Shock waves in plasmas with charge fluctuating dust of opposite polarity

S. K. Paul[a], G. Mandal[b], A. A. Mamun[c] and M. R. Amin[d]
[a]Department of Mathematics, The University of Asia Pacific, Dhanmondi, Dhaka, Bangladesh
[b,d]Department of Electronics & Communications Engineering, East West University, Mohakhali, Dhaka 1212, Bangladesh
[c]Department of Physics, Jahangirnagar University, Savar, Dhaka 1342, Bangladesh

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A dusty plasma system containing Boltzmann-distributed electrons and ions, mobile charge fluctuating positive dust and charge fluctuating stationary negative dust has been considered. The nonlinear propagation of the dust-acoustic (DA) waves in such a dusty plasma has been investigated by employing the reductive perturbation method. It has been found that the dust charge fluctuation is a source of dissipation, and is responsible for the formation of DA shock waves in such a dusty plasma. The basic features of the DA shock waves have been identified in this investigation which could be useful in understanding the properties of localized space dusty plasmas. It has been proposed to design a new laboratory experiment, which will be able to identify the basic features of the dust-acoustic shock waves predicted in this theoretical investigation.

I. INTRODUCTION

The wave propagation in dusty plasmas has received much attention in the recent years because of its vital role in understanding different types of collective processes in space environments, namely, lower and upper mesosphere, cometary tails, planetary rings, planetary magnetosphere, interplanetary spaces, interstellar media, etc. [1-6]. The dusty plasmas have also noticeable applications in laboratory devices [7-10]. The consideration of charge dust grains in plasmas does not only modify the existing plasma wave spectra [11-13], but also introduces a number of novel eigenmodes, such as the dust ion-acoustic (DIA) waves, the dust-acoustic (DA) waves, the dust lower-hybrid (DLH) waves, the dust lattice (DL) waves, etc. [14-18]. Most of the studies in dusty plasmas have been confined in considering the dust as negatively charged grains in addition to electrons and positively charged ions as the plasma species [4-6], [19-24]. It has been found that there are some plasma systems, particularly in space plasma environments, namely, cometary tails [1-3,25,26], upper mesosphere [27], Jupiter's magnetosphere [28], etc. where positively charged dust grains play significant roles.

There are basically three mechanisms by which the dust grains in the plasma systems mentioned above can be positively charged. These mechanisms are the following: (i) photo emission in the presence of a flux of ultraviolet (UV) photons; (ii) thermionic emission induced by radiative heating; and (iii) secondary emission of electrons from the surface of the dust grains. Mamun and Shukla [29] have considered a very simple dusty plasma containing positively and negatively charged dust and studied non linear properties of DA solitary waves. The simple dusty plasma model of Mamun and Shukla [29] is only valid if a complete depletion of background electrons and ions is possible. Recently Sayeed and Mamun [30] generalized the work of Mamun and Shukla [29] by including the effects of Maxwellian electrons and ions. It has been found that Mamun and Shukla [29] and Sayeed and Mamun [30] considered the plasma system with positive and negative dust and they considered dust of constant charge. Very recent researchers are paying more attention for investigating the non linear properties of DA waves by considering the charge fluctuation. It is found from the work of Mamun [31] and S. S. Duha and Mamun [32] that when charge fluctuation is considered in dusty plasmas then shock waves are generated.

In this paper, we have considered a dusty plasma containing mobile charge fluctuating positive dust, charge fluctuating stationary negative dust, Boltzmann-distributed electrons and ions, and have studied the nonlinear propagation of DA waves.

This paper is organized as follows. The basic equations describing our dusty plasma model are presented in Section II. The Burgers equation is derived in Section III. The basic features of the shock wave solution of the Burgers equation is analyzed in Section IV. Finally, a brief discussion is given in Section V.

