Optical bistable and multistable response from a ferroelectric Fabry-Perot resonator

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A ferroelectric Fabry-Perot (FP) interferometer with dielectric mirrors partially coated on its interfaces is considered. The ferroelectric (FE) material, capable of exhibiting nonlinearity under an application of high electric field, is assumed to have an intensity-dependent refractive index where the third order nonlinear susceptibility $\chi^{(3)}_{ilmn}$ acts like a Kerr coefficient. The response of the ferroelectric medium is modeled by the Landau-Khalatnikov (LK) dynamical equation, often used for a discussion of nonlinear optical (NLO) behavior near a resonance frequency. Within a single frequency approximation, the nonlinear wave equation can be written in terms of polarization P rather than the electric field E as the dependent variable. The resulting nonlinear polarization equation is numerically integrated across the FP interfaces. The behavior of the resulting polarization as a function of the field input intensity is investigated. With the application of boundary conditions at both interfaces, transmitted and reflected waves are also plotted as a function of the incident intensity. This approach is applicable in frequency ranges where the nonlinear response of the ferroelectric material is strongly resonant. The possibility of obtaining an optical switch from such system is explored.

I. INTRODUCTION

Recently Kerr bistability has attracted many authors [1,2] due to its various applications in nonlinear optics such as quantum nondemolition measurements, quantum logic gates and quantum state teleportation. Bistable devices are also useful in the field of optical communications, such as all-optical switching and memory devices. More specifically, optical bistable response of a Fabry-Perot (FB) interferometer containing a nonlinear material with Kerr nonlinearity has attracted a great deal of interest [3-5]. The standard analysis in studying this problem in nonlinear optics is to expand the optical polarization P as a Taylor series in the field E. However, Goldstone and Garmire [6] pointed out that the standard analysis is not suitable to describe this type of intrinsic optical bistability. K-H Chew *et al*. [7] considered a dielectric nonlinear FP interferometer following the approach used in Ref [6]. They developed an alternative analysis based on the Duffing oscillator which is found to be more suitable to frequency ranges where the nonlinear response of materials like the FE materials is strong and resonant. However, the work in Ref [7] studied only the transmittance coefficient of a nonlinear dielectric FP interferometer. Following Chew's approach in [7], we have studied recently in greater details the polarization behavior, reflectance and transmittance of a nonlinear ferroelectric (FE) FP interferometer [8].

Recently, Rajan *et al.* [9] have derived expressions of the tensor elements for various second- and third-order nonlinear optical effects including optical Kerr bistability for bulk FE having various symmetries. They have shown that many of these elements have large linear and nonlinear optical coefficients even in the visible and near-infrared frequency regions. They have found that it is the combination of the temperature divergence and the resonant frequency, which is typically in the THz region, dependence that underlies their large values. For these reasons we are interested in investigating theoretically into the possibility of generating optical bistability effect from FE materials. Moreover, because of the development of high-power THz sources [10,11] it may now be possible to demonstrate experimentally bistability phenomenon in these materials.

In this work we apply the Duffing model analysis to a Fabry-Perot (FP) interferometer containing a FE material coated with partially-reflecting mirrors. Since the material is FE, the Landau-Khalatnikov (LK) dynamical equation and the Landau-Devonshire free energy are used in the formulation [12]. A nonlinear polarization equation is derived and integrated across the etalon thickness. The resultant nonlinear polarization, reflection and transmission are plotted as functions of the input intensity of the field.

II. MATHEMATICAL FORMALISM

We consider a FP interferometer system consisting of a FE etalon (medium 2) of thickness L as shown in Fig. 1, which includes partially-reflecting mirrors at its interfaces. The mirrors can be modeled by a thin metallic layer of thickness δ_M permittivity ε_M and conductivity σ_M [13]. For simplicity, a far-infrared (FIR) radiation with a single frequency ω is assumed incident normally on the etalon. The fields in medium 1 and 3 (considered vacuum) may be written in a form of a plane wave $\mathcal{E}_1 = E_0\left[\exp(-ik_1 z - i\omega\boldsymbol{t}) + r\exp\left(ik_1 z - i\omega\boldsymbol{t}\right)\right]$ and $\mathcal{E}_3 = tE_0\exp[-ik_3(z+L) - i\omega\boldsymbol{t}]$ respectively. The reflection $R = |r|^2$ and transmission $T = |t|^2$ are evaluated following the standard analysis in linear optics [14]. The nonlinear Duffing potential is obtained from Landau-Devonshire free energy *F* for bulk FE exhibiting second-order phase transitions [12], which is

