Bound state spectra of $Q\bar{Q}$-systems: relativistic and non-relativistic

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Power law potential models with effective power $\nu \propto m_Q$ and $\nu \propto \mu$ are investigated as quark confining potentials in Schrödinger and the Dirac equations. Here m_Q is the mass of the constituent quarks (in GeV) and μ is the reduced mass of the meson (in GeV). It is shown that the same power law potentials with equal admixture of scalar and vector parts acquire relativistic consistency in generating Dirac bound-states of $Q\bar{Q}$-systems, in agreement with the Schrödinger spectroscopy. The models are capable of predicting the meson spectroscopy encompassing both light and heavy quark-antiquark systems in a unified way.

I. INTRODUCTION

One of the greatest challenges and triumphs of the last century physics has been the understanding of quarks as the possible ultimate building blocks of matter at the fundamental level. This has been made possible by tremendous development in the particle accelerator technology [1]. Many low-lying phenomena such as M1 transitions cannot ordinarily be explained by first principle applications of quantum chromodynamics (QCD) which is the underlying theory of strong interactions at the basic structural level of hadrons. Attempts have therefore been made to construct phenomenological models incorporating the basic features of QCD and in fact these models have proven to be quite useful. The relativistic quark model formulated by Godfrey *et al*. [2] provided a phenomenological frame-work to describe all mesons in a unified way.

It is well-known that in order to explain subatomic phenomena, field theoretical methods have been quite successful and thus QCD has emerged as a correct theory of strong interactions. Though QCD is qualitatively successful in accounting for all subatomic phenomena, we have not yet obtained a basic understanding of quark confinement. The usual method to explain the subatomic phenomena is to develop potential models phenomenologically. More often than not one has to borrow ideas from QCD to motivate the asymptotic behaviour of such a potential for large and small r. This approach is more appealing because of its simplicity when compared with QCD. Yet potential model based on QCD predictions must necessarily have a coulomb plus a linear part [3] or be logarithmic [4] which then must be solved numerically. A linear potential can be solved exactly only if $l = 0$ [5]. Many non-relativistic calculations have been made along this line. Moreover, a good potential model should be able to explain not only mass spectra but electromagnetic properties as well viz., the decay-widths.

The power law potential models have been quite successful in explaining the meson spectroscopy with great success [6-8]. Kang and Shnitzer [9], using linear potential and Ram and Halasa [10], using quadratic potential computed the bound-state energies of ψ, ϕ and ρ mesons. The spectra of Υ and ψ mesons were explored by Martin [8], Barik and Jena [7] and by Barik and Barik [6], using a fractional power potential viz. $V(r) = Ar^{0.1} + V_0$.

Quigg and Rosener [11] observed that when the effective power $\nu = Ar^{0.1} \pm 0.05$, the quark masses for ψ and Υ mesons have weak dependence on level spacing $(E_1 - E_2)$. In a non-relativistic study of power law potential, it is observed that the effective power be given approximately as $\nu = 0.08 \pm 0.05$. Further studies [3], on level density of the bound systems of ψ and Υ, have revealed that the power should be $\nu = 0.20 \pm 0.06$ for the ψ family and $\nu = 0.33 \pm 0.23$, for the Υ family. Sharma and Sharma [12] have used a power law potential with power $\nu \propto \frac{1}{m_Q}$ in the Dirac equation. Their model could predict the meson spectroscopy encompassing both light and heavy $Q\bar{Q}$ system in a unified way.

Sharma [13,14] have further proposed a relativistic potential model to calculate energy levels and electromagnetic properties of a few mesons. Work [15-17] dealing with the calculation of meson spectra of yet to be experimentally observed meson $t\bar{t}$ (toponium), has also been published.

The present investigation is motivated with a desire to predict the entire meson spectroscopy (including leptonic decay-widths) for both equal quark-antiquark masses and also for the unequal quark-antiquark masses. For this we use two power law potentials viz.

