Lattice Design of Energy Sweep Compact Rapid Cycling Hadron Therapy (ESCORT)

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Abstract. A next generation cancer therapy driver by the fast-cycling and induction synchrotron is given. The design of the lattice parameters with the zero momentum-dispersion and high-flat momentum-dispersion region as the essential parameters must be fulfilled and the optimization process with results are discussed here.

Keywords: Energy Compact Rapid Cycling Hadron Therapy (ESCORT), lattice, cancer therapy, induction synchrotron, momentum dispersion

I. INTRODUCTION

3D spot scanning of hadron beams to cancer tissues of human organs is of most concern in this society [1]. Therefore, an Energy Compact Rapid Cycling Hadron Therapy (ESCORT) driver for future cancer therapies based on the induction synchrotron concept is designed as published by Leo et al [1, 2] and as shown in Table 1 and Fig. 1. This lattice has the two-fold symmetry with a circumference of 74.8 m, 4 m-long dispersion-free straight section, and 4 m-long large flat dispersion straight section. By assuming a 1.5 T bending magnet, the ring can deliver heavy ions of 400 MeV/au at 10 Hz.

TABLE 1. ESCORT's design and its technical specifications

Parameters	Specification
Energy	400 MeV/nucleon for A/Q = 2 ion
C_0	74.8 m
Ion Species	Gaseous/Metal ions
Ion Source (IS)	Laser ablation IS, Electron Cyclotron Resonance IS
Injector	200 kV (electrostatic)

Ring	Fast cycling (10 Hz)
	B_{max} = 1.5 Tesla
	ρ = 4.42928 m
	FODOF cell with edge focus of B
	Mirror symmetry
	ν_x/ν_y = 1.88024/1.69714
	4m long dispersion-free region
	4m long flat large dispersion region
	α_p=0.0396646
	γ_T=5.0211
	Induction cells driven by SPS employing SiC-MOSFET
	$V_{acc} = \rho C_0 dB/dt$ (max 7 kV)
Acceleration	Induction cells driven by SPS employing SiC-MOSFET
	$V_{acc} = \rho C_0 dB/dt$ (max 7 kV)
Vacuum	10^{-8} Pascal

In order to realize a slow extraction technique and fast extraction in a fast cycling synchrotron, which allows the energy sweep beam scanning, the essential features [1,2] are required as:

i. Dispersion-free region for the induction acceleration devices and injection device
ii. Localized large flat dispersion region for the slow extraction with the length of 4 m
iii. Local betatron phase advance of $\pi/2$ for the fast extraction