$$F(\mathcal{P}) = \frac{\alpha}{2\varepsilon_0}\mathcal{P}^2 + \frac{\beta_1}{4\varepsilon_0^2}\mathcal{P}^4 - \mathcal{E}_2.\mathcal{P}\,. \tag{1}$$

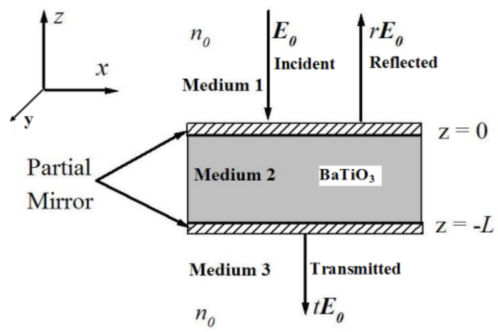

FIG. 1. Geometry of the Fabry-Perot interferometer system.

Eq. (1) is the simplest binding potential that leads to bistable behavior where $\alpha = a(T - T_C)$ with *a* being the inverse of the Curie constant and T_C is the Curie temperature, and β_1 is a material dependent parameter. The term $\mathcal{E}_2.\mathcal{P}$ accounts for the coupling of the *FIR* radiation to the driving field. The response of a FE material exposed to a high-intensity FIR radiation may be described by the LK dynamical equation of motion in terms of polarization, $\mathcal{P}$,

$$M(d^2\mathcal{P}/d\boldsymbol{t}^2) + \gamma(d\mathcal{P}/d\boldsymbol{t}) = -\partial F/\partial\mathcal{P} \tag{2}$$

M and γ are inertial and damping parameters respectively. The term $\gamma d\mathcal{P}/d\boldsymbol{t}$ represents a linear loss. We assume $\mathcal{E}_2$ and $\mathcal{P}$ to be time harmonic plane waves $\mathcal{P}(z,\boldsymbol{t}) = P(z)\exp(-i\omega\,\boldsymbol{t})$ and $\mathcal{E}_2(z,\boldsymbol{t}) = E_2(z)\exp(-i\omega\boldsymbol{t})$ with ω being the operating frequency. Using (1) and (2), it is possible to derive *E* in terms of *P*, and convert the resulting equation into dimensionless form

$$E_d = \frac{3\sqrt{3}}{2}\left[[-M_d f^2 - i\gamma_d f + (T_d - 1)]p + \frac{3}{4}|p|^2\,p\right] \tag{3}$$

where the above equation was derived under single frequency approximation and the following scaling factors are used: $f = \omega/\omega_0$ with ω_0 being the resonance frequency. $p = P/P_0$, with $P_0 = \sqrt{aT_C\varepsilon_0/\beta_1}$ being the spontaneous polarization of the FE material at $T = 0$. $E_d = E_2/E_c$ with $E_c = \sqrt{4a^3T_C^3/27\varepsilon_0\beta_1}$ being the coercive field of the FE material at $T = 0$. Other scaled variables are $T_d = T/T_C$, $\gamma_d = [\gamma\omega_0\varepsilon_0/aT_C]$ and $M_d = [M\omega_0^2\varepsilon_0/aT_C]$. In the high frequency limit, the scaled form of the electromagnetic (EM) wave equation for waves propagating in the *z*-direction is

$$-\frac{d^2E_d}{du^2} = f^2\varepsilon_\infty E_d + [P_0/E_c\varepsilon_0]f^2 p\,. \tag{4}$$

