wave solution with the variation of the mass ratios (Q) of negative to positive ions for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, χ = 0.17, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, Z = 1, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, μ = 0.15, ν = 0.85, n_{jo} = 0.05, n_{io} = 0.88 .

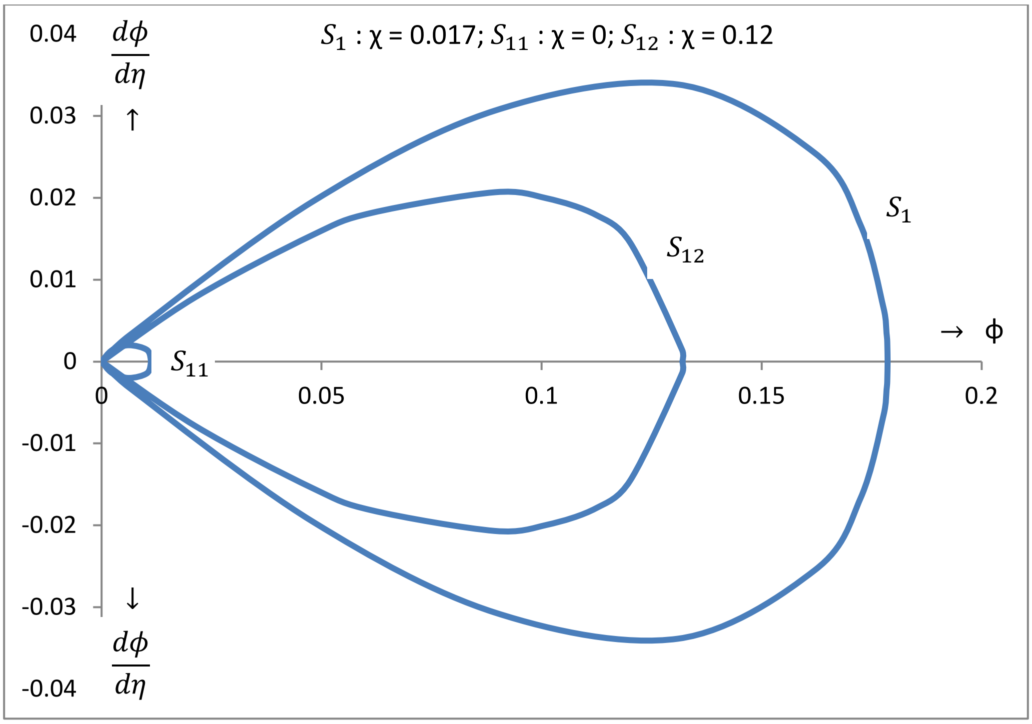


FIGURE 5. Bounded region of solitary wave solution with the variation of the concentration of positron (χ) for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, Q = 4, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, Z = 1, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, μ = 0.15, ν = 0.85, n_{jo} = 0.05, n_{io} = 0.88 .

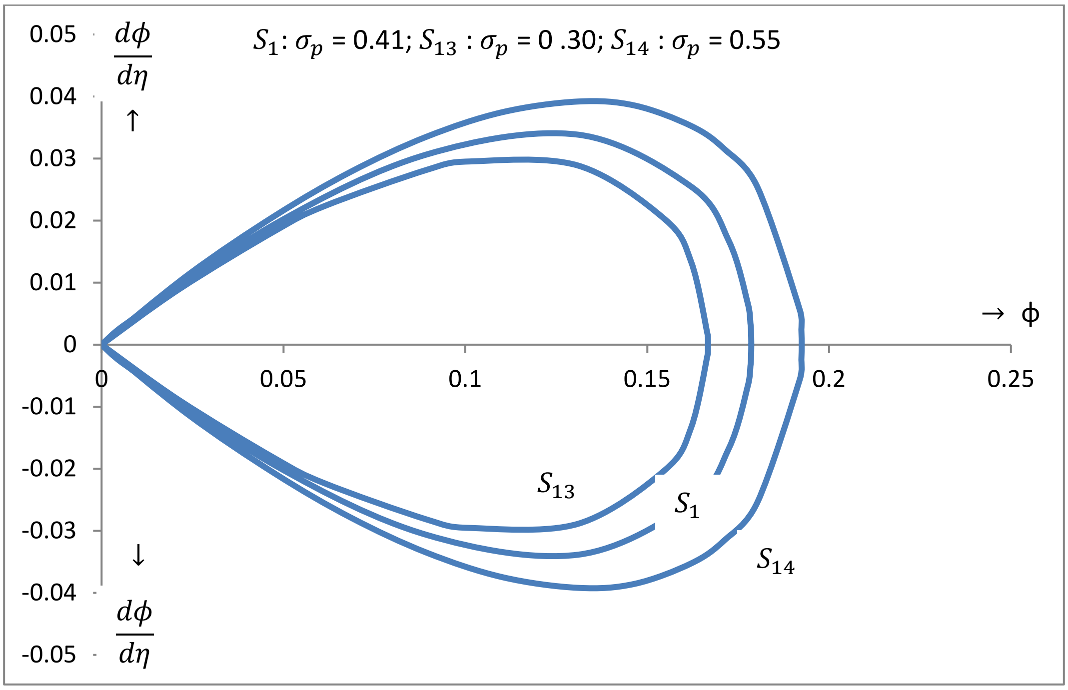


FIGURE 6. Bounded region of solitary wave solution with the variation of the temperature ratio of electron and positron (σ_p) for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, χ = 0.17, Q = 4, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, Z = 1, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, μ = 0.15, ν = 0.85, n_{jo} = 0.05, n_{io} = 0.88.