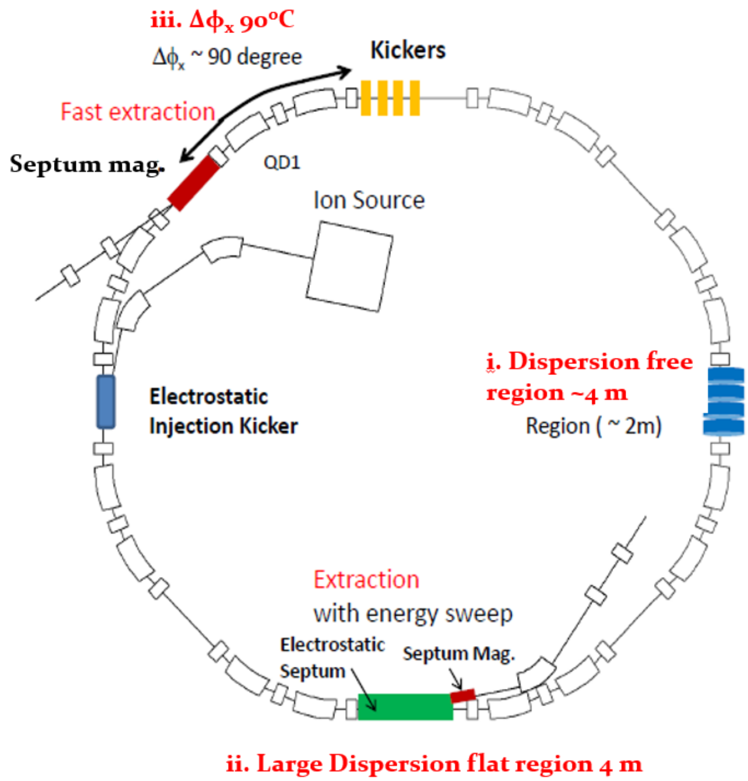

FIGURE 1. ESCORT Driver System

Details of the lattice parameters and optimization method are discussed in next section.

II. LATTICE: BASIC AND CONFIGURATION

The lattice of ESCORT is designed based on the Proton Ion Medical Machine Study (PIMSS) [3]. A lattice is an array of bending magnets and focusing magnets. Whereby the coefficient of restoring force, K(s), in the betatron equation is uniquely determined by this lattice. Hence, the individual common components can be treated with 3 X 3 matrices with the following equations [4,5,6].

Drift space transfer matrix,

$$M_D = \begin{bmatrix} 1 & \ell & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Focusing magnet transfer matrix,

$$M_f = \begin{bmatrix} \cosh\sqrt{K_F}\,\ell_{Q_F} & \dfrac{\sinh\sqrt{K_F}\,\ell_{Q_F}}{\sqrt{K_F}} & 0 \\ \sqrt{K_F}\sinh\sqrt{K_F}\,\ell_{Q_F} & \cosh\sqrt{K_F}\,\ell_{Q_F} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Defocusing magnet transfer matrix,

$$M_d = \begin{bmatrix} \cos\sqrt{K_D}\,\ell_{Q_D} & \frac{\sin\sqrt{K_D}\,\ell_{Q_D}}{\sqrt{K_D}} & 0 \\ -\sqrt{K_D}\sin\sqrt{K_D}\,\ell_{Q_D} & \cos\sqrt{K_D}\,\ell_{Q_D} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Bending magnet transfer matrix,

$$M_B = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\sin\theta/\rho & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \quad (4)$$

Horizontal edge effect transfer matrix of bending magnet,

$$M_{HE} = \begin{pmatrix} 1 & 0 & 0 \\ \tan\varepsilon/\rho & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (5)$$

Vertical edge effect transfer matrix of bending magnet,

$$M_{VE} = \begin{pmatrix} 1 & 0 & 0 \\ 1/\rho\left({}^{b}/_{6\rho\cos\varepsilon} - \tan\varepsilon\right) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (6)$$

K- value,

$$K = \frac{B'}{B\rho} \quad (7)$$

Where:

$\theta =$ Bending angle of bending magnet

$\rho =$ Curvature of of bending magnet

$\varepsilon =$ Edge angle of bending magnet

$b =$ Distance over which the fringe field drops to zero of bending magnet

$K_{D,F} =$ K − value of the related magnets

ℓ or $\ell_{Q_{D,F}} =$ Length of the related space or magnet

The lattice configuration of ESCORT is shown in Fig. 2 with the half ring configuration of Cell-1 X Cell-2 X Drift Space-4 X Cell-2 X Cell-1 with length of 38.2 m and the elements parameters are shown in Table 2 and Table 3.

