Space-time trajectory approach in nuclear relativistic Coulomb excitation and its recoil correction

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We implement a space-time trajectory approach to study the relativistic Coulomb excitation of the giant dipole resonance and double giant dipole resonance in nuclei. The results of calculation are compared with some experimental results (~700MeV). Besides that we will extend the space-time trajectory approach to the intermediate and low energy Coulomb excitation by including recoil effects which start to become significant at this range of energy.

I. INTRODUCTION

Coulomb excitation is a process that occurs when the electromagnetic field of, say, a projectile nucleus induces an inelastic excitation in a target nucleus. The Coulomb excitation process induced by the collisions of heavy-ions has been studied both experimentally and theoretically for over half a century [1]. With the advent of higher energy accelerators, the large relative velocity of the ions necessitates the use of a relativistic treatment [2,3]. In addition, the retardation of the electromagnetic field of the relativistic nuclei greatly enhances the excitation energy induced in the target nuclei. An interesting phenomenon that can happen in the nucleus is the giant dipole resonance (GDR) excitation. The GDR state is recognized as a collective excitation that is caused by the electric dipole field ($E1$) of the projectile.

A theory of relativistic Coulomb excitation was first formulated by Winther and Alder [2]. In this formalism, the trajectory of the projectile nucleus is approximated by a classical path while the internal degrees of freedom of the target nucleus are treated quantum-mechanically. The target nucleus can be taken to be stationary since the projectile nucleus moves at high speed. In this approach, the Schrödinger equation is solved in the frequency domain, ω, by first-order perturbation theory to obtain the excitation amplitude. A full description involves a multipole expansion of the interacting potentials in terms of Fourier transforms of the time-dependence of the retarded electromagnetic coupling. However, this approach entails serious numerical difficulties, especially if the excitation has to be expressed in terms of a multi-step description. Alternatively, Bertulani *et al.* [4] have developed a formalism by which the excitation amplitude may be calculated from a coupled-channel equation that is solved in the time domain, t. Compared with the previous formalism the latter method can give a better approximation to the excitation probabilities. Recently, Dasso *et al.* [5] have applied the time-representation to the relativistic Coulomb excitation and modeled the Coulomb excitation by a three-dimensional spring joining the centres of mass of the Z_T protons and N_T neutrons of the target nucleus.

In this paper, we will present the formulation of the coupled-channel approach as well as the space-time trajectory approach. Beside this we will also discuss the recoil correction to the classical spring model and obtain its expression in the intermediate energy region where the projectile path is no longer a straight line. Using the two approaches above, we studied a particular case of the Coulomb excitation in a projectile nucleus ^{136}Xe incident on ^{208}Pb target at E_{lab} = 690 MeV/A, which has been measured by experiment [6].

II. COUPLED-CHANNEL APPROACH

Using quantum mechanics the time-dependent Schrödinger equation can be written, in the coupled-channel description, following Bertulani *et al*. [4], as

$$i\hbar \frac{da_n}{dt} = \sum_n e^{(i/\hbar)(E_n - E_m)} V_{nm}(t) a_m(t) \,. \tag{1}$$

Here, excitation amplitudes of the electromagnetic interaction are a_n where n represent the states with energies, E_n. Further, $V_{nm}(t)$ are the time-dependent matrix elements for the electromagnetic excitation

$$V_{nm}(t) = \left\langle I_n M_n \left| \left[\rho(\vec{r}\,') - \frac{\vec{v}}{c^2} \cdot \vec{J}(\vec{r}\,') \right] \phi(\vec{r}\,', t) \right| I_m M_m \right\rangle , \tag{2}$$

from the initial state $\left| I_m M_m \right\rangle$ to the final state $\left| I_n M_n \right\rangle$. The nuclear states are specified by the spin quantum numbers I and the corresponding magnetic quantum numbers M, respectively. The interaction in Eq. (2) can be written as