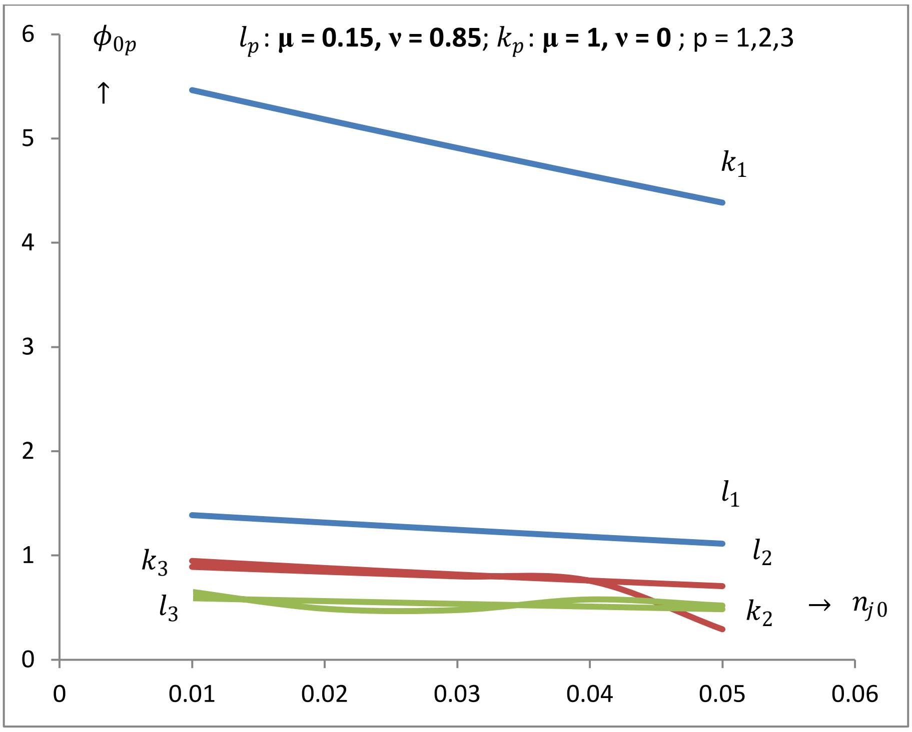


FIGURE 7. Comparison of first(ϕ_{01}), second (ϕ_{02}) and third (ϕ_{03})order amplitudes of the solitary wave solutions versus negative ion concentration (n_{j0}) between two-temperature and single temperature non-isothermal electron plasma for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3,Q = 4, μ = 0.15, ν = 0.85, n_{j0} = 0.05, n_{i0} = 0.88, $\boldsymbol{\sigma_i = 0}$, $\boldsymbol{\sigma_j = 0}(\sigma_i = \sigma_j)$, χ = 0.17, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.40, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, Z = 1.

Figure 8 shows the first (ϕ_{01}), second (ϕ_{02}) and third (ϕ_{03}) order amplitudes of the solitary wave solutions versus negative ion concentration (n_{j0}) for a definite temperature of positive (σ_i) and negative (σ_j) ions. Here ϕ_{01}, ϕ_{02} and ϕ_{03} are shown respectively by l_4 , l_5, l_6 and k_4 , k_5, k_6 for $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$ $(\sigma_i < \sigma_j)$ when μ = 0.15 , ν = 0.85 and μ = 1 , ν = 0 . As concentration of negative ions (n_{j0}) are increasing, the first (ϕ_{01}), second (ϕ_{02}) and third (ϕ_{03}) order amplitudes are decreasing with increasing temperature of positive and negative ions than cold ions (σ_i = 0, σ_j = 0) and are increasing for single temperature than two-temperature non-isothermal electron at the same temperature of positive and negative ions.

The comparison of first (ϕ_{01}), second (ϕ_{02}) and third (ϕ_{03}) order amplitudes of the solitary wave solutions versus negative ion concentration (n_{j0}) for a particular value of positive and negative ion temperature ($\sigma_i = \frac{1}{25}$, $\sigma_j = \frac{1}{30}$; $\sigma_i > \sigma_j$) in presence of positron are shown in Fig. 9. In this case l_7 , l_8 , l_9 (when μ = 0.15, ν = 0.85) and k_7, k_8 and k_9 (when μ = 1, ν = 0) are represented by ϕ_{01}, ϕ_{02} and ϕ_{03} for $\sigma_i = \frac{1}{25}$, $\sigma_j = \frac{1}{30}$. It is evident from the figure that the first (ϕ_{01}), second (ϕ_{02}) and third (ϕ_{03}) order amplitudes are normally decreasing [k_7>l_7, k_8 >l_8 , k_9 >l_9] but for single temperature ϕ_{01}, ϕ_{02} , ϕ_{03} take larger values than two-temperature non-isothermal electron plasma when temperature of positive ion (σ_i) is greater than the temperature of negative ion (σ_j).

In Fig. 10, the comparison of first (ϕ_{01}) , second (ϕ_{02}) and third (ϕ_{03}) order amplitudes of the solitay wave solutions versus negative ion concentration (n_{jo}) for a definite equal positive and negative ion temperatures ($\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{30}$; $\sigma_i = \sigma_j$) in presence of positron are shown that are represented by the respective figures l_{10} , l_{11}, l_{12} (when μ = 0.15, ν = 0.85) and k_{10},

k_{11} , k_{12} (when μ = 1, ν = 0) for $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{30}$ $(\sigma_i = \sigma_j)$. When negative ion concentration (n_{jo}) is increasing then the first (ϕ_{01}), second (ϕ_{02}) and third (ϕ_{03}) order amplitudes are decreasing but when we observe two-temperature and single temperature non-isothermal amplitude result it is found that $k_{10} > l_{10}$, $k_{11} > l_{11}$, $k_{12} > l_{12}$ for the equal temperature of positive(σ_i) and negative (σ_j) ion ($\sigma_i = \sigma_j = \frac{1}{30}$).

Besides these three types of amplitudes, we are now discussing the phase velocity for slow ion-acoustic mode (V_S) in presence of positron which are represented by the figs.11 - 14.

Figure 11 represents the phase velocity (V_S) for slow ion-acoustic mode against positron density (χ) with the variation of the ratio (σ_p) of electron (T_e) and positron (T_p) temperature. As σ_p increases from 0.3 to 0.9, the slow ion-acoustic phase velocity (V_S) increases i.e. $V_{S_5} > V_{S_4} > V_{S_3} > V_{S_2} > V_{S_1}$ where V_{S_1}, V_{S_2}, V_{S_3}, V_{S_4} and V_{S_5} are respectively the phase velocities at σ_p = 0.3, 0.4, 0.55, 0.7 and 0.9. Again when χ increases for a fixed value of σ_p , the said slow mode phase velocity (V_S) then increases gradually.

In Fig. 12, the phase velocity (V_S) for slow ion-acoustic mode of solitary waves versus positron density (χ) with the variation of positive ion temperature (σ_i) is shown. When temperature of positive ion (σ_i) increases, phase velocity (V_S) of slow mode then decreases $[V_{S_1} > V_{S_2} > V_{S_3} > V_{S_4} > V_{S_5}]$ where V_{S_1} is the phase velocity at $\sigma_i = \frac{1}{20}$, V_{S_2} is the phase velocity at $\sigma_i = \frac{1}{30}$, V_{S_3} is the phase velocity at $\sigma_i = \frac{1}{40}$, V_{S_4} is the phase velocity at $\sigma_i = \frac{1}{100}$ and V_{S_5} is the phase velocity at $\sigma_i = 0$. For a particular positive ion temperature (σ_i), phase velocity (V_S) is increasing for increasing values of χ.