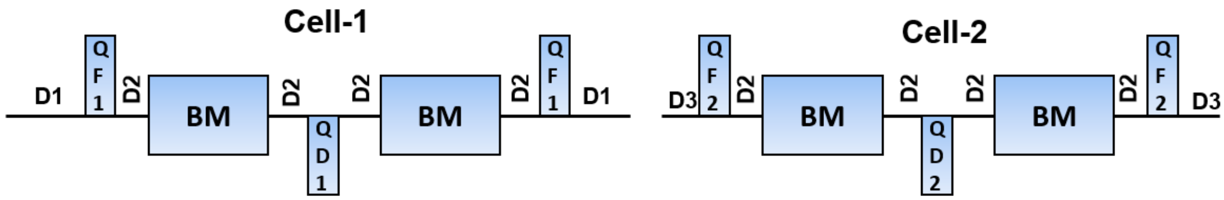

FIGURE 2. Configuration of Cell-1 and Cell-2

TABLE 2. Parameters of drift space and quadrupole elements

Elements	Length, l [m]	K-value [m^{-2}]
Drift space 1, D1	2.0	---
Drift space 2, D2	0.3	---
Drift space 3, D3	0.5	---
Drift space 4, D4	3.0	---
Focusing Quadrupole magnet 1, QF1	0.3	0.69064*
Defocusing Quadrupole magnet, 1 QD1	0.3	- 0.76969*
Focusing Quadrupole magnet 2, QF22	0.3	0.5889*
Defocusing Quadrupole magnet 2, QD1	0.3	- 0.8470*

(*After optimization)

TABLE 3. Parameters of bending magnet

Magnetic Flux Density, B [T]	Bending Angle, θ [°]	Edge Angle, θ_E [°]	Curvature, ρ [m]	Length, l [m]
1.5	22.5	11.25	4.4429	1.74

By referring to Eqs. (1) to (6) with parameters of Table 2 and Table 3, the transfer matrix of individual cell can be expressed as follow:

For horizontal transfer matrix of Cell 1,

$$M_{Hcell1} = M_{D1}.M_{f1}.M_{D2}.M_{HE}.M_{B}.M_{HE}.M_{D2}.M_{d1}.M_{D2}.M_{HE}.M_{B}.M_{HE}.M_{D2}.M_{f1}.M_{D1} \quad (8)$$

For horizontal transfer matrix of Cell 2,

$$M_{Hcell2} = M_{D3}.M_{f2}.M_{D2}.M_{HE}.M_{B}.M_{HE}.M_{D2}.M_{d2}.M_{D2}.M_{HE}.M_{B}.M_{HE}.M_{D2}.M_{f2}.M_{D3} \quad (9)$$

For vertical transfer matrix of Cell 1,

$$M_{Vcell1} = M_{D1}.M_{d1}.M_{D2}.M_{VE}.M_{DB}.M_{VE}.M_{D2}.M_{f1}.M_{D2}.M_{DB}.M_{VE}.M_{DB}.M_{VE}.M_{d1}.M_{D1} \quad (10)$$

For vertical transfer matrix of Cell 2,

$$M_{Vcell2} = M_{D3}.M_{d2}.M_{D2}.M_{VE}.M_{DB}.M_{VE}.M_{D2}.M_{f2}.M_{D2}.M_{VE}.M_{DB}.M_{VE}.M_{D2}.M_{d2}.M_{D3} \quad (11)$$

Whereby the total transfer matrix can be expressed as:

For horizontal transfer matrix of half ring,

$$M_{HTotal} = M_{Hcell1}.M_{Hcell2}.M_{D4}.M_{Hcell2}.M_{Hcell1} \tag{12}$$

For vertical transfer matrix of half ring,

$$M_{VTotal} = M_{Vcell1}.M_{Vcell2}.M_{D4}.M_{Vcell2}.M_{Vcell1} \tag{13}$$

For example, the horizontal transfer matrix of Cell 1 can be expressed as:

$$M_{Hcell1} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ 0 & 0 & 1 \end{pmatrix}_{Hcell1} \tag{14}$$