$\nu = \lambda \dfrac{m_Q}{m_0}$, for the case of equal quark-antiquark masses, where a parameter m_0 has been introduced to make the power ν dimensionless (m_0 = 1 GeV). For retaining the flavour type of interaction, the authors instead of using the reduced mass μ, have considered only the individual mass of a quark for the case of equal quark-antiquark system. However, for unequal quark-antiquark system we have considered an effective power $\nu = \lambda \dfrac{\mu}{m_0}$, where μ is the reduced mass in GeV of the mesons under study. The calculated masses have been compared with the experimental data [18].

In section 2, we investigate a potential with power $\nu \propto m_Q$ in the Schrödinger equation to explain the meson spectroscopy of ψ and Υ states. In section 3, the same potential is studied for generating Dirac bound states, of both heavy and light mesons viz., Υ, ψ, ϕ and ρ mesons. It should be noted that following the prescription of Magyari [19], the calculation of Dirac bound states for the $Q\overline{Q}$-system have been done by considering Lorentz structure of our potential in the form of an approximately equal admixture of scalar and vector parts.

In section 4, replacing the potential with a power $\nu \propto \mu$ for equal-unequal quark-antiquark masses, we first explain the meson spectroscopy of D^0, $D_S{}^+$, B^-, B_S^0 in the Shrödinger frame-work and then using the same potential we generate Dirac bound states for the above mentioned mesons.

Finally section 5 of this paper concludes our results.

II. SCHRÖDINGER BOUND- STATES (MESONS WITH EQUAL QUARK-ANTIQUARK MASSES)

The potential considered by us has the form

$$V(r) = Gr^{\lambda \frac{m_Q}{m_0}} + V_0 \,. \tag{1}$$

Here m_Q is the mass of the constituent quark (in GeV), m_0 is also expressed in GeV. First of all we obtain an expression for the spin-averaged mass of non-relativistic $Q\overline{Q}$-bound-state system from the Schrödinger equation

$$\frac{d^2\psi(r)}{dr^2} + \left[m_Q(E - V(r)) - \frac{l(l+1)}{r^2} \right]\psi(r) = 0 \tag{2}$$

$(\hbar = c = 1)$

Inserting the value of $V(r)$ from Eq. (1) and setting $\rho = (r/r_0)$ with the scale factor r_0 chosen such that

$$r_0 = (m_Q G)^{-\frac{1}{\lambda \frac{m_Q}{m_0} + 2}} \,, \tag{3}$$

Eq. (2) assumes the form

$$\frac{d^2\psi(\rho)}{d\rho^2} + \left[\varepsilon - \rho^{\lambda \frac{m_Q}{m_0}} - \frac{l(l+1)}{\rho^2} \right]\psi(\rho) = 0 \,. \tag{4}$$

Here

$$\varepsilon = m_Q(E - V_0)[m_Q G]^{-\frac{2}{\lambda \frac{m_Q}{m_0} + 2}} \,. \tag{5}$$

Eq. (4) has been solved for $\nu > 0$, using the semi-classical W.K.B method [11,20]. This would give $\varepsilon = \varepsilon_{nl}$ (say) corresponding to any particular radial and orbital state (n,l) with the quark-antiquark binding energy $E = E_{nl}$, obtained from Eq. (5) as

$$E_{nl} = g_1 (g_1 / m_Q)^{\lambda m_Q/(\lambda m_Q + 2m_0)} \varepsilon_{nl} + V_0 \tag{6}$$

where

$$g_1 = (G)^{\frac{1}{\left(\lambda \frac{m_Q}{m_0}\right)}} \,. \tag{7}$$

From above, the non-relativistic Schrödinger bound-state masses of $Q\overline{Q}$-system are obtained as

$$M_{nl}^{Shr}(Q\overline{Q}) = 2m_Q + V_0 + g_1 \left(\frac{g_1}{m_Q} \right)^{\frac{\lambda m_Q}{(M_Q + 2M_0)}} \varepsilon_{nl} \,. \tag{8}$$

