Finding Casmir Force Klein-Gordon Field by Using Energy–Momentum Tensor

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(Received: 17 March 2014; published: 15 October 2014)

Abstract. In the present article we investigate Casimir force for field Klein-Gordon by using energy-momentum tensor that blades are considered non-penetration. Eigefunction was found by using border condition and solution of Klein-Gordon field Eventually by finding expectation value energy momentum Tensor inside and outside of blades that the difference between these amounts is Casimir force.

Keywords: Casmir force, Klein-Gordon field, energy – momentum tensor

I. INTRODUCTION

One of the inherent characteristics of the quantum zero point energy fields is the Hamiltonian of quantum fields put in zero point energy appears that has infinite energy, We present the theory using the Hamiltonian operator that involves procedures such as normal ordering [1]. In the world of modern physics with quantum field describing the quanta field of the fermion and boson(s), the square is the zero point energy in vacuum.[2]. To solve this issue and whether normal ordering of the zero point energy can be removed and the zero point quantum fluctuations in the field remains as energy is non trivial. Casimir energy of the zero point fluctuation can be achieved as shown by Henrik Cazimir on 29 May 1948, in his in influential article in the field of physics. In this article, from the standpoint of Casimir in this two-page statement for a non conductor and the neutral in charge instead of in free space, there is attraction force between each other [3]. Later, this force of attraction was called the energy resources by Casimir. Electrodynamics of conductive plates involves the explanation that is not classic, can be connected also to the force of gravity as the energy resources because the types of forces are are weaker than the strong nuclear force because the distance is in the macroscopic scale.

II. QUANTUM ELECTRODYNAMICS AND EXPLANATIONS OF CASIMIR EFFECT

The ideal conditions that Casimir has considered was whether there is any force for very, very small distance between two conductive plates at micrometer scale and at zero temperature (T = 0) in the space of two parallel plates where in this case there is just the basic mode or quantum electrodynamics vacuum [4]. The Casimir force for two conductive plates has been provided in the next book based on calculations he had provided.

$$F_c = \frac{\hbar c \pi^2}{240a^4} \tag{1}$$

Here, we calculate the Casimir energy by using the Energy-Momentum Tensor approach.

III. THE FIELD EQUATION OF KLEIN-GORDON

$$L\left(\varphi, \varphi^*, \frac{\partial\varphi}{\partial x^\mu}, \frac{\partial\varphi^*}{\partial x^\vartheta}\right) = \frac{\hbar^2}{2m_0} g^{\mu\vartheta} \frac{\partial\varphi^*}{\partial x^\vartheta}\frac{\partial\varphi}{\partial x^\mu} - \frac{m_0 c^2}{2}\varphi\varphi^* \tag{2}$$

$$g_{\mu\vartheta} = g^{\mu\vartheta} = \begin{Bmatrix} 1 \ \mu = \vartheta = 0 \\ -1 \ \mu = \vartheta \neq 0 \\ 0 \ \mu \neq \vartheta \end{Bmatrix}$$

$$\mu, \vartheta = 0,1,2, \dots$$

The corresponding canonical conjugate field Φ is

$$\pi(x,t) = \frac{\partial L}{\partial\dot{\varphi}(x,t)} = \frac{\hbar^2}{2m_0 c^2}\dot{\varphi}^*(x,t) \tag{3}$$

The commutation relation can be defined thus

$$[\hat{\varphi}(x,t), \hat{\pi}(x,t)] = i\hbar\delta(x - x') \tag{4}$$

$$[\hat{\varphi}(x,t), \hat{\varphi}(x',t)] = [\hat{\pi}(x,t), \hat{\pi}(x',t)] = 0 \tag{5}$$

The Hamiltonian can be as follows:

$$H = \pi\dot{\varphi} + \pi^*\dot{\varphi}^* - L \tag{6}$$

Placement (3) in (6) we have:

$$\hat{H} = \int \frac{\hbar^2}{2m_0}(\frac{1}{c^2}\dot{\hat{\varphi}}^\dagger\dot{\hat{\varphi}} + (\nabla\hat{\varphi}^\dagger).(\nabla\hat{\varphi}) + \frac{m_0^2 c^2}{\hbar^2}\hat{\varphi}^\dagger\hat{\varphi})d^3x \tag{7}$$