Here, ε_∞ is the high-frequency limit of the complex dielectric function $\varepsilon(\omega)$ and $u = z\omega_0/c$ is the scaled thickness. Eqs. (3) and (4) give the following ODE in polarization,

$$\begin{aligned}&\left[\psi_d + \frac{3}{2}|p|^2\right]\frac{d^2p}{du^2} + \frac{3}{4}p^2\frac{d^2p^*}{du^2} + 3p\frac{dp}{du^2}\frac{dp^*}{du}\\ &+\frac{3}{2}p^*\left(\frac{dp}{du}\right)^2 = -f^2\varepsilon_\infty\left[\psi_d + \frac{3}{4}|p|^2 + w\right]p\end{aligned} \tag{5}$$

with $w = 1/aT_c\,\varepsilon_\infty$ and $\psi_d = [-M_d f^2 - if\gamma_d + (T_d - 1)]$. The term d^2p^*/du^2 may be eliminated from Eq. (5); and the resulting form may be integrated numerically as an initial value problem to evaluate the desired polarization. To evaluate the reflectance *R*, we apply the boundary conditions of *E* at *z = 0*.

$$R = |r|^2 = \left|\frac{3\sqrt{3}}{2e_0}\left[\frac{3}{4}|p_t|^2 + \psi_d\right]p_t - 1\right|^2 \tag{6}$$

The quantities p_t and $e_0 = E_0/E_c$ are the polarization value at $u = 0$ and the scaled incident intensity respectively. Application of boundary conditions of *H* at *z = 0* and utilizing (6) leads to

$$\begin{aligned}e_0 = i\frac{27}{4f}&\left[\left(\psi_d + \frac{3}{2}|p_t|^2\right)\frac{dp_t}{du} + \frac{3}{4}p_t^2\frac{dp_t^*}{du}\right]\\ &+\frac{3\sqrt{3}}{4}[1 - \eta_d]\left[\psi_d + \frac{3}{4}|p_t|^2\right]p_t\end{aligned} \tag{7}$$

The parameter $\eta_d = \eta_{d,a} - i\eta_{d,b}$ accounts for the mirror in a scaled form [13] with $\eta_{d,a} = c\mu_0\delta_M\sigma_M$ and

$\eta_{d,b} = \omega\delta_M \varepsilon_M / c$. In this work a dielectric mirror is considered ($\sigma_M = 0$). Since η_d does not have a very direct physical significance, we label graphs with both $\eta_{d,b}$ and the equivalent frequency-dependent reflectivity R_M [7,8]. The transmittance T is obtained by applying the boundary conditions of E at $z = -L$.

$$T = |t|^2 = \left| \frac{3\sqrt{3}}{2e_0} \left[\psi_d p_b + \frac{3}{4} | p_b |^2 \; p_b \right] \right|^2 \qquad (8)$$

The quantity p_b is the polarization value at $u = -l$ with $l = \omega_0 L/c$ being the dimensionless thickness. Application of boundary conditions of H at $z = -L$ and utilizing (8) leads to

$$\frac{dp_b}{du} = \frac{\sqrt{3}}{3} f p_b \left[\frac{\eta_{s,b} \left[\frac{2}{3} |\psi_d|^2 + \psi_d |p_b|^2 + \frac{3}{8} |p_b|^4 \right]}{\left[|\psi_d|^2 + \frac{3}{2} \left(\psi_d + \psi_d^* \right) |p_b|^2 + \frac{27}{16} |p_b|^4 \right]} - i \frac{2}{3} \right] \qquad (9)$$