Again, the phase velocity (V_S) for slow ion-acoustic mode of the solitary waves versus positron density (χ) with the variation of the drift velocity of positive ion (u_{io}) is shown in Fig. 13. When drift velocity of positive ion (u_{io}) is increasing, the phase velocity (V_S) is then increasing for increasing values of χ. Here $V_{S_3} > V_{S_2} > V_{S_1}$ where V_{S_1} is the phase velocity at u_{io}= 1.05, V_{S_2} is the phase velocity at u_{io}= 1.051 and V_{S_3} is the phase velocity at u_{io}= 1.052. For a particular drift velocity (u_{io}), phase velocity (V_S) is increasing for increasing values of χ.

In Fig. 14, the phase velocity (V_S) for slow ion-acoustic mode of the solitary waves versus positive ion drift velocity(u_{io}) with the variation of positron density (χ) is shown. The phase velocity (V_S) increases for increasing drift velocity of positive ion (u_{io}) at a particular χ. Again when χ increases, the phase velocity (V_S) increases for increasing drift velocity of positive ion (u_{io}) i.e. $V_{S_1} < V_{S_2} < V_{S_3}$ where V_{S_1} is the phase velocity at χ = 0, V_{S_2} is the phase velocity at χ = 0.17 and V_{S_3} is the phase velocity at χ = 0.22 .

Finally in order to determine the nature of the solitary wave solutions, we are now trying to draw the Sagdeev pseudopotential $[\psi(\phi)]$ curves under the variation of some plasma parameters. These are represented by the Figs. 15 – 18.

Fig. 15 shows the profiles of the Sagdeev potential curves [ψ(ϕ)] against ϕ for three different values of the concentration of negative ions (n_{j0}). In this case, the amplitudes of the respective solitary waves are decreasing when concentration of negative ions (n_{j0}) are increasing (n_{j0} = 0, 0.05, 0.09). Out of these three concentrations of negative ions, the magnitude of the amplitude is largest for cold negative ion concentration (n_{j0} = 0) and in this situation, the depth of the well is much larger than those of the two. The three curves are denoted by $a_1(n_{j0} = 0)$, $a_2(n_{j0} = 0.05)$ and $a_3(n_{j0} = 0.09)$ and from this, the effect of negative ion is clearly visible.

In Fig. 16, the profiles of the Sagdeev potential curves [ψ(ϕ)] verses ϕ for the following three values of negative to positive ion mass ratios [Q = 4(He^+ O^-) plasma, Q = 8.875 (He^+, Cl^-) plasma, Q = 35.5 (H^+, Cl^-) plasma] are shown. It is observed from these curves that the amplitudes are gradually increasing for higher values of Q (Q = 35.5) and is least for small value of Q (Q = 4) when all other related plasma parameters have a definite value. The profiles a_4, a_5 and a_6 are represented for Q = 4, Q = 8.875 and Q = 35.5 respectively.

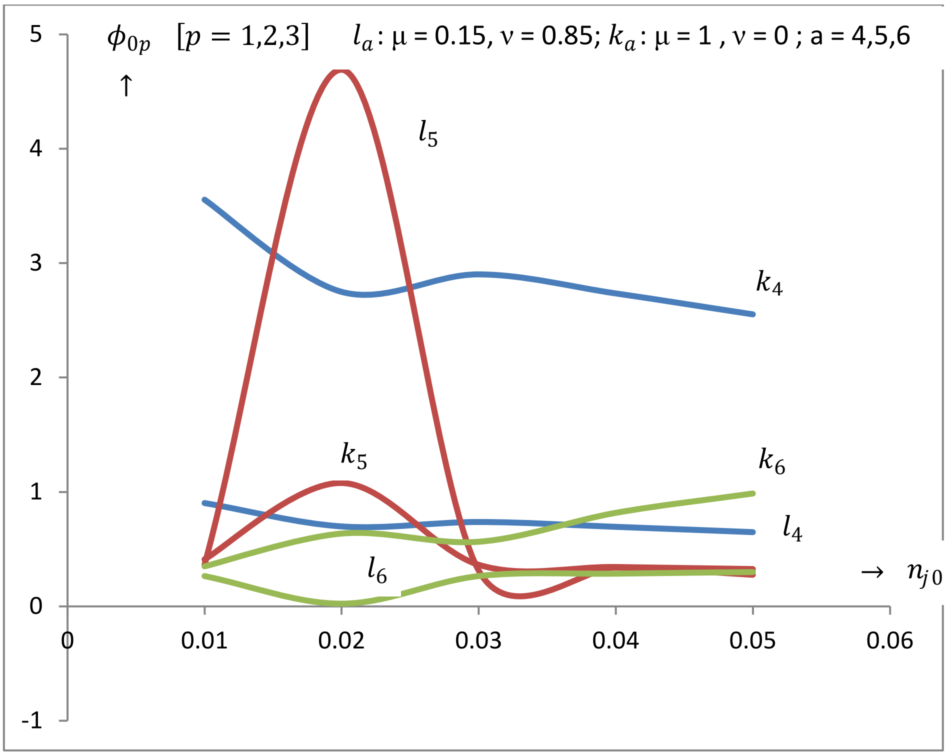


FIGURE 8. Comparison of first, second and third order amplitudes of the solitary wave solutions versus negative ion concentration between two-temperature and single temperature non-isothermal electron plasma for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3,Q = 4, μ = 0.15, ν = 0.85, n_{j0} = 0.05, n_{i0} = 0.88, $\boldsymbol{\sigma_i} = \frac{1}{30}$, $\boldsymbol{\sigma_j} = \frac{1}{25}$ $(\sigma_i < \sigma_j)$, χ = 0.17, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.40, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, Z = 1.