Changes in the field of time π (x, t) and φ (x, t) using the Heisenberg equation in this case would be:

$$\dot{\hat{\varphi}}(x,t) = \frac{1}{i\hbar}\left[\hat{\varphi}(x,t), \hat{H}\right] = \frac{2m_0 c^2}{\hbar^2}\hat{\pi}^*(x,t) \tag{8}$$

$$\dot{\hat{\pi}}(x,t) = \frac{1}{i\hbar}\left[\hat{\pi}(x,t),\hat{H}\right] = \left(\frac{\hbar^2}{2m_0}\nabla^2 - \frac{m_0c^2}{2}\right)\hat{\varphi}^*(x,t) \tag{9}$$

And finally, the field equations of Klein - Gordon was extract as follows:

$$\ddot{\hat{\varphi}}(x,t) = \left(c^2\nabla^2 - \frac{m_0^2c^4}{\hbar^2}\right)\hat{\varphi}(x,t) \tag{10}$$

IV. QUANTIZED OF KLEIN-GORDON FIELD

Klein – Gordon. Field in one dimension:

$$\left(\frac{\partial^2}{c^2\,\partial t^2} - \frac{\partial^2}{\partial z^2} + \frac{m_0^2c^4}{\hbar^2}\right)\hat{\varphi}(z,t) = 0 \tag{11}$$

Entities connected with particle physics φ (z, t) to make up for the expansion of

$$\hat{\varphi}(z,t) = \frac{1}{\sqrt{2\pi\hbar}}\int dp\ N_p\left(u\,\hat{a}_p(t) + \hat{a}_p^\dagger(t)u^\dagger\right) \tag{12}$$

$u_p(z) = \frac{1}{\sqrt{2\pi\hbar}}e^{i\frac{p}{\hbar}z}$ (13)

Using the relationships of displacement, the equation of motion for **a**:

$$\hat{a}_p(t) = \hat{a}_p(0)e^{\mp i\omega t} \tag{14}$$

$$\omega_p = \sqrt{\frac{p^2c^2}{\hbar^2} + \frac{m_0^2c^4}{\hbar^2}} \tag{15}$$

$$\ddot{\hat{a}}_p(t) = -\omega_p^2\hat{a}_p(t) \tag{16}$$

However, placing the relation (14) in (13) we have:

$$\hat{\varphi}(z,t) = \frac{1}{\sqrt{2\pi\hbar}}\int dpN_p(\hat{a}_p e^{i\left(\frac{p}{\hbar}z-\omega_p t\right)} + \hat{a}_p^\dagger\, e^{-i\left(\frac{p}{\hbar}z-\omega_p t\right)} \tag{17}$$

After being normalized by φ (z, t) we have:

$$\hat{\varphi}(z,t) = \int_{-\infty}^{+\infty} dp(\frac{m_0c^2}{4\pi\omega_p\hbar^2})^{1/2}\left[\hat{a}_p e^{i\left(\frac{p}{\hbar}z-\omega_p t\right)} + \hat{a}_p^\dagger e^{-i\left(\frac{p}{\hbar}z-\omega_p t\right)}\right] \tag{18}$$

Quantum field Klein - Gordon are certain special functions with the following audio to the normal coefficient will be:

$$\hat{\varphi}(z,t) = \int_{-\infty}^{+\infty} dp\,(\frac{m_0c^2}{4\pi\omega_p\hbar^2})^{1/2}\left[\hat{a}_p(t)u_p(z) + \hat{a}_p^\dagger(t)u_p^*(z)\right] \tag{19}$$

Using the above relationships and relationships to transiting Hamiltonian can be thus wrote:

$$\widehat{H} = \int_{-\infty}^{+\infty} dp\hbar\omega_p(\hat{a}_p^\dagger \hat{a}_p + \tfrac{1}{2}) = \int_{-\infty}^{+\infty} dp\hbar\omega_p(\hat{n}_p + \tfrac{1}{2}) \tag{20}$$

$$n_p = \hat{a}_p^\dagger \hat{a}_p$$

$$n = 0,1,2, \dots$$

Field Hamiltonian Klein - Gordon mathematically equivalent to a simple harmonic oscillator Hamiltonian is set [5].