III. RESULTS OF NUMERICAL CALCULATION

Since there is no incoming wave in medium 3, it is more convenient to integrate the ODE in Eq. (5) from $u = -l$ to $u = 0$. Our numerical strategy is as follows: we assume an arbitrary value of $p_b = p_b^* = p_0$ at the boundary $u = -l$ and evaluate dp_b / du using (9). We then integrate Eq. (5) as an initial value problem from $u = -l$ to $u = 0$. This gives p_t , dp_t / du and their complex conjugates. Subsequently, values of $| e_0 |$, r and t are calculated. The procedure is repeated for a large number of arbitrary p_b values. The available experimental data of $BaTiO_3$ is used in our computation to obtain more realistic results [15]. The operating frequency $f = 1.5$ is selected. We also use relatively small value of thickness $l = 1$ appropriate for ionic solid in the FIR. Fig. (2) shows plot of $| p_b |$ and $| p_t |$ as a function of $| e_0 |$ while Fig. (3) shows both reflectance R and transmittance T as a function of $| e_0 |$. In general, these graphs have typical features found in materials exhibiting bistability and multistability behaviour. It is observed that the system responds linearly for small values of $|e_0|$ and after reaching a certain threshold value, a bistable behavior is exhibited as observed in both figures. Here, the threshold value is $| e_0 | \approx 380$ which is $| E_0 | \approx 1.28 \times 10^{10} Vm^{-1}$, a physically realizable value. It is also observed that as $| e_0 |$ increases, a multistability phenomenon sets in. The stable operating points at a certain value of $| e_0 |$ can be determined by drawing vertical lines on the graphs at that particular value. For example at $| e_0 | = 500$, the vertical line drawn in Figs. (2) and (3) intersects with all curves at 5 operating points although only 3 of them are stable. Interestingly, it is also observed from our generated graphs that the nonlinear response of the system exists only for a certain range of input intensity $| e_0 |$. Above this range, the system response becomes linear again.

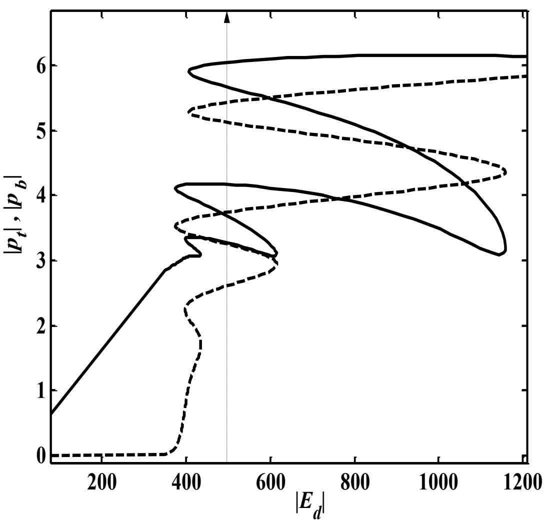

FIG. 2. $| p_t |$ (solid line) and $| p_b |$ (dashed line) versus $| e_0 |$ for $BaTiO_3$ at f = 1.5 with $\eta_{s,b}$ = 0.5 corresponding to mirror reflectance of R_M = 0.1, l = 1, and ε_∞ = 3.

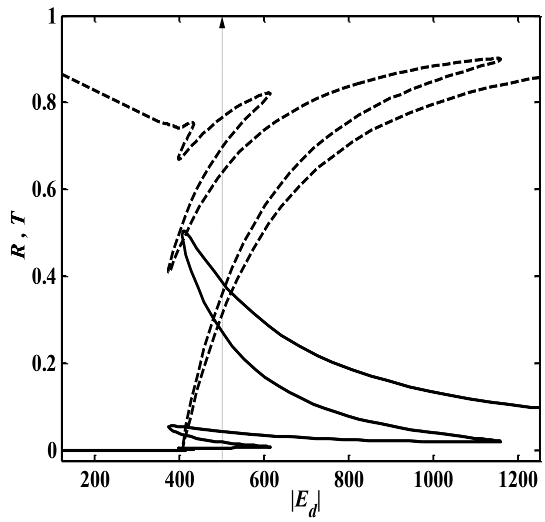

FIG. 3. R (dashed line) and T (solid line) versus $| e_0 |$. Other parameters remain as in Fig. 2.

IV. CONCLUSIONS

Our results show that it is possible to generate bistability and multistability phenomena from a FE Fabry-Perot etalon. We have demonstrated that the approach we adopted is more suitable for ferroelectrics particularly at frequency ranges where the nonlinear response of the material is strong and resonant. From the application point of view, such FE Fabry-Perot interferometer has the potential to be applied as an optical switch if a low threshold value of input intensity could be realized. It may be possible to achieve a lower threshold value by varying the operating frequencies or varying other system parameters like etalon thickness and mirror reflectivity, which we are currently investigating.

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