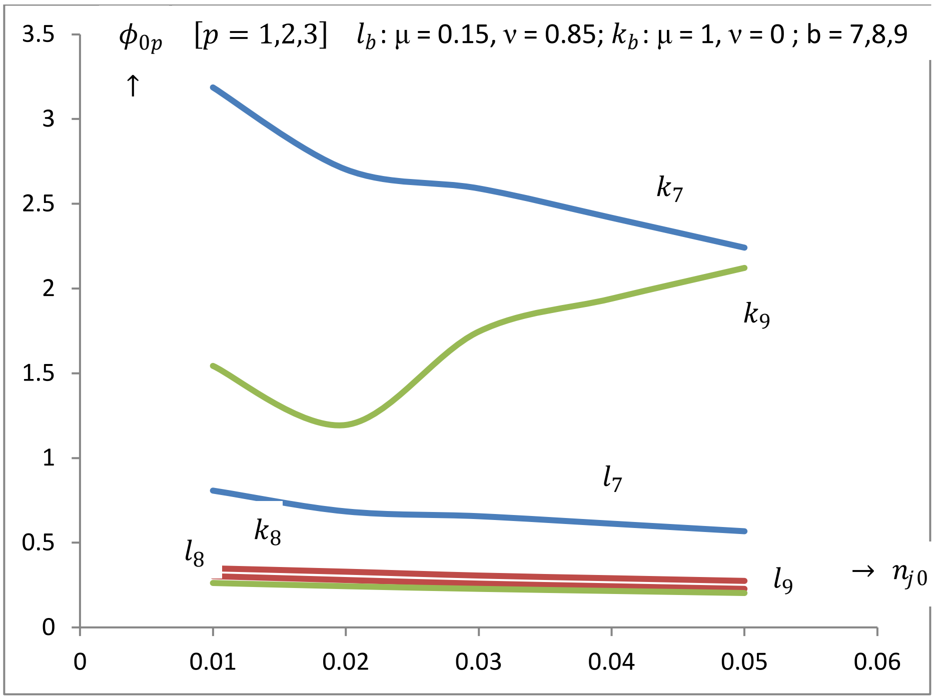


FIGURE 9. Comparison of first, second and third order amplitudes of the solitary wave solutions versus negative ion concentration between two-temperature and single temperature non-isothermal electron plasma for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3,Q = 4, μ = 0.15, ν = 0.85, n_{j0} = 0.05, n_{i0} = 0.88, $\boldsymbol{\sigma_i} = \frac{1}{25}$, $\boldsymbol{\sigma_j} = \frac{1}{30}$ $(\sigma_i > \sigma_j)$, χ = 0.17, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.40, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, Z = 1.

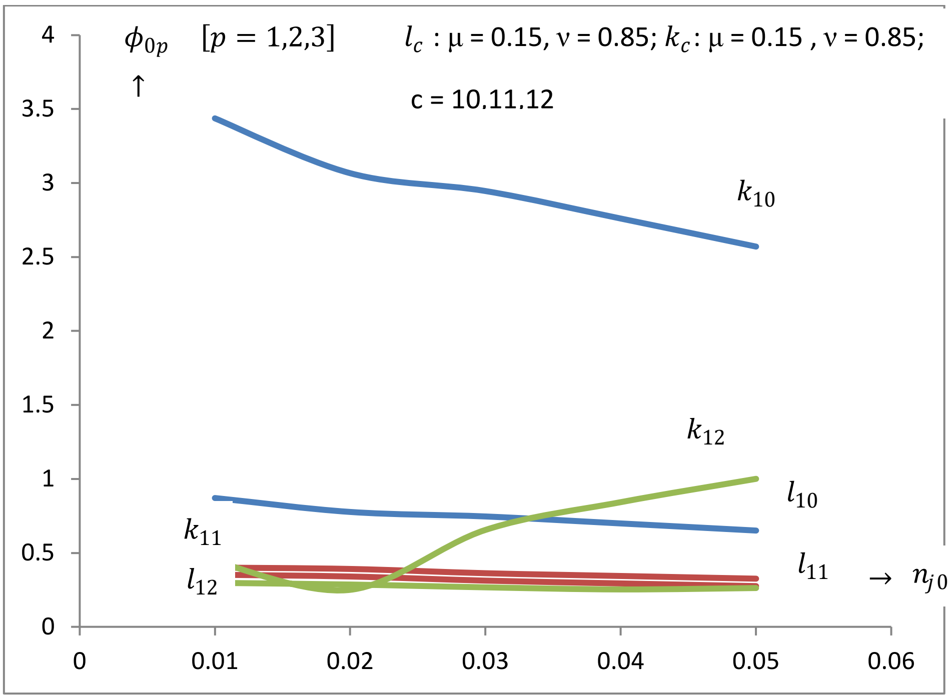


FIGURE 10. Comparison of first, second and third order amplitudes of the solitary wave solutions versus negative ion concentration between two-temperature and single temperature non-isothermal electron plasma for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3,Q = 4, μ = 0.15, ν = 0.85, n_{j0} = 0.05, n_{i0} = 0.88, $\boldsymbol{\sigma_i} = \frac{1}{30}$, $\boldsymbol{\sigma_j} = \frac{1}{30}$ $(\sigma_i = \sigma_j)$, χ = 0.17, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.40, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, Z = 1.