If we consider the semi-classical W.K.B. results [11] of Eq. (4), the values of ε_{nl} and the absolute square of wave functions at the origin $\left|\psi_{ns}^{(0)}\right|^2$ have the following form

$$\varepsilon_{nl} = \left[A \left(\lambda \frac{m_Q}{m_0} \right) \left(n + \frac{l}{2} - \frac{1}{4} \right) \right]^{2\lambda m_Q/(\lambda m_Q + 2m_0)} \tag{9}$$

$$|\psi_{ns}(0)|^2 = \frac{1}{2\pi^2}\left(\frac{\lambda m_Q}{\lambda m_Q + 2m_0}\right) \times \left[g_1 m_Q \left\{g_1 A\left(\lambda\frac{m_Q}{m_0}\right)\right\}^{\lambda\frac{m_Q}{m_0}}\right]^{3m_0/(\lambda m_Q+2m_0)} \times \left(n-\frac{1}{4}\right)^{2(\lambda m_Q - m_0)/(\lambda m_Q+2m_0)} \quad (10)$$

where

$$A\left(\lambda\frac{m_Q}{m_0}\right) = \frac{2(\pi)^{\frac{1}{2}}\Gamma\left(\frac{3}{2}+\frac{m_0}{\lambda m_q}\right)}{\Gamma\left(1+\frac{m_0}{m_q}\right)}. \quad (11)$$

Using the above results, we compute the gross features like spin-averaged bound- state masses and the ratio of leptonic decay- widths of $Q\bar{Q}$-system. Eq. (8) combined with Eqs. (9) and (11), furnishes the Schrödinger bound state masses of $Q\bar{Q}$-system corresponding to various values of radial $(n = 1,2,3\ldots)$ and orbital $(l = 0,1,2,3\ldots)$ states.

The leptonic decay widths can be obtained using the following well-known Von-Royen-Weisskopf formula [21]

$$\Gamma(ns \to e^+e^-) = \frac{16\pi\alpha^2 e_Q^2}{M_{ns}^2(Q\bar{Q})}|\psi_{ns}(0)|^2. \quad (12)$$

However, this formula should not be relied upon too much on its absolute value, as is has bearing on the correction factors not quite certain. To avoid this short coming, we calculate instead the leptonic decay width ratio. The leptonic decay width ratio is given by

$$R_{ns} = \frac{\Gamma(ns \to e^+e^-)}{\Gamma(1s \to e^+e^-)} = \left[\frac{M_{1s}(Q\bar{Q})}{M_{ns}(Q\bar{Q})}\right]^2 \times \left[\frac{|\psi_{ns}(0)|^2}{|\psi_{1s}(0)|^2}\right]. \quad (13)$$

Using the parameters mentioned in the following Table I, we calculate the bound-state mass spectrum and leptonic decay-width ratios of $c\bar{c}$ and $b\bar{b}$ systems. These results along with their experimental values are shown in Tables II and III respectively.

III. DIRAC BOUND STATES

Next we consider the static potential of Eq. (1) to obtain spin-averaged mass of $Q\bar{Q}$-bound system from the Dirac equation written in the independent particle model of quarks. Following the procedure of Magyari [19] the given potential is considered as an equal admixture of scalar and vector components like

$$V(r) = a_v V(r) + (1-a_v)V(r), \quad (14)$$

with the vector fraction $a_v = \frac{1}{2}$, then in the independent particle model of quarks, the potential would be $V'(r) = \frac{1}{2}V(r) = V_s(r) + V_v(r)$. Hence the scalar part $V_s(r)$ and the vector part $V_v(r)$ are simultaneously present. The Dirac equation can then be written as $(\hbar = c = 1)$

$$[\alpha.\vec{P} + m_Q\beta]\psi(r) = [E' - V_\upsilon(r) - V_s(r)\beta]\psi(r). \quad (15)$$

TABLE I. Values of the parameters m_Q, m_0, λ, G and V_0.