II. GOVERNING EQUATIONS

We consider an unmagnetized collisionless dusty plasma system consisting of charge fluctuating positively charged mobile dust, charge fluctuating stationary negative dust and Boltzmann-distributed electrons and ions. Thus at equilibrium, we have $n_{eo} + z_{do}^{-} n_{do}^{-} = n_{io} + z_{do}^{+} n_{do}^{+}$ where n_{eo} (n_{io}) is the equilibrium electron (ion) number density, n_{do}^{-} is the negative dust number density, n_{do}^{+} is the positive dust number density, z_{do}^{+} is the equilibrium charge state of the positive dust component, z_{do}^{-} is the equilibrium charge state of the negative dust component. The dynamics of the DA waves of such a dusty plasma system in one-dimensional form is given by

$$\frac{\partial n_d^+}{\partial t}+\frac{\partial}{\partial x}(n_d^+ u_d^+)=0\,, \tag{1}$$

$$\frac{\partial u_d^+}{\partial t}+u_d^+\frac{\partial u_d^+}{\partial x}=-\frac{z_d^+ e}{m_d}\frac{\partial \varphi}{\partial x}\,, \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2}=4\pi e\left[n_e-n_i-z_d^+ n_d^+ + z_d^- n_d^-\right], \tag{3}$$

where n_j is the number density of the plasma species j (j equals i for ions, e for electrons), $n_d^+(n_d^-)$ is the number density of positive (negative) dust. u_d^+ is the positive dust fluid speed. $z_d^+(z_d^-)$ is the charge state of the positive (negative) dust component, φ is the electrostatic wave potential. The electron and the ion densities are assumed to follow the Boltzmann distribution:

$$n_e=n_{eo}\exp\left(\frac{e\varphi}{k_B T_e}\right), \tag{4}$$

$$n_i=n_{io}\exp\left(-\frac{e\varphi}{k_B T_i}\right), \tag{5}$$

where k_B is the Boltzmann constant and T_e is the electron temperature, T_i is the ion temperature. We assume that dust is charged by photo-emission current $\left(I_P^+\right)$, the thermionic emission current $\left(I_t^+\right)$ and the electron absorption current $\left(I^-\right)$ for the positive charged dust while the electron current $\left(I_e\right)$ and the ion current $\left(I_i\right)$ for negative charged dust only. All other charging processes are neglected. The charge state z_d^+ and z_d^- components are not constant, but vary according to the following equations:

$$\frac{\partial z_d^+}{\partial t}+u_d^+\frac{\partial z_d^+}{\partial t}=\frac{I_P^+ + I_t^+ + I^-}{e}c\,, \tag{6}$$

$$\frac{\partial z_d^-}{\partial t}=-\left(\frac{I_e+I_i}{e}\right), \tag{7}$$

where

$$I_p^+=\pi r_d^2 e\, JY\exp\left(-\frac{z_d^+ e^2}{k_B r_d T_{ph}}\right), \tag{8}$$

$$I_t^+ = 2\pi r_d^2 e\left(\frac{m_e k_B T_P}{2\pi\hbar^2}\right)^{\frac{3}{2}}\left(\frac{8k_B T_P}{\pi m_e}\right)^{\frac{1}{2}}\left(1+\frac{z_d^+ e^2}{k_B r_d T_p}\right) \times\exp\left(-\frac{z_d^+ e^2}{k_B r_d T_p}-\frac{W_e}{k_B T_P}\right), \tag{9}$$

$$I^- = -\pi r_d^2 e\, n_{eo}\, e^{\frac{e\varphi}{k_B T_P}}\left(\frac{8k_B T_P}{\pi m_e}\right)^{\frac{1}{2}}\left(1+\frac{z_d^+ e^2}{k_B r_d T_p}\right), \tag{10}$$

$$I_e=-4\pi r_d^2\, n_{eo} e^{\frac{e\varphi}{k_B T_e}} e\left(\frac{k_B T_e}{2\pi m_e}\right)^{\frac{1}{2}}\exp\left(-\frac{z_d^- e^2}{k_B r_d T_e}\right), \tag{11}$$

$$I_i=4\pi r_d^2\, n_{io} e^{-\frac{e\varphi}{k_B T_i}} e\left(\frac{k_B T_i}{2\pi m_i}\right)^{\frac{1}{2}}\left(1+\frac{z_d^- e^2}{k_B r_d T_i}\right). \tag{12}$$

where $\hbar$ is the Planck's constant, T_{ph} is the photon temperature, W_e is the work function, J is the UV photon flux, Y is the yield of photons (typical values of W_e, J and Y are 2.2 eV, 5.0×10^{14} photons/cm^2/s, and 0.1, respectively), and r_d is the dust radius. Introducing the following normalized variables:

$$N_d^+ = n_d^+/n_{do}^+\,,\ U_d^+ = u_d^+/C_d\,,\ \Phi = e\phi/k_B T_e\,,$$
$$Z_d^- = z_d^-/z_{do}^-\,,\ Z_d^+ = z_d^+/z_d^+\,,\ Z_d^- = z_d^-/z_{do}^-\,,\ X = x/\lambda_{Dd}\,,$$
$$T = t\,\omega_{pd}\,,\ \lambda_{Dd}=\left(k_B T_e/4\pi z_{do}^+ n_{do}^+ e^2\right)^{1/2},$$
$$C_d=\left(z_{do}^+ k_B T_e/m_d\right)^{1/2}\ \text{and}\ \omega_{pd}=\left(4\pi z_{do}^{+\,2} n_{do}^+ e^2/m_d\right)^{1/2},$$

one can reduce Eq. (1) to Eq. (7) as

$$\frac{\partial N_d^+}{\partial T}+\frac{\partial}{\partial X}\left(N_d^+ U_d^+\right)=0\,, \tag{13}$$

$$\frac{\partial U_d^+}{\partial T}+U_d^+\frac{\partial U_d^+}{\partial X}=-Z_d^+\frac{\partial \Phi}{\partial X}\,, \tag{14}$$

$$\frac{\partial^2 \Phi}{\partial X^2}=\left(1+\mu_i-\gamma\right)e^{\Phi}-\mu_i e^{-\sigma\Phi}-Z_d^+ N_d^+ + \gamma Z_d^-\,, \tag{15}$$

$$\frac{\partial Z_d^+}{\partial T}+U_d^+\frac{\partial Z_d^+}{\partial X} =\mu^+\left[Pe^{-\alpha Z_d^+}+Q\left(1+\beta Z_d^+\right)e^{-\beta Z_d^+}-\mathrm{R}e^{\Phi}\left(1+\beta Z_d^+\right)\right], \tag{16}$$

$$\frac{\partial Z_d^-}{\partial T} = \mu^- \left[X_e e^{\Phi - \alpha_e Z_d^-} - X_i \left(1+\alpha_i Z_d^-\right) e^{-\sigma\Phi} \right], \tag{17}$$

where

$$\mu^- = \frac{4\pi r_d^2}{z_{do}^-\omega_{pd}}, \quad \mu^+ = \frac{\pi r_d^2}{z_{do}^+\omega_{pd}}, \quad \alpha_e = \frac{z_{do}^- e^2}{r_d k_B T_e}, \quad \alpha_i = \frac{z_{do}^- e^2}{r_d k_B T_i},$$

$$\alpha = \frac{z_{do}^+ e^2}{r_d k_B T_{ph}}, \quad \beta = \frac{z_{do}^+ e^2}{r_d k_B T_p}, \quad \gamma = \frac{z_{do}^- n_{do}^-}{z_{do}^+ n_{do}^+}, \quad \mu_e = \frac{n_{eo}}{z_{do}^+ n_{do}^+},$$

$$\mu_i = \frac{n_{io}}{z_{do}^+ n_{do}^+}, \quad \sigma = \frac{T_e}{T_i}, \quad X_e = n_{eo}\left(\frac{k_B T_e}{2\pi m_e}\right)^{1/2},$$

$$X_i = n_{io}\left(\frac{k_B T_i}{2\pi m_i}\right)^{1/2}, \quad P = YJ,$$

$$Q = 2\left(\frac{m_e k_B T_P}{2\pi\hbar^2}\right)^{3/2}\left(\frac{8k_B T_P}{\pi m_e}\right)^{1/2} e^{\frac{-W_e}{k_B T_P}}, \quad R = n_{eo}\left(\frac{8k_B T_p}{\pi m_e}\right)^{1/2}.$$