$$\begin{aligned} M_{Hcell1} = & \begin{pmatrix} 1 & \ell_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\sqrt{K_F}\,\ell_{Q_F} & \dfrac{\sin\sqrt{K_F}\,\ell_{Q_F}}{\sqrt{K_F}} & 0 \\ -\sqrt{K_F}\sin\sqrt{K_F}\,\ell_{Q_F} & \cos\sqrt{K_F}\,\ell_{Q_F} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ \dfrac{\tan\varepsilon}{\rho} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ & \times \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\dfrac{\sin\theta}{\rho} & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ \dfrac{\tan\varepsilon}{\rho} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ & \times \begin{pmatrix} \cosh\sqrt{K_D}\,\ell_{Q_D} & \dfrac{\sinh\sqrt{K_D}\,\ell_{Q_D}}{\sqrt{K_D}} & 0 \\ \sqrt{K_D}\sinh\sqrt{K_D}\,\ell_{Q_D} & \cosh\sqrt{K_D}\,\ell_{Q_D} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ \dfrac{\tan\varepsilon}{\rho} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ & \times \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\sin\theta/\rho & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ \tan\varepsilon/\rho & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ & \times \begin{pmatrix} 1 & \ell_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\sqrt{K_F}\,\ell_{Q_F} & \dfrac{\sin\sqrt{K_F}\,\ell_{Q_F}}{\sqrt{K_F}} & 0 \\ -\sqrt{K_F}\sin\sqrt{K_F}\,\ell_{Q_F} & \cos\sqrt{K_F}\,\ell_{Q_F} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \tag{15}$$

Therefore, Eqs. (1) to (15) the essential parameters of lattice for betatron tune, phase advanced and dispersion function can be expressed as follow:

For horizontal or vertical phase advanced of half ring,

$$\mu_{HTotal,Vtotal} = \left\{\cos^{-1}\left((m_{11}+m_{22})/2\right)\right\}_{HTotal,VTotal} \tag{16}$$

Fractional part of horizontal or vertical tune with unknown integer part,

$$\Delta\nu_{HTotal,Vtotal} = \left\{\frac{\mu}{2\pi}\right\}_{HTotal,Vtotal} \tag{17}$$

For dispersion function at particular transfer location,

$$\begin{pmatrix} D \\ D' \end{pmatrix}_i = \left\{\frac{1}{2-(m_{11}+m_{22})}\begin{pmatrix} m_{13}+m_{12}m_{23}-m_{22}m_{13} \\ m_{23}+m_{21}m_{13}-m_{11}m_{23} \end{pmatrix}\right\}_i \tag{18}$$

Whereby, Eqs. (16), (17) and (18) are essential to optimize the K-value of individual quadrupole elements as described at next section.

III. LATTICE: OPTIMIZATION PROCESS AND RESULT

The lattice of ESCORT can be simplified with four FODO cells and treated as a super-period. They are grouped into two types of cells, CELL-1 and CELL-2, and aligned as CELL-1, CELL-2, Drift4, CELL-2 and CELL-1 as shown in Fig. 3.

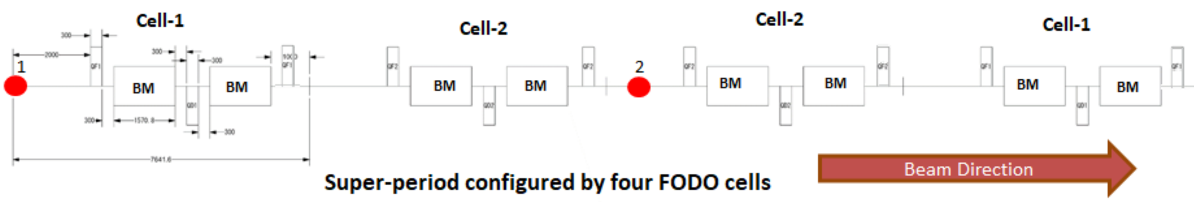

FIGURE 3. Configuration of half ring (1 denote as large flat dispersion region and 2 denote as free dispersion region)

The crucial requirements of this lattice are noted in Fig. 3 with large flat dispersion and free dispersion region. Therefore, the aim is to optimize the K-value of quadrupole to achieve the (D, D')~ (arbitrary number , 0) at location 1 and (D, D') ~ (0, 0) at location 2 (Fig. 3) by scanning the vertical tune ν_V , while ν_H is fixed (D may be slightly modified). The detail process and successive iteration are shown in Fig. 4 and Fig. 5.