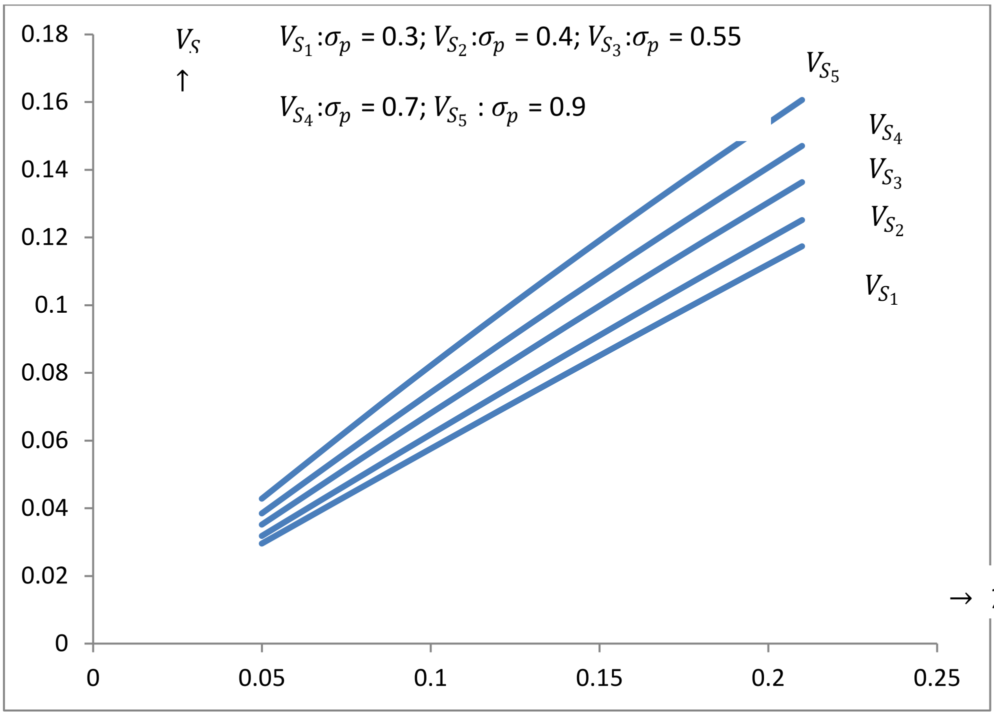


FIGURE 11. Slow ion – acoustic phase velocity(V_s) versus positron density(χ) with the variation of the ratio of the temperature (σ_p) of electron and positron for $u_{i0} = 1.05$, $\sigma_i = \frac{1}{30}$, $\sigma_p = 0.3$, 0.4 , 0.55 , 0.7 , 0.9 .

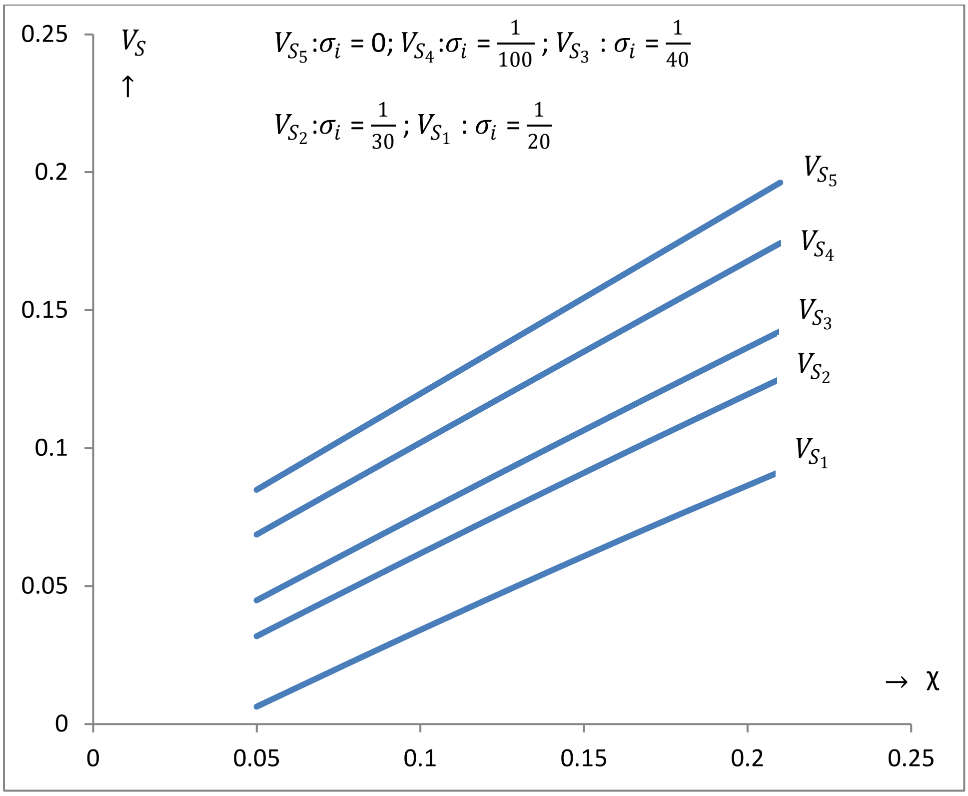


FIGURE 12. Slow ion – acoustic phase velocity(V_s) versus positron density(χ) with the variation of the temperature of positive ion (σ_i) for $u_{i0} = 1.05$, $\sigma_p = 0.4$, $\sigma_i = 0, \frac{1}{100}, \frac{1}{40}, \frac{1}{30}, \frac{1}{20}$.

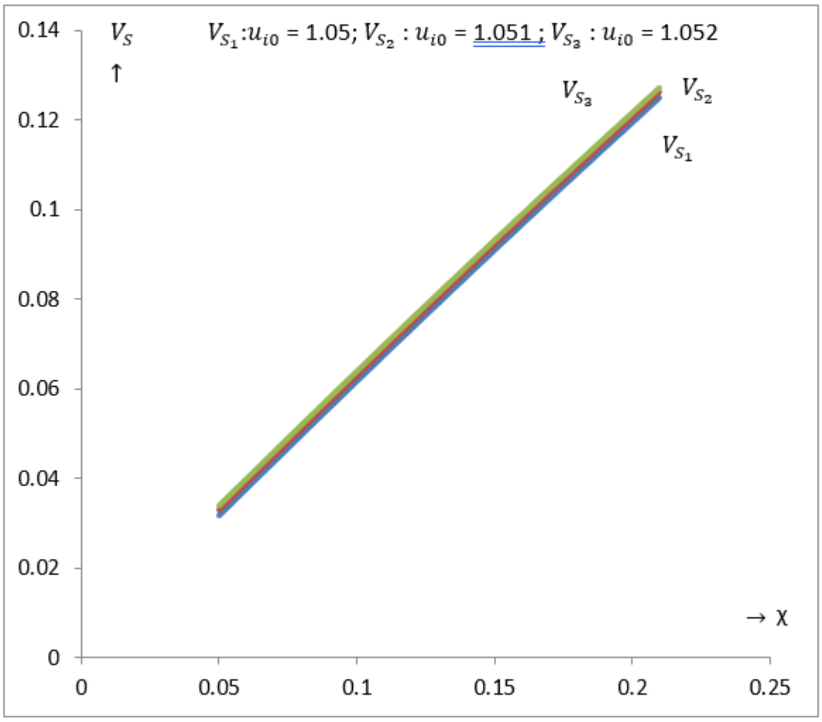


FIGURE 13. Slow ion – acoustic phase velocity(V_s) versus positron density(χ) with the variation of drift velocity of positive ion (u_{i0}) for σ_p = 0.4, $\sigma_i = \frac{1}{30}$, u_{i0} = 1.05 , 1.051, 1.052.