III. DERIVATION OF BURGERS EQUATION

To derive a dynamical equation for the nonlinear propagation of the DA shock waves in a dusty plasma, we use Eqs. (13)-(17) and employ the reductive perturbation technique (RPT) [33]. We introduce the stretched coordinates [34]:

$$\xi = \in\left(X - V_0 T\right), \tag{18}$$

$$\tau = \in^2 T, \tag{19}$$

where $\in$ is a smallness parameter $(0 < \in < 1)$ measuring the weakness of the dispersion and V_0 is the Mach number (the phase speed of DA shock waves normalized by C_d). We expand N_d^+, U_d^+, Φ, Z_d^+ and Z_d^- about their equilibrium values in power series of $\in$

$$N_d^+ = 1 + \in N_d^{+(1)} + \in^2 N_d^{+(2)} + \cdots, \tag{20}$$

$$U_d^+ = \in U_d^{+(1)} + \in^2 U_d^{+(2)} + \cdots, \tag{21}$$

$$\Phi = \in \Phi^{(1)} + \in^2 \Phi^{(2)} + \cdots, \tag{22}$$

$$Z_d^+ = 1 + \in Z_d^{+(1)} + \in^2 Z_d^{+(2)} + \cdots, \tag{23}$$

$$Z_d^- = 1 + \in Z_d^{-(1)} + \in^2 Z_d^{-(2)} + \cdots. \tag{24}$$

Now, substituting Eqs. (18)-(24) into Eqs. (13)-(17), we develop equations in various power of $\in$. We have for the lowest order of $\in$:

$$U_d^{+(1)} = \frac{\Phi^{(1)}}{V_0}, \tag{25}$$

$$N_d^{+(1)} = \frac{\Phi^{(1)}}{V_0^2}, \tag{26}$$

$$Z_d^{+(1)} = f\Phi^{(1)}, \tag{27}$$

$$Z_d^{-(1)} = \rho\Phi^{(1)}, \tag{28}$$

$$V_0^2 = \frac{1}{(1+\mu_i-\gamma) + \sigma\mu_i - f + \gamma\rho}, \tag{29}$$

where

$$f = \frac{R(1+\beta)}{P\alpha^2 - P\alpha + \frac{3}{2}Q\beta^3 - Q\beta^2 - R\beta},$$

and

$$\rho = \frac{X_e - X_e\alpha_e + X_e\alpha_e^2 + X_i\sigma + X_i\sigma\alpha_i}{X_e\alpha_e + X_i\alpha_i - \frac{1}{2}X_e\alpha_e^2}.$$

Eq. (29) represents the linear dispersion relation for the DA waves which is significantly modified by the presence of charge fluctuating positive dust. It may be noted here that the dust charge fluctuation, which is due to the absorption of plasma particles (electrons and ions) by the dust, can lead to the damping of the linear DA waves [35]. One can show by the linear mode analysis of the DA waves that the dust charge fluctuation due to photoemission and thermionic emission also leads to the damping of the DA waves. However, the detailed linear mode analysis of the DA waves in our case is beyond the scope of this paper.

To the next higher order of $\in$, one obtains

$$\frac{\partial N_d^{+(1)}}{\partial\tau} - \frac{\partial}{\partial\xi}\left(N_d^{+(1)}U_d^{+(1)}\right) - V_0\frac{\partial N_d^{+(2)}}{\partial\xi} + \frac{\partial U_d^{+(2)}}{\partial\xi} = 0, \tag{30}$$

$$\frac{\partial U_d^{+(1)}}{\partial\tau} - V_0\frac{\partial U_d^{+(2)}}{\partial\xi} + U_d^{+(1)}\frac{\partial U_d^{+(1)}}{\partial\xi} = -\frac{\partial\Phi^{(2)}}{\partial\xi} - Z_d^{+(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}, \tag{31}$$

$$\left(1+\mu_i-\gamma\right)\Phi^{(2)} + \frac{1}{2}\left(1+\mu_i-\gamma\right)\left[\Phi^{(1)}\right]^2 + \mu_i\sigma\Phi^{(2)} - \frac{1}{2}\sigma^2\mu_i\left[\Phi^{(1)}\right]^2 = Z_d^{+(1)}N_d^{+(1)} + Z_d^{+(2)} + N_d^{+(2)} - \gamma Z_d^{-(2)}, \tag{32}$$