Choose the arbitrary K-value of the quadrupole elements

↓

Evaluate 3 χ^2's (QF1 is scanned by an appropriate small value e, while QD1 is fixed)

$$\chi_1^2 = (\Delta\nu_{H1} - \Delta\nu_{H0})^2 + (\Delta\nu_{V1} - \Delta\nu_{V0})^2$$
$$\chi_2^2 = (\Delta\nu_{H2} - \Delta\nu_{H0})^2 + (\Delta\nu_{V2} - \Delta\nu_{V0})^2$$
$$\chi_3^2 = (\Delta\nu_{H3} - \Delta\nu_{H0})^2 + (\Delta\nu_{V3} - \Delta\nu_{V0})^2$$

Find optimized QD1 (=<QF1>) by parabola fitting to 3 χ^2's

↓

Evaluate 3 χ^2's (QD1 is scanned by an appropriate small value e, while QF1 is fixed)

$$\chi_1^2 = (\Delta\nu_{H1} - \Delta\nu_{H0})^2 + (\Delta\nu_{V1} - \Delta\nu_{V0})^2$$
$$\chi_2^2 = (\Delta\nu_{H2} - \Delta\nu_{H0})^2 + (\Delta\nu_{V2} - \Delta\nu_{V0})^2$$
$$\chi_3^2 = (\Delta\nu_{H3} - \Delta\nu_{H0})^2 + (\Delta\nu_{V3} - \Delta\nu_{V0})^2$$

Find optimized QD1 by parabola fitting to 3 χ^2's

↓

Test change of χ^2. If test value is larger than some small value (~0.001 or less), repeat optimization procedure by setting QF1=<QF1> , QD1=<QD1> and $\chi^2_{0,previous} = \chi^2_0$.

$$test = \frac{\left|\chi_0^2 - \chi_{0.previous}^2\right|}{\chi_{0,previous}^2}$$

FIGURE 4. Optimization procedure of K-value.

For each stage, the dispersion function at location 1 and 2 must be evaluated according to the Eq. (18). In additional, the QD_2 is used for fine tuning to fulfill the 3 lattice requirements of the ESCORT.

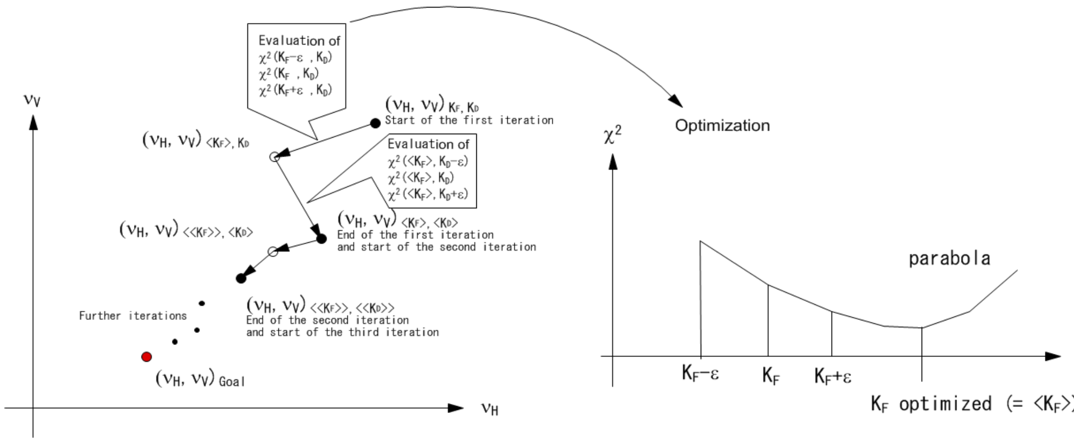

FIGURE 5. Schematic of tune drift during successive iteration

According to the process as described in Fig. 4, the K-value of quadrupole elements have been optimized with the result as shown in Table 4. Meanwhile, the dispersion function at location 1 and location 2 must be evaluated as well to confirm the desirable dispersion magnitude. If the large flat and zero dispersion regions haven't been realized by this optimization process. Then the optimization process should be restarted from the beginning.