Meson	Mass of the constituent quark m_Q(GeV)	m_0 (GeV)	λ	$G\ [(\mathrm{GeV})]^{\frac{\lambda m_Q}{m_0}+1}$	V_0 (GeV)
$\Upsilon(b\bar{b})$	5.11	1.0	0.03914	3.2407	−4.4900
$\psi(c\bar{c})$	1.75	1.0	0.03914	9.3529	−10.3677
$\phi(s\bar{s})$	0.63875	1.0	0.03914	35.011237	−36.4385
$\rho(u\bar{u})$	0.38325	1.0	0.03914	84.4627	−86.1710

TABLE II. Schrödinger bound state spectrum of ψ and Υ mesons in GeV. [Input values have been underlined.]

Meson	Theory	Experiment [18]	Meson	Theory	Experiment [18]
Υ(1s)	9.460	9.460	Υ(1p)	9.7998	–
Υ(2s)	10.0133	10.02	Υ(2p)	10.2351	–
Υ(3s)	10.3447	10.355	Υ(3p)	10.5320	–
Υ(4s)	10.6080	10.557	–	–	–
Υ(5s)	10.8600	–	–	–	–
$\psi(1s)$	3.096	3.096	$\psi(1p)$	3.446	–
$\psi(2s)$	3.684	3.684	$\psi(2p)$	3.8413	–
$\psi(3s)$	4.025	4.03	$\psi(3p)$	4.135	–
$\psi(4s)$	4.2097	4.159	–	–	–
$\psi(5s)$	4.421	4.415	–	–	–

TABLE III. Ratio of leptonic decay widths for ψ and Υ mesons.

n	l	ψ (mesons)		Υ (mesons)	
		Theory	Experiment [18]	Theory	Experiment [18]
1	s	1.0	1.0	1.0	1.0
2	s	0.33	0.45 ± 0.09	0.479	0.44 ± 0.06
3	s	0.184	0.156	0.322	0.32 ± 0.04
4	s	0.125	0.102	0.245	0.2 ± 0.06

Since $V_v(r)$ and $V_s(r)$ are spherically symmetric, Eq. (15) can be written as [22]

$$[E' - V_\upsilon - V_s - m_Q] f(r) + \left[\frac{K+1}{r} + \frac{d}{dr}\right] g(r) = 0 \qquad (16)$$

$$[E' - V_\upsilon + V_s + m_Q] g(r) + \left[\frac{K-1}{r} - \frac{d}{dr}\right] f(r) = 0 \qquad (17)$$

where $K = l+1$, when $j = l + 1/2$ and $K = -l$ when $j = l - 1/2$. $f(r)$ and $g(r)$ are the radial parts of the large and small components of the spinor $\psi(r)$ respectively.

Now setting $f(r) = \frac{\phi(r)}{r}$ and $V_s(r) = V_\upsilon(r) = \frac{1}{4}\left[G r^{\lambda \frac{m_Q}{m_0}} + V_0\right]$, we get from Eqs. (16) and (17) the expression for the large component $f(r)$ in the form

$$\frac{d^2\phi(r)}{dr^2} + \left[(E' + m_Q)(E' - m_Q - 2V_s(r)) - \frac{l(l+1)}{r^2}\right]\phi(r) = 0. \qquad (18)$$

Setting

$$\frac{r}{r_0'} = \rho \quad \text{with} \quad r_0' = \left[(m_Q + E')\frac{G}{2}\right]^{-\frac{1}{\lambda \frac{m_Q}{m_0} + 2}}$$

Eq. (18) reduces to the following form similar to Schrödinger equation

$$\frac{d^2\phi(\rho)}{d\rho^2} + \left[\varepsilon' - \rho^{\lambda \frac{m_Q}{m_0}} - \frac{l(l+1)}{\rho^2}\right]\phi(\rho) = 0\,, \qquad (19)$$

where

$$\varepsilon' = \left[E' - m_0 - \frac{1}{2}V_0\right]\left[(m_Q + E')\left(\frac{2}{G}\right)^{2\frac{m_0}{\lambda m_Q}}\right]^{\frac{\lambda m_Q}{(\lambda m_Q + 2m_0)}}. \qquad (20)$$