$$-V_0\frac{\partial Z_d^{+(1)}}{\partial\xi}=\mu^+\Bigg[Z_d^{+(2)}\left\{-P\alpha+P\alpha^2-Q\beta^2+\frac{3}{2}Q\beta^3-R\beta\right\}+\left\{\frac{1}{2}P\alpha^2-\frac{1}{2}Q\beta^2+\frac{3}{2}Q\beta^3\right\}[Z_d^{+(1)}]^2-R\beta Z_d^{+(1)}\Phi^{(1)}-R\Phi^{(2)}-R\beta\Phi^{(2)}-\frac{1}{2}R(\Phi^{(1)})^2-\frac{1}{2}R\beta(\Phi^{(1)})^2\Bigg], \tag{33}$$

$$-V_0\frac{\partial Z_d^{-(1)}}{\partial\xi}=\mu^-\Bigg[X_e\Phi^{(2)}+\frac{1}{2}X_e(\Phi^{(1)})^2-\alpha_eX_e\Phi^{(2)}-\frac{1}{2}\alpha_eX_e(\Phi^{(1)})^2-\alpha_eX_e\Phi^{(1)}Z_d^{-(1)}-\alpha_eX_eZ_d^{-(2)}+\frac{1}{2}\alpha_e^2X_e\Phi^{(2)}+\frac{1}{4}\alpha_e^2X_e(\Phi^{(1)})^2+\frac{1}{2}\alpha_e^2X_e(Z_d^{-(1)})^2+\alpha_e^2X_eZ_d^{-(1)}\Phi^{(1)}+\alpha_e^2X_e(Z_d^{-(2)})+X_i\sigma\Phi^{(2)}-\frac{1}{2}\sigma^2X_i(\Phi^{(1)})^2+\alpha_iX_i\sigma\Phi^{(2)}-\frac{1}{2}\sigma^2X_i\alpha_i(\Phi^{(1)})^2+\sigma X_i\alpha_i\Phi^{(1)}Z_d^{-(1)}-X_i\alpha_iZ_d^{-(2)}\Bigg]. \tag{34}$$

Now using Eqs. (30)-(34), one can eliminate $N_d^{+(2)}$, $U_d^{+(2)}$, $Z_d^{+(2)}$, $Z_d^{-(2)}$ and $\Phi^{(2)}$, and can finally obtain the following equation:

$$\frac{\partial\Phi^{(1)}}{\partial\tau}+A\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}=C\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}, \tag{35}$$

where the nonlinear coefficient A and the dissipation coefficient C are given by

$$A=\frac{B'}{A'}, \tag{36}$$

$$C=-\frac{C'}{A'}, \tag{37}$$

$$A'=-\frac{2}{V_0^3}, \tag{38}$$

$$B'=\left(1+\mu_i-\gamma\right)-\sigma^2\mu_i-\frac{3f}{V_0^2}+\frac{f^2\left(P\alpha^2-Q\beta^2+\frac{3}{2}Q\beta^3\right)}{P\alpha^2-Q\beta^2+\frac{3}{2}Q\beta^3-P\alpha-R\beta}-\frac{2R\beta f+R+R\beta}{P\alpha^2-Q\beta^2+\frac{3}{2}Q\beta^3-P\alpha-R\beta}-\frac{3}{V_0^4}+\left(\frac{2\gamma}{\alpha_iX_i-X_e\alpha_e^2+X_e\alpha_e}\right)H, \tag{39}$$

$$H=\frac{1}{2}X_e-\frac{1}{2}X_e\alpha_e-X_e\alpha_e\rho+\frac{1}{2}\alpha_e^2X_e\rho^2+\alpha_e^2X_e\rho-\frac{1}{2}\sigma^2X_i+\frac{1}{4}\alpha_e^2X_e-\frac{1}{2}\sigma^2X_i\alpha_i+\sigma^2X_i\alpha_i,$$

$$C'=\frac{V_0f}{\mu^+\left(P\alpha^2-Q\beta^2+\frac{3}{2}Q\beta^3-P\alpha-R\beta\right)}+\frac{V_0\rho\gamma}{\mu^-\left(\alpha_iX_i-X_e\alpha_e^2+X_e\alpha_e\right)}. \tag{40}$$

Eq. (35) is the well-known Burgers equation describing the nonlinear propagation of the DA shock waves in the dusty plasma under consideration. It is obvious from Eqs. (35) and (37) that the dissipative term, i.e. the right-hand side of Eq. (35) is due to the presence of the charge fluctuating dust.