TABLE 4. Optimization result (i, ii, iii denoted as the essential lattice features of ESCORT)

Parameters	**Specifications**	**Remarks**
K-Value of QF_1 [m^{-2}]	0.69064	Optimized
K-Value of QD_1 [m^{-2}]	-0.76969	Optimized
K-Value of QF_2 [m^{-2}]	0.5889	Optimized
K-Value of QD_2 [m^{-2}]	-0.8470	Optimized
D_2 [m]	$-7.35\text{X}10^{-6}$	Dispersion free region (i)
D_2' [m]	1.94×10^{-15}	Sustainable for along 4 m (i)
D_1 [m]	6.64	Large flat dispersion region (ii)
D_1' [m]	-2.17×10^{-15}	Sustainable along 4 m (ii)
Phase advance	$\pi/2$	For fast extraction (iii)

The result shown in Table 4, the lattice requirements of ESCORT: dispersion region, large flat dispersion region and betatron phase advance for fast extraction has been fulfilled with the optimized K-value of quadrupole elements. Hence, the beta function can be plotted by the Courant-Snyder parameters, which is discussed in the next section.

IV. COURANT-SNYDER PARAMETERS AND ESCORT BETA FUNCTION

A beam comprise of particles will goes along with different movement along the design orbit in a transport line, where magnets are aligned. All particles in a beam are positioned in a phase space. If such a phase space is scanned along the beam orbit, there revealed a surface enclosing all particles. This surface is called as Beam Envelope. From stand point of accelerator design, it is important to know a beam behavior rather than a behavior of each particle. In the phase space, the beam envelope is given by an ellipse as shown in Fig. 6, which is defined by the following formula [4,5,6].

$$\beta x'^2 + 2\alpha x x' + \gamma x^2 = \varepsilon \tag{19}$$

where,
β, α and γ = Courant-Snyder Parameter coefficients depending on the orbit position s.
ε = Emittance and defined by an ellipse area divided by π.