$$V_{nm}^{E1}(t)=\sqrt{\frac{2\pi}{3}}\,\gamma\{\xi_1(\tau)[M_{nm}(E1,-1)-M_{nm}(E1,1)] + \sqrt{2}\,\gamma\tau\left[\xi_1(\tau)-i\frac{\omega v}{c^2\gamma}\times(1+\tau^2)\xi_2(\tau)\right]M_{nm}(E1,0)\}, \tag{3}$$

where $\tau=\gamma vt/b$, while $\xi_1(\tau)=\frac{Z_T e}{b^2\left[1+\tau^2\right]^{3/2}}$ and $\xi_2=\frac{Z_T e\tau}{b\left[1+\tau^2\right]^{3/2}}$ are the transverse and longitudinal electric fields respectively generated at the target nucleus with charge $Z_T e$; $M_{nm}(E1,\mu)=\langle I_n M_n\,|\,M(E1,-1)\,|\,I_m M_m\rangle$ where $\mu=-1$, 0, or 1, with $M(E1,\mu)$ denoting the usual electric dipole moment defined as

$$M(E1,\mu)=\int d^3r'\rho(r')rY_{1\mu}(r'). \tag{4}$$

By inserting Eq. (3) into the coupled-channel equation, and by numerical calculation, the amplitude of *E*1 excitation, a_{1st}, can be calculated.

III. SPACE-TIME TRAJECTORY APPROACH

As mentioned in the introduction above, Dasso *et al.* [5] have modeled the Coulomb excitation in the target nucleus by a three-dimensional spring connecting the center of mass of N_T neutrons to the center of mass of Z_T protons. For simplicity, the projectile nucleus is taken to be a point charge $Z_P e$ moving in the *y*-direction with velocity v_p and producing a retarded electromagnetic field on the target charge as

$$E_x(t)=\frac{Z_P e\gamma b}{(b^2+\gamma^2 v_P^2 t^2)^{3/2}},\ E_y(t)=\frac{Z_P e\gamma v_P t}{(b^2+\gamma^2 v_P^2 t^2)^{3/2}} \tag{5}$$

and $B_z(t)=\frac{v_P}{c^2}E_x(t)$.

The spring has zero elongation initially. By assuming that the target center of mass is at rest, the equation of motion for the vector $\vec{r}(t)\equiv(x(t),y(t),0)$ representing the displacement between protons and neutrons as a function of time is

$$D\frac{d^2\vec{r}(t)}{dt^2}+C\vec{r}(t)=Z_T e\delta\left[\vec{E}(t)+\delta\frac{d\vec{r}}{dt}\times\vec{B}(t)\right]. \tag{6}$$

Here,

$$D=\frac{N_T Z_T}{N_T+Z_T}m \text{ and } C=D\omega^2 \tag{7}$$

are, respectively, the mass and restoring force parameters corresponding to the effective one-body problem associated with the mode and *m* is the mass of a nucleon. The factor $\delta=N_T/(N_T+Z_T)$ gives the scale between the actual displacement in space of the center of the charge and the relative coordinate, $\vec{r}$. The electromagnetic field as expressed in Eq. (6) felt by the target nucleus is only appreciable in a time interval of which is in the order of Δt, where $\Delta t\approx b/(\gamma v)$.

As the projectile speed increases, the field strength is further enhanced due to Lorentz contraction effect although the time interval Δt decreases. Therefore in the calculation only the effect of the electromagnetic field in the order of this time interval needs to be considered. Once the effects of the driving forces in Eq. (6) cease to exist the harmonic mode reaches an asymptotic state with excitation energy E_∞. In this situation, the r.h.s. of Eq. (6) becomes zero, and the total energy is

$$E_\infty=\frac{1}{2}C\left|\vec{r}_\infty\right|^2+\frac{1}{2}D\left|\frac{d\vec{r}_\infty}{dt}\right|^2. \tag{8}$$

Here, $\vec{r}_\infty$ and its time derivative denote the displacement and velocity of the spring-mass system. The energy E_∞ can be converted into an average number of phonons $N_\infty=E_\infty/\hbar\omega$. Following Dasso *et al.* [5] we neglect the magnetic field term $B_z(t)$ in Eq. (6) since it is much weaker than $E_x(t)$. N_∞ is interpreted as the excitation probability.