IV. SOLUTION OF THE BURGERS EQUATION

We are now interested in looking for the stationary shock wave solution of Eq. (35) by introducing the variables $\varsigma=\xi-U_0\tau'$ and $\tau'=\tau$, where U_0 is the shock wave speed (in the reference frame) normalized by C_d, ς is normalized by λ_{Dd}, and τ is normalized by ω_{pd}^{-1}. This leads us to write Eq. (35), under the steady state condition $(\partial/\partial\tau=0)$, as

$$-U_0\frac{\partial\Phi^{(1)}}{\partial\varsigma}+A\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\varsigma}=C\frac{\partial^2\Phi^{(1)}}{\partial\varsigma^2}. \tag{41}$$

It can be easily shown [36], [37] that Eq. (41) describes shock waves whose speed U_0 (in the reference frame) is related to the extreme values $\Phi^{(1)}(-\infty)$ and $\Phi^{(1)}(\infty)$ by $\Phi^{(1)}(-\infty)-\Phi^{(1)}(\infty)=2U_0/A$. Thus, under the condition that $\Phi^{(1)}$ is bounded at $\varsigma = \pm\infty$, the shock wave solution of Eq. (41) can be written as

$$\Phi^{(1)} = \Phi_0[1 - \tanh(\varsigma/\Delta)], \tag{42}$$

where

$$\Phi_0 = U_0 / A, \tag{43}$$

and

$$\Delta = 2C / U_0 c \tag{44}$$

are respectively, the height and thickness of the shock waves moving with the speed U_0. It is obvious from Eq. (41) to Eq. (44) that the shock waves are due to the presence of the charge fluctuating dust, and the shock structures are associated with the negative potential $(A < 0)$ as well as with positive potential $(A > 0)$. To find the parametric regimes for which positive and negative shock wave (potential) profiles exist, we have numerically analyzed A and obtain A = 0 (2-D) curves for γ = 0.2 to 0.6 and μ_i = 0 to 3.8. The A = 0 curve is shown in Fig. 1. It shows that we can have positive shock wave (potential) profiles for the parameters whose values lie above A = 0 curve and negative shock wave (potential) profiles for the parameters whose value lie below the A = 0 curve. These are shown in Figs. 2-3. Figs. 2 and 3 show the positive and negative shock potential profiles respectively.

V. DISCUSSION

We have studied the nonlinear propagation of DA waves in an unmagnetized dusty plasma containing Boltzmann-distributed electrons and ions, mobile charge fluctuating positive dust and charge fluctuating stationary negative dust. We have shown here how the basic features of the nonlinear DA waves are modified by the presence of the charge fluctuating dust in dusty plasmas. The results, which have been obtained from this investigation, can be summarized as follows:

The dust charge fluctuation is a source of dissipation and is responsible for the formation of DA shock waves in the dusty plasma. The shock structures are associated with the negative potential $(A < 0)$ as well as positive potential $(A < 0)$. It is shown that the height (normalized by $k_B T_e / e$) of the potential structures in the form of the shock waves is directly proportional to the shock speed U_0, and it is also found that the thickness (normalized by λ_{Dd}) of these shock structures is inversely proportional to the shock speed U_0.

The parametric regimes for the existence of positive as well as negative shock structures are shown in Fig. 1. Figs. 2 and 3 show the positive and negative shock potential profiles of shock waves respectively.

It is to be mentioned here that the parameters we have chosen in our numerical analysis are very much relevant to the plasma in the mesosphere [27]. We stress that the results of the present investigation could be useful in understanding the properties of localized DA waves of dusty plasmas in the mesosphere.