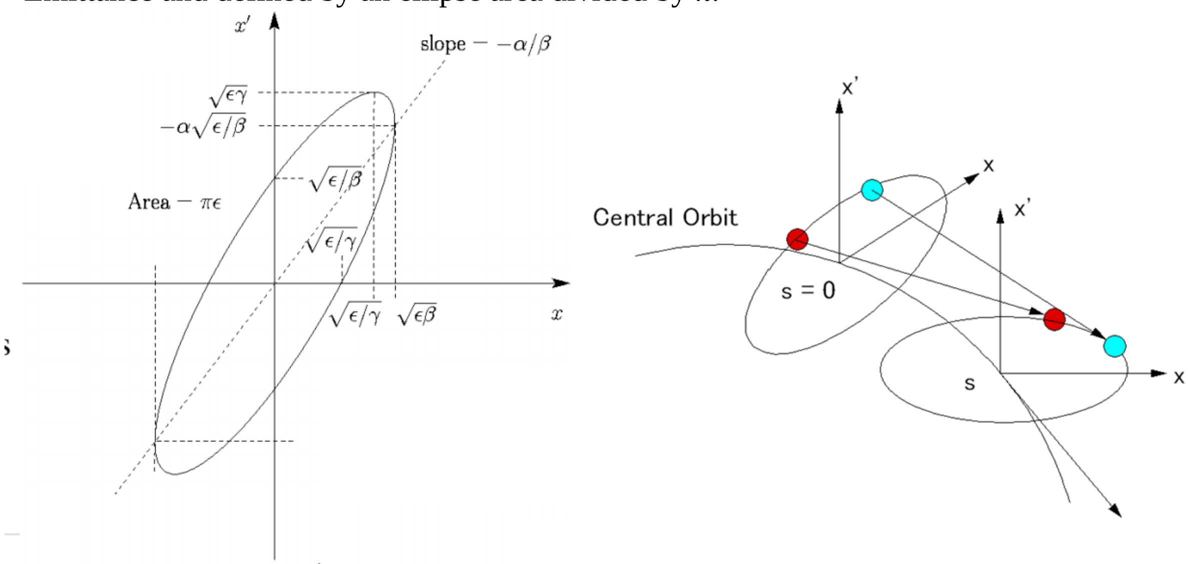

FIGURE 6. Courant-Snyder parameters with phase space revolution along the ring

Since courant-Snyder parameter satisfies the following relation, the number of independent parameters is two.

$$\beta\gamma = 1 + \alpha^2 \tag{20}$$

Twiss parameter is transferred by a matrix constructed by a corresponding transfer matrix elements. Such a matrix is shown in the following. Here, m_{ij} is a component of the transfer matrix.

$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_s = M\begin{pmatrix}\beta_0\\ \alpha_0\\ \gamma_0\end{pmatrix} \quad , \quad M \equiv \begin{pmatrix} {m_{11}}^2 & -2m_{11}m_{12} & {m_{12}}^2 \\ -m_{11}m_{21} & m_{11}m_{22}+m_{12}m_{21} & -m_{12}m_{22} \\ {m_{21}}^2 & -2m_{21}m_{22} & {m_{22}}^2 \end{pmatrix} \tag{21}$$

Whereby, the dispersion function is constructed by a corresponding transfer matrix elements. Such a matrix is shown in the following. Here, m_{ij} is a component of the transfer matrix.

$$\begin{pmatrix} D \\ D' \\ 1 \end{pmatrix}_s = M \begin{pmatrix} D_0 \\ D'_0 \\ 1 \end{pmatrix}_s \quad , \quad M \equiv \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ 0 & 0 & 1 \end{pmatrix} \tag{22}$$

From Eqs. (21, 22) with the lattice parameters from Table 2., the beta and dispersion functions of the ESCORT is plotted as shown in Fig. 7.