As Eq. (18) is similar to its non-relativistic counterpart, it would therefore yield the value of $\varepsilon' = \varepsilon_{nl} > 0$ corresponding to the confined bound states of quarks. Thus we realize that there exists bounded solutions describing confined Dirac bound states of quarks with energy eigenvalues $E' = E'_{nl} > \left|\left(m_Q + \frac{1}{2}V_0\right)\right|$ which can be obtained from Eq. (20) with $\varepsilon' = \varepsilon_{nl}$.

If we substitute $G = (g_1)^{(\lambda m_Q + m_0)/m_0}$, $g_1\chi_{nl} = \left(E'_{nl} - m_Q - \frac{1}{2}V_0\right)$ and $g_1 b = \left(2m_Q + \frac{1}{2}V_0\right)$, Eq. (20) can be put in the form

$$\chi_{nl}^{(\lambda m_Q + 2m_0)/\lambda m_Q}(\chi_{nl} + b) = (\,)^{m_0/\lambda m_Q}(\varepsilon_{nl})^{(\lambda m_Q + 2m_0)/\lambda m_Q}. \quad (21)$$

Thus the equation for the Dirac bound masses can be written as

$$M_{nl}^{Dir}(Q\overline{Q}) = (g_1\chi_{nl} + 2m_Q + V_0), \quad (22)$$

where χ_{nl} is the positive root of the Eq. (21). The value of ε_{nl} is given by Eq. (9) and $b = \frac{\left(2m_Q + \frac{V_0}{2}\right)}{g_1}$.

Choosing same values of parameters G, V_0, m_Q, m_0 and λ as.mentioned in Table I, Dirac bound-state masses calculated for Υ, ψ, ϕ and ρ mesons have been displayed in Table IV below.

TABLE IV. Dirac bound state masses of Υ, ψ, ϕ and ρ mesons in GeV. [The input values have been underlined.]

Meson	Theory	Richardson Potential [23]	Experiment [18]	Meson	Theory	Richardson Potential [23]
Υ (1s)	<u>9.46</u>	9.46	9.46	Υ (1p)	9.778	9.935
Υ (2s)	10.014	10.02	10.02	Υ (2p)	10.203	–
Υ (3s)	10.3556	10.3487	10.355	Υ (3p)	10.492	–
Υ (4s)	10.600	10.604	10.577	–	–	–
Υ (5s)	10.807	10.824	–	–	–	–
ψ (1s)	3.115	3.097	3.097	ψ (1p)	3.449	3.556
ψ (2s)	3.675	3.661	3.684	ψ (2p)	3.8467	–
ψ (3s)	3.998	4.055	4.03	ψ (3p)	4.093	–
ψ (4s)	4.205	4.383	4.159	–	–	–
ψ (5s)	4.40	4.673	4.415	–	–	–
ϕ (1s)	<u>1.020</u>	1.020	1.202	ϕ (1p)	1.409	1.564
ϕ (2s)	1.675	1.706	1.685	ϕ (2p)	1.872	–
ϕ (3s)	2.0315	–	–	ϕ (3p)	2.167	–
ρ (1s)	<u>0.77</u>	0.759	0.77	ρ (1p)	1.252	1.353
ρ (2s)	1.587	1.509	1.59	ρ (2p)	1.844	–
ρ (3s)	2.056	–	–	ρ (3p)	2.234	–

Crater & Alstine (1981) [23]

IV. SCHRÖDINGER AND DIRAC BOUND-STATES (FOR MESONS WITH UNEQUAL QUARK-ANTIQUARK MASSES)

As stated in section 1, we now replace the effective power in our potential (1) by $\nu = \lambda \frac{\mu}{m_0}$ for the case of unequal quark-antiquark masses. Thus our potential now takes the form

$$V(r) = Gr^{\lambda \frac{\mu}{m_0}} + V_0 , \qquad (23)$$

where μ, is the reduced mass in GeV given by $\mu = \left[\frac{(m_{q_1} m_{q_2})}{(m_{q_1} + m_{q_{12}})} \right]$, with $m_{q_1} \neq m_{q_2}$ for the mesons understudy. The procedure, as adopted for equal quark-antiquark mesons, is followed. Finally the results so obtained are compared with the experimental data [18].