IV. DEFLECTED PATH OF THE PROJECTILE

In the two approaches above the projectile is approximated as moving in a straight line. This approximation is valid in the high region when the interaction does not involve strong interaction. However in the intermediate energy regime, consideration of the deflected path of the projectile, which is known to be a hyperbolic curve, is important [7,8]. The equation of motion can be expressed using the parameter χ as given in Ref. [9] as

$$r(\chi)=\frac{a_0}{\gamma}(\varepsilon\cosh\chi+1),\ t(\chi)=\frac{a_0}{\gamma v}(\varepsilon\sinh\chi+\chi),$$
$$x=\frac{a_0}{\gamma}(\cosh\chi+\varepsilon) \text{ and } r(\chi)=\frac{a_0}{\gamma}\sqrt{\varepsilon^2-1}\sinh\chi. \tag{9}$$

Here, a_0 denotes the half-distance of closest approach for a head-on collision and ε represents the eccentricity of the hyperbola. By using the parameterization above and Eq. (8), the excitation probability can be shown to be

$$P = \frac{E_\infty}{\hbar\omega} = \frac{Z_T^2 e^2 \delta^2 \pi}{2D\hbar\omega} \left| E(\omega_0) \right|^2 , \tag{10}$$

where

$$E(\omega_0) = \frac{Z_p e}{a_0} \int_{-\infty}^{\infty} e^{i\xi(\varepsilon \sinh\chi + \chi)} \times \frac{\cosh\chi + \varepsilon + i(\varepsilon^2 - 1)^{\frac{1}{2}} \sinh\chi}{(\varepsilon\cosh\chi + 1)^2} , \tag{11}$$

and

$$\xi = \frac{\omega a_0}{\gamma v} . \tag{12}$$

This expression in Eq. (10) above is equivalent to the non-relativistic calculations of the dipole Coulomb excitation obtained by using a semi-classical approach in Ref. [1] except that, in the non-relativistic case, $\gamma = 1$.

In the first approximation of a straight line, $x \approx \gamma vt$, the equation of motion in Eq. (6) yields

$$y = \sqrt{\frac{(\gamma vt)^2}{(\varepsilon^2 + 1)} + (\frac{a_0}{\gamma})^2} + \varepsilon(\frac{a_0}{\gamma}) \approx b + \frac{a_0}{2\gamma} . \tag{13}$$

This mean for high energy collisions the recoil correction can be taken in account by replacing the impact parameter b with the term $b + \frac{a_0}{2\gamma}$.

V. RESULTS AND CALCULATION

We applied the two approaches above to study the case where ^{136}Xe is incident on ^{208}Pb at E_{lab} = 690 MeV/A. The experimental dipole excitation cross-section is 1485±100 mb [6]. The results of the dipole excitation cross-section calculation are given in the table below:

Spacetime trajectory approach	Semi-classical approach
1440 mb	1390 mb

In the calculations, the closest distance between the two nuclei is taken to be 13 fm, based on the prescription $R = 1.5(A_p^{1/3} + A_T^{1/3})$ fm. The results show that both the spacetime trajectory and the semi-classical approaches are able to reproduce the experimental dipole excitation cross-section to within the experimental error. The spacetime trajectory approach yields results that are in quantitative agreement with a full quantum-mechanical treatment. Hence, the accuracy of the spacetime trajectory approach as well as its computational simplicity is expected to lead to its adoption for dipole Coulomb excitation calculations.

VI. CONCLUSION

From the equation of motion that takes into account the recoil of the projectile we have shown that the expressions obtained, Eqs. (10-12), agree with those given in the non-relativistic semi-classical approach as $v/c \to 0$. We have also shown that the recoil correction as derived from the hyperbolic curve can be taken into account by replacing the impact parameter b with $b + \frac{a_0}{2\gamma}$. Finally our calculation shows that the space-time trajectory approach gives a result that is in close agreement with experimental measurements.

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