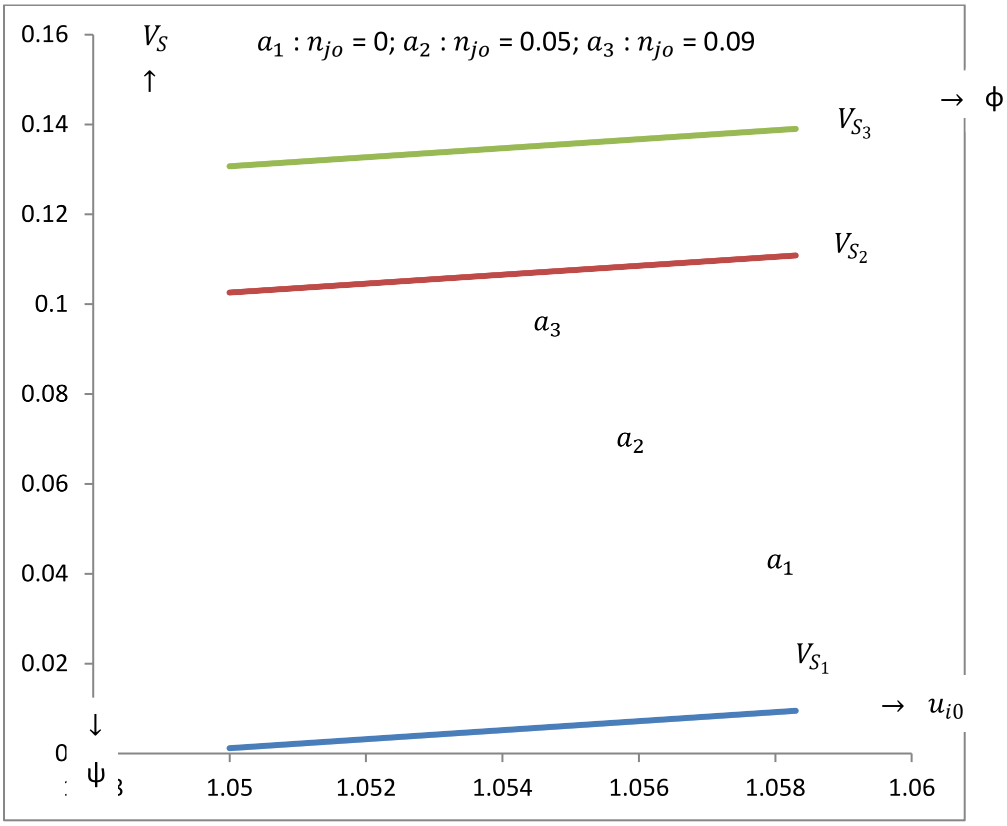


FIGURE 14. Slow ion – acoustic phase velocity(V_s) versus positive ion drift velocity (u_{i0}) with the variation of positron density (χ) for σ_p = 0.4, $\sigma_i = \frac{1}{30}$, χ = 0 , 0.17 , 0.22.

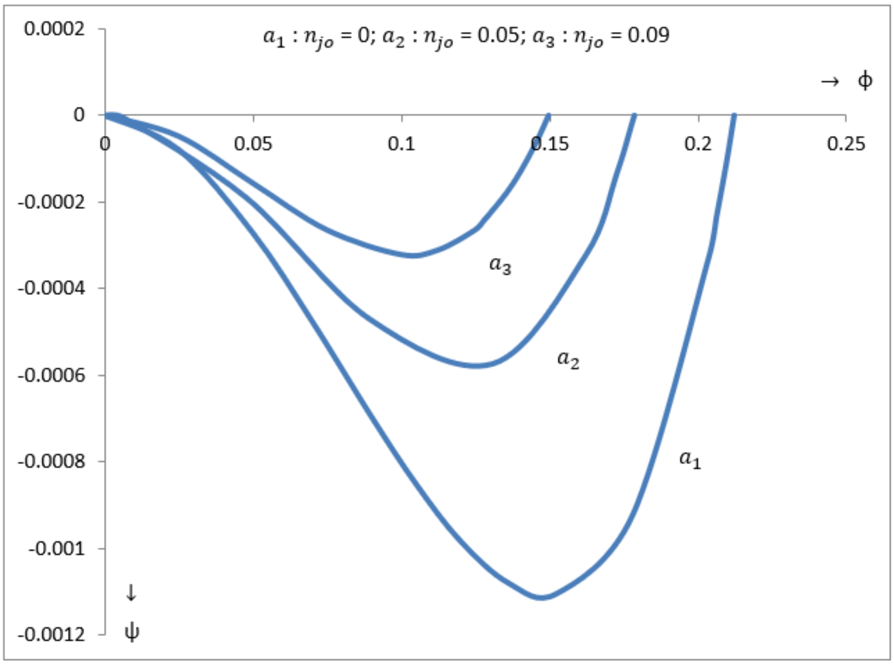


FIGURE 15. Profiles of Sagdee potential function [ψ(ϕ)] versus electrostatic potential (ϕ) with the variation of the negative ion concentration (n_{jo}) in two-temperature non-isothermal electron plasma for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, χ = 0.17, Q = 4, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.24, $b_h^{(1)}$ = 0.51, Z = 1, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, μ = 0.15, ν = 0.85 .

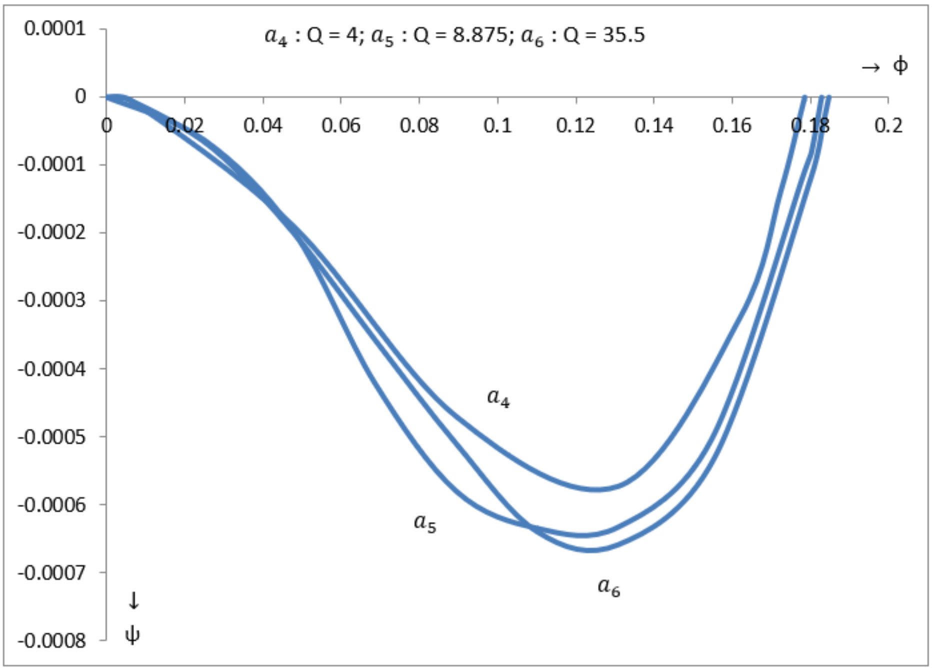


FIGURE 16. Profiles of Sagdeev potential function [ψ(φ)] versus electrostatic potential (φ) in two-temperature non-isothermal electron plasma with the variation of the ratios of negative to positive ion masses (Q) for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, n_{j0} = 0.05, n_{i0} = 0.88, χ = 0.17, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.24, $b_h^{(1)}$ = 0.51, Z = 1, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, μ = 0.15, ν = 0.85.