We first use the potential with the above effective power in the Schrödinger frame work to explain meson spectroscopy of D^0, D_s^+, B^-, $\overline{B}_s^0$ and B_c^-. Then we consider the same potential to generate Dirac bound-states for the above mentioned mesons. For the changed effective power, the adjusted parameters have been depicted in Table V. The results obtained have been shown in Tables VI and VII for the Schrödinger and the Dirac's frame-work, respectively.

TABLE V. Values of the parameters μ, m_0, λ, G and V_0.

Meson	μ (GeV)	m_0 (GeV)	λ	G (GeV)	V_0
$D^0(c\overline{u})$	0.3144	1.112	0.03997	93.392	−96.088
$D_s^+(c\overline{s})$	0.4037	1.112	0.03997	70.552	−73.444
$B^-(b\overline{u})$	0.3565	1.112	0.03997	88.4731	−89.8862
$\overline{B}_s^0(b\overline{s})$	0.5678	1.112	0.03997	39.8246	−41.3622
$B_c^-(b\overline{c})$	1.3036	1.112	0.03997	5.8442	−6.9661

TABLE VI. Schrödinger bound state masses for selected unequal quark-antiquark mesons. [The input values have been underlined.]

Meson	Theory (GeV)	Experiment [18] (GeV)
$D^0(1s)$	<u>1.862</u>	1.864 ± 0.004
$D^0(2s)$	2.458	2.458
$D^0(3s)$	2.644	–
$D_s^+(1s)$	<u>1.965</u>	1.968 ± 0.005
$D_s^+(2s)$	2.660	2.660
$D_s^+(3s)$	2.832	–
$B^-(1s)$	<u>5.275</u>	5.279 ± 0.005
$B^-(2s)$	5.600	–
$B^-(3s)$	5.777	–
$\overline{B}_s^0(1s)$	<u>5.365</u>	5.367 ± 0.002
$\overline{B}_s^0(2s)$	5.555	–
$\overline{B}_s^0(3s)$	5.730	–
$B_c^-(1s)$	<u>6.283</u>	6.286 ± 0.005
$B_c^-(2s)$	7.001	–
$B_c^-(3s)$	7.333	–

TABLE VII. Dirac bound state masses for selected unequal quark-antiquark mesons. [Input values have been underlined.]

Meson	Theory (GeV)	Experiment [18] (GeV)
$D^0(1s)$	$\underline{1.862}$	1.864 ± 0.004
$D^0(2s)$	2.458	2.458
$D^0(3s)$	2.642	–
$D_s^+(1s)$	$\underline{1.964}$	1.968 ± 0.005
$D_s^+(2s)$	2.660	2.660
$D_s^+(3s)$	2.836	–
$B^-(1s)$	$\underline{5.275}$	5.279 ± 0.005
$B^-(2s)$	5.601	–
$B^-(3s)$	5.779	–
$\bar{B}_s^0(1s)$	$\underline{5.367}$	5.367 ± 0.002
$\bar{B}_s^0(2s)$	5.548	–
$\bar{B}_s^0(3s)$	5.731	–
$B_c^-(1s)$	$\underline{6.282}$	6.286 ± 0.005
$B_c^-(2s)$	7.000	–
$B_c^-(3s)$	7.336	–