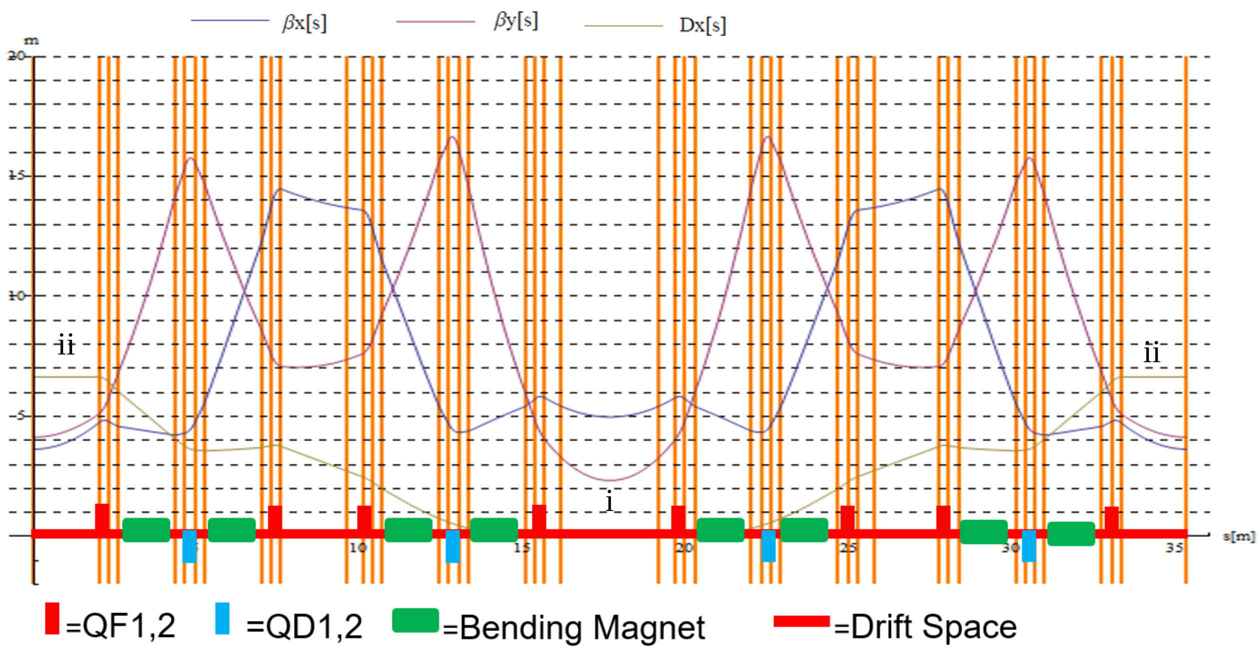

FIGURE 7. Beta function of ESCORT

Fig. 7 shows the requirements of i. Dispersion free region, ii. Large flat dispersion region with sustainability of 4 m is fulfilled ideally.

V. DISCUSSION

In the design of a Synchrotron, the lattice's parameters is essential. Those parameters optimization is an initial stage to determine the ring size and arrangement of the magnets along the Synchrotron ring. By this optimized parameters, the magnet designed and parameters will be studied and optimized as well. Meanwhile, those parameters will be used to study the perturbation term of the particles which might be caused by the errors of magnet magnitude or alignment of the magnets. In this paper the method of optimization has been well-developed with the desired parameters that fulfilled the essential requirements of the ESCORT. Therefore, the lattice parameters can be applied to the hadron driver with the compatible machine parameters and the method could be expanded to design and study the lattice parameters of Synchrotron for various applications.

VI. CONCLUSION

With the optimization method of tune scanning, the lattice parameters of the ESCORT have been defined. The requirements of the dispersion free region, large flat dispersion region and phase advance of 90^{o} for fast extraction have been fulfilled as well. For further consideration, non-linear perturbation terms must be studied and the magnet components design must be able to realize with the designed lattice. Hence, the lattice should be modified to comply with the engineering aspects of the magnet components.

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REFERENCES

1. K. W. Leo, T. Monma, T. Adachi, T. Kawakubo, T. Dixit and K. Takayama. Compact Hadron Driver for Cancer Therapies Using Continuous Energy Sweep Scanning. *Phys. Rev. ST AB* **19**, 042802 (2016).
2. K. W. Leo, T. Adachi, T. Kawakubo, T. Monma, T. Dixit and K. Takayama. A Compact Hadron Driver for Cancer Therapies Using Continuous Energy Sweep Scanning. *Proceedings of the 13th International Conference on Heavy Ion Accelerator Technology* 2015, Yokohama.
3. Accelerator Complex Study Group, CERN. Proton-Ion Medical Machine Study (PIMMS) Part 1. *CERN/PS 99-010 (DI)* (2 March 1999).
4. Helmut Wiedemann. *Particle Accelerator Physics*. ISBN-13 978-3-540-49043-2 3rd ed. Springer Berlin Heidelberg New York.
5. T. Adachi. *KEK-Nuclear Malaysia-Sokendai Accelerator School in Nuclear Malaysia.* 1st-4th December 2014.
6. Stephen Peggs and Todd Satogata. *Introduction to Accelerator Dynamics* 1st Edition. Cambridge University Press; 1st Edition (August 7, 2017).