Figure 17 shows the profiles of the Sagdeev potential curves [ψ(ϕ)] against ϕ for three different values of the concentration of positron (χ = 0, 0.12, 0.17) when temperature of positive(σ_i) and negative (σ_j) ions are $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$ and concentration of two-temperature electron is μ = 0.15 , ν = 0.85 for the mass ratio Q = 4. It is found that the amplitudes are increasing for increasing values of χ and will be largest for χ = 0.17. In this case, the curve a_7 is marked for χ = 0, a_8 is for χ = 0.12 and finally a_9 is represented for χ = 0.17. Comparing a_8 and a_9 with a_7, we see the effect of positron which gives an important result.

In Fig. 18 the profiles of the Sagdeev potential function [ψ(ϕ)] against electrostatic potential (ϕ) for four different values of the temperatures of positive (σ_i) and negative (σ_j) ions when concentration of two-temperature electron (μ = 0.15 ,ν = 0.85) and temperature of positron (σ_p = 0.41) have a definite value. Since temperatures of positive (σ_i) and negative (σ_j) ions are an important parameter for our non-isothermal plasma, we see from the curves of the respective Sagdeev potential that the temperatures of positive (σ_i) and negative ions (σ_j) will affect the Sagdeev potential profile very distinctly which are represented by $a_{10}(\sigma_i = 0,\ \sigma_j = 0)$, a_{11} ($\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$) and a_{12} ($\sigma_i = \frac{1}{20}$, $\sigma_j = \frac{1}{30}$) and a_{13} ($\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{30}$).Observing the curves a_{11} , a_{12}, a_{13} and comparing these curves with a_{10} , one can easily see the effect of the temperature of positive (σ_i) and negative (σ_j) ions on these curves.

Tagare et al [24,36] & Das et al [11] took cold positive ($\sigma_i = 0$) and cold negative $(\sigma_j = 0)$ ion plasma with non-isothermal & isothermal electron. But in this paper, we have taken different temperatures of positive (σ_i) and negative (σ_j) ions in two-temperature non-isothermal plasma under some conditions on σ_i and σ_j ($\sigma_i = \sigma_j$, $\sigma_i > \sigma_j$, $\sigma_i < \sigma_j$) which are more general than Ref.[11,24,36] and gives a new idea.

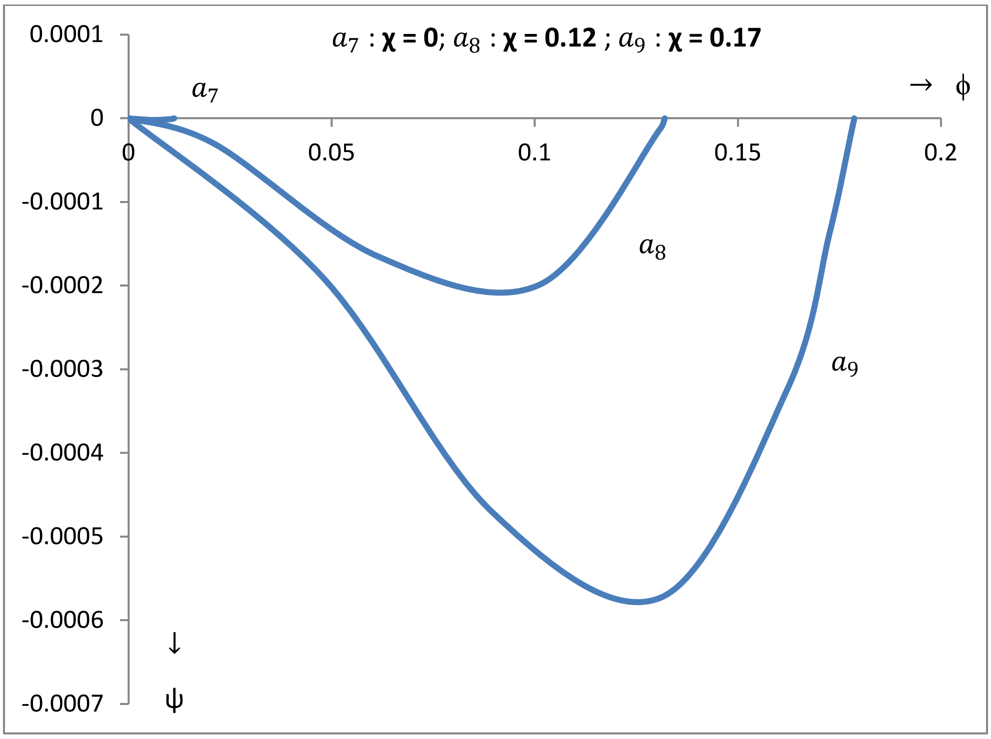
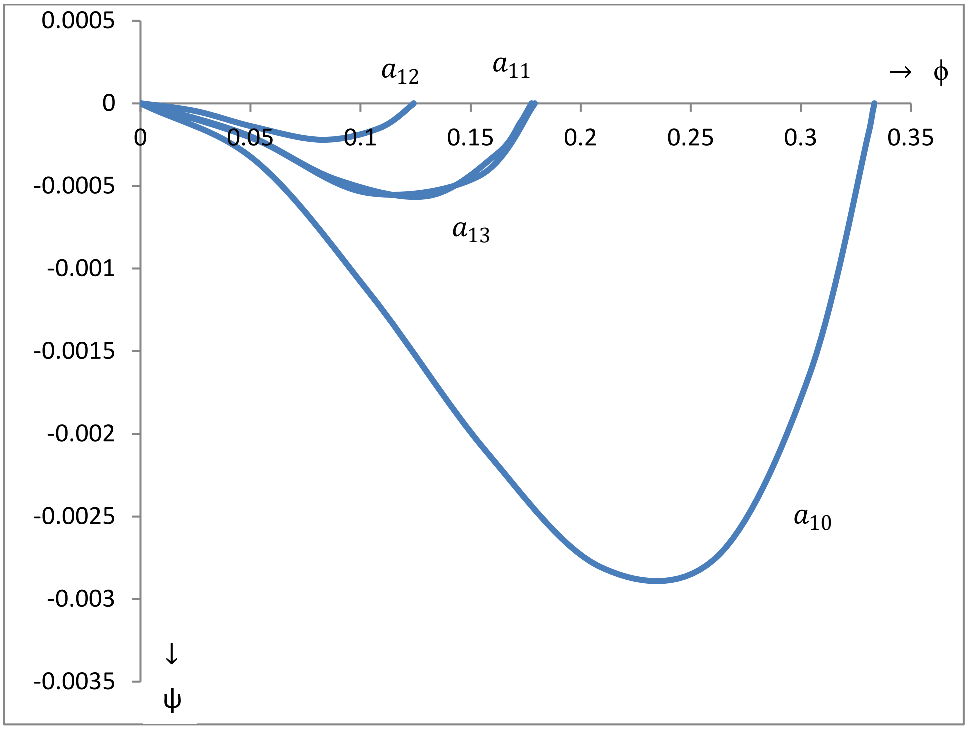