V. CONCLUSION

We conclude that our potential model with power $\nu = \frac{\lambda m_Q}{m_0}$ generates bound- state mass spectrum of $c\bar{c}$ and $b\bar{b}$ system in the Schrödinger equation and gives excellent agreement with the experimental values. The prediction for the ratio of the leptonic decay-width is also comparable to their corresponding experimental values. The same potential also predicts Dirac bound-states for most of the mesons in the unified way. It would also be worthwhile to note that that there is a close agreement in the values of bound-state masses of $c\bar{c}$ and $b\bar{b}$ system as obtained by us in both the relativistic and non-relativistic approaches. Similarly the potential possessing an effective power $\nu = \frac{\lambda \mu}{m_0}$, also generates bound-state mass spectrum of $c\bar{u}$, $c\bar{s}$, $b\bar{u}$, $b\bar{s}$ and $b\bar{c}$ system in the Schrödinger equation and gives excellent agreement with the experimental values [18] after appropriate adjustments in the parameters. The same effective power also predicts.

Dirac bound-.states for most of the mesons in a unified way. Here too there is a close agreement in the values of bound-state masses of the considered mesons in the relativistic and non-relativistic framework.

Our results for both the light and the heavy meson systems are thus in reasonably good agreement with the experimental data. Thus, in spite of its discomforting non-coulombic nature which contradicts the predictions of the QCD, our simple power law potentials are capable of mimicking the spectra of all mesons simultaneously. Finally, it may be concluded with the remark that the potential models proposed in this paper represent fairly well the actual interaction between $Q\bar{Q}$ systems for a wide range of quark-antiquark separation, probed in the study of entire meson spectra. Further it may be of interest to note that that a similar type of work has been done by Jena *et al*. [24] in connection with the baryon spectra.

REFERENCES

[1] C. Campagnari and M. Franklin, Rev. Mod. Phys., **69**,137 (1997).
[2] S. Godfrey and N. Isgur, Phys. Rev., **D32**, 189 (1985).
[3] R. S. Kaushal and H. J. W. Müller-Kirsten, J. Math. Phys., **20**, 2233 (1979).
[4] C. Quigg and J. L. Rosener, Phys. Lett., **B71**, 153 (1977).
[5] J. W. Norbury and F. F Badavi, Am. J. Phys., **54**, 1031 (1986).
[6] N. Barik and K. Barik, Pramana. **17,** 489-497(1981).
[7] N. Barik and S. N. Jena, Phys. Lett., **B97**, 261 (1980).
[8] A. Martin, Phys.Lett., **B93**, 338 (1980).

[9] J. J. Kang and H. J.Schnitzer, Phys. Rev., **D12**, 841 (1975).
[10] B. Ram and R. Halasa, Phys. Rev., **D19**, 3467 (1979).
[11] C. Quigg and J. L. Rosner, Phys. Rep., **56**, 167 (1979).
[12] G. S. Sharma and L. K. Sharma, Pramana, **22**, 533 (1984).
[13] L. K. Sharma, J. Fizik. Malaysia, **15**, 47 (1994).
[14] L. K. Sharma, Bots. J. of Tech., **12**, 14 (2003).
[15] V. R. Mundembe and L. K. Sharma, J. Fizik. Malaysia, **18**, 113 (1998).
[16] J. O. Fiase and L. K. Sharma, Nig. J. of Math. Phys., **6**, 81 (2002).
[17] L. K. Sharma, J. O. Fiase and W. Whende, Chiang Mai J. Sc., **31**, 171 (2003).
[18] W,- M. Yao, J. Phys., **G33**, 1 (2006).
[19] E. Magyari, Phys. Lett., **B95**, 295 (1980).
[20] G. Feldman, T. Futon and A. Devoto, Nucl. Phys., **B154**, 441 (1979).
[21] Van Royen and E. Weisskopf, Nuovo Cim., **50**, 617 (1967).
[22] L. I. Schiff, *Quantum Mechanics*, 3rd ed., Chap. 13 (1968).
[23] H. Crater and P. V. Alstine, Phys. Lett., **B100**, 166 (1981).
[24] S. N. Jena, P. Panda and T. C. Tripathy, Jounl. Phys., **G27**, 227 (2001).