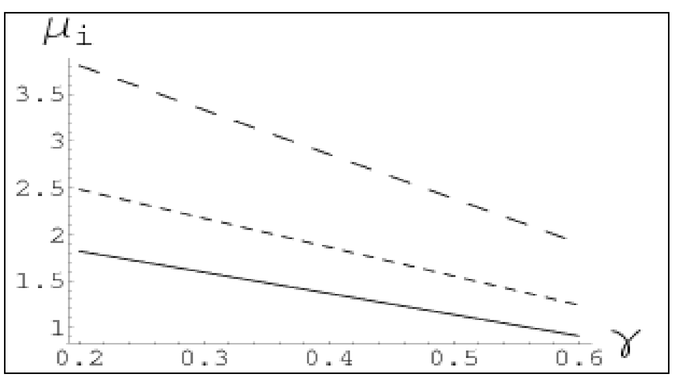

FIG. 1. Showing A = 0 (γ vs. μ_i) curves for the parameters $P = 5.0 \times 10^{13}$ cm^{-2}, $Q = 1.93 \times 10^{28}$ cm^{-2}s^{-1}, $R = 2.48 \times 10^{28}$ cm^{-2}s^{-1} with $\alpha_i = 4.77$, $\beta = 38.6$, $\sigma = 1.2$ (Solid Curve), $\sigma = 1.15$ (Dotted curve) and $\sigma = 1.1$ (Dashed curve).

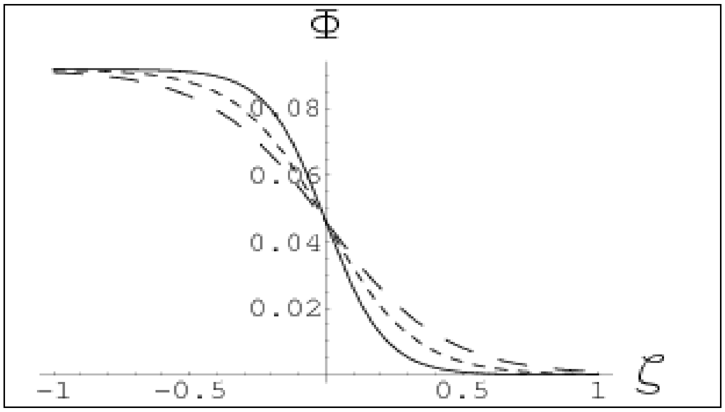

FIG. 2. Showing positive potential (Φ vs. ς) curves for the parameters $P = 5.0 \times 10^{13}$ cm^{-2}, $Q = 1.93 \times 10^{28}$ cm^{-2}s^{-1}, $R = 2.48 \times 10^{28}$ cm^{-2}s^{-1} with $\alpha_i = 4.77$, $\beta = 38.6$, $\mu_i = 4.5$ (Solid Curve). $\mu_i = 5.6$ (Dotted curve), $\mu_i = 7.5$ (Dashed curve).

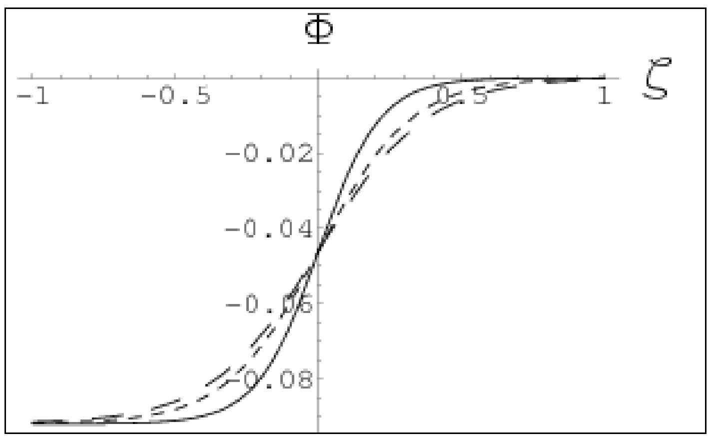

FIG. 3. Showing negative potential (Φ vs. ς) curves for the parameters $P = 5.0 \times 10^{13}$ cm^{-2}, $Q = 1.93 \times 10^{28}$ cm^{-2}s^{-1}, $R = 2.48 \times 10^{28}$ cm^{-2}s^{-1} with $\alpha_i = 4.77$, $\beta = 38.6$, $\mu_i = 0.5$ (Solid Curve). $\mu_i = 1.0$ (Dotted curve), $\mu_i = 1.3$ (Dashed curve).

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