FIGURE 17. Sagdeev potential function [ψ(ϕ)] versus electrostatic potential (ϕ) with the variation of the concentration of positron (χ) in two-temperature non-isothermal electron plasma for Q = 4, V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, n_{j0} = 0.05, n_{i0} = 0.88, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.40, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, Z = 1.



a_{10} : $\boldsymbol{\sigma_i = 0, \sigma_j = 0}$; a_{11} : $\boldsymbol{\sigma_i = \frac{1}{30}, \sigma_j = \frac{1}{20}}$

a_{12} : $\boldsymbol{\sigma_i = \frac{1}{20}, \sigma_j = \frac{1}{30}}$; a_{13} : $\boldsymbol{\sigma_i = \frac{1}{30}, \sigma_j = \frac{1}{30}}$

FIGURE 18. Sagdeev potential function [ψ(ϕ)] versus electrostatic potential (ϕ) with the variation of the temperature of positive (σ_i) and negative(σ_j) ions in two-temperature non-isothermal electron plasma for χ = 0.17, Q = 4, V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, n_{j0} = 0.05, n_{i0} = 0.88, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.40, $b_l^{(1)}$ = 0.24, $b_h^{(1)}$ = 0.51, Z = 1.

V. CONCLUSION

In this paper we have studied theoretically the stable phase plane trajectories or the boundary regions of compressive solitary wave solutions, the first (ϕ_{01}), second (ϕ_{02})and third (ϕ_{03}) order amplitudes of the corresponding solitary wave solutions, two types of phase velocities(V_F , V_S) with special attention on slow mode phase velocity(V_S) and the profiles of Sagdeev pseudopotential function [ψ(ϕ)] under the variation of different plasma parameters.

The energy equation gives the velocity $\left(\frac{d\phi}{d\eta}\right)$ of the pseudoparticle in the potential well and this velocity $\left(\frac{d\phi}{d\eta}\right)$ is symmetrical with respect to the line of the electrostatic potential ϕ for different values maintaining some restrictions on ϕ. The corresponding potential energy regarded as Sagdeev potential function ψ(ϕ) and the velocity $\left(\frac{d\phi}{d\eta}\right)$ of the pseudoparticle in the potential well against ϕ are thus found out easily that gives the boundary region of the compressive solitary wave solution.

The feasible solution of solitary wave equation with special discussion on the bounded solution region of compressive solitary wave solution by equation (14c) is one of our important motivations shown by the figs. 1 – 6. Actually this shows the interaction of trapped electrons on the ion-acoustic soliton propagation in multicomponent plasma where amplitudes may be increased or decreased due to the increase of the mach number or higher order trapping effect (or non-isothermality). The bounded or interior region of compressive solitary wave solution for this two-temperature non-isothermal electron plasma is our principal objectivity and the condition for its appearance is satisfied whereas outside boundary, the condition for solitary wave solution is not satisfied. For the interior region of the separator, we have periodic solutions while for exterior or outside region of the separatrix an aperiodic solutions are observed. The separatrix gives a physically acceptable ion-acoustic solitary wave observed in space plasma.

Our next intention is to compare the first, second and third order amplitudes ($\phi_{01}, \phi_{02}, \phi_{03}$) between two-temperature and single temperature non-isothermal electron plasma for different values of the temperatures of positive (σ_i) and negative (σ_j) ions when μ = 0.15, ν = 0.85 and μ = 1, ν = 0 shown by the figs. 7 – 10. By this way the present author thus tried to show the effects of positron on the amplitudes of solitary waves and comparison of amplitudes between two-temperature and single temperature non-isothermal electron plasma which are most probably a new findings that previous authors [29,30] did not consider.

On the other hand, slow ion-acoustic phase velocity (V_S) is also considered and analysed properly with the variation of different plasma parameters and they are depicted in Figs. 11 - 14.

In order to determine the existence or non-existence of soliton solution, the form or profile of the Sagdeev pseudopotential function $\psi(\phi)$ in graphical representation is inevitable. The nature of Sagdeev pseudopotential [$\psi(\phi)$] curves against electrostatic potential (ϕ) is thus shown in Figs. 15 – 18 under the variation of negative ion concentration (n_{j0}), mass ratios (Q) of negative to positive ions, positron concentration (χ) and temperatures of positive (σ_i) and negative (σ_j) ions by which the effect of these respective parameters are easily observed.

In our study, the plasma under consideration and their results can support the formation of the boundary region of the compressive solitons with certain restricted values of the plasma parameters. The results thus obtained are important in the context of ionospheric and magnetospheric plasmas.

Actually in space, the plasma acoustic modes are generated by the non-isothermality of space plasma. The well known and renowned Freja satellite observations could not be able to find spiky and explosive solitary waves due to higher order trapping of electrons in the potential well. This Freja Scientific Satellite observations could be motivated through the satellite for the new findings in space plasma.

This present work with its associated results which may occur in laboratory and space plasma, is applicable for understanding the wave phenomena in presence of warm positive ion, warm negative ion and warm positron.

Our future plan is to solve the two-temperature isothermal electron plasma with positron along with relativistic warm positive and warm negative ions.

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