V. CALCULATED TENSOR ENERGY-MOMENTUM QUANTUM FIELD KLEIN-GORDON

We use the invariance of Lagrangian under the transformation to calculate the energy tensor - the momentum. Using the field Klein - Gordon Lagrangian and the insertion of quantum field instead of the classical field, the tensor energy - momentum quantum field Klein - Gordon can be calculated as follows:

$$T_\vartheta^\mu = \frac{\hbar^2}{2m_0}\left[g^{\sigma\mu}\frac{\partial\varphi^*}{\partial x^\sigma}\frac{\partial\varphi}{\partial x^\vartheta} + g^{\sigma\mu}\frac{\partial\varphi^*}{\partial x^\vartheta}\frac{\partial\varphi}{\partial x^\sigma} - (g^{\sigma l}\frac{\partial\varphi^*}{\partial x^\sigma}\frac{\partial\varphi}{\partial x^l} - \frac{m_0^2c^2}{\hbar^2}\varphi\varphi^*)g_\vartheta^\mu\right] \tag{21}$$

Energy-momentum tensor in one dimension to form would be the following:

$$T_{33} = \frac{\hbar^2}{2m_0}\left[\frac{\partial\varphi^*}{\partial x^3}\frac{\partial\varphi}{\partial x^3} + \frac{\partial\varphi^*}{\partial x^0}\frac{\partial\varphi}{\partial x^0} - \frac{m_0^2c^2}{\hbar^2}\varphi\varphi^*\right] \tag{22}$$

Now, it can be compiled on a force level, using the relationship between $\vec{f}_\theta = \vec{T}_\theta^\mu \overrightarrow{ds_\mu}$ was calculated,

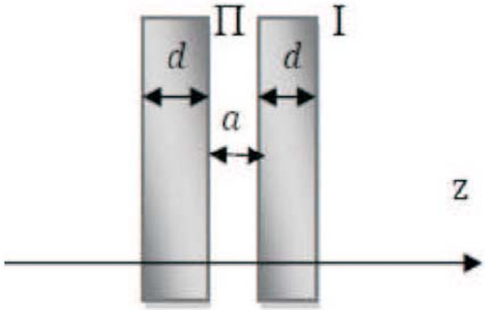

FIGURE 1. Schematic for parallel plates.

VI. CALCULATE THE CASIMIR FORCE BETWEEN TWO BLADES IN ONE DIMENSION

To calculate the Casimir force by simulating the infinite symmetric potential well of the walls of the well thickness d of infinite potential and the potential is zero outside and inside walls we refer to Fig. 1.

$$V(z) = 0 \quad z < -\frac{a}{2} - d \ , -\frac{a}{2} < z < \frac{a}{2} \ , d + \frac{a}{2} < z$$

$$V(z) = \infty \quad -\frac{a}{2} > z > -\frac{a}{2} - d \ , \frac{a}{2} < z < \frac{a}{2} + d$$

The following function is a special case in the region I

$$u_p(z) = N_p e^{i\frac{p}{\hbar}x} + N'_p e^{-i\frac{p}{\hbar}x} \tag{23}$$

By applying the boundary conditions $u\left(\frac{a}{2} + d\right) = 0$

$$u_p(z) = \frac{1}{\sqrt{4\pi\hbar}} (e^{i\frac{p}{\hbar}z} - e^{2i\frac{p}{\hbar}\left(\frac{a}{2}+d\right)} e^{-i\frac{p}{\hbar}z}) \tag{24}$$

As a result, the field φ (z, t) will be as follows:

$$\hat{\varphi}(z,t) = \int_{-\infty}^{+\infty} dp \left(\frac{m_0^2 c^2}{8\pi\hbar^2 \omega_p}\right)^{1/2} \times \begin{bmatrix} \left(e^{i\frac{p}{\hbar}z} - e^{2i\frac{p}{\hbar}\left(\frac{a}{2}+d\right)} e^{-i\frac{p}{\hbar}z}\right) \hat{a}_p e^{-i\omega_p t} \\ +(e^{-i\frac{p}{\hbar}z} - e^{-2i\frac{p}{\hbar}\left(\frac{a}{2}+d\right)} e^{i\frac{p}{\hbar}z} \end{bmatrix} \tag{25}$$

In region II, along with the boundary conditionsu $\left(\frac{a}{2}\right) = 0$ and $u\left(-\frac{a}{2}\right) = 0$

$$u_p^-(z) = \sqrt{\frac{2}{a}} \sin\frac{p}{\hbar} z \ , p = \frac{n\pi\hbar}{a} \ , \ n = \cdots, -2, 0, 2, \ldots \tag{26}$$

Thus the field φ (z, t) in region 2 will [6]

$$\hat{\varphi}(z,t) = \left(\frac{m_0 c^2}{a\hbar}\right)^{1/2} \left[\sum_{\text{even n}} \frac{1}{\sqrt{\omega_p}} \sin\frac{p}{\hbar} z(\hat{a}_{pn} e^{-i\omega_p t} + \hat{a}_{pn} e^{+i\omega_p t}) + \sum_{\text{odd n}} \frac{1}{\sqrt{\omega_p}} \cos\frac{p}{\hbar} z(\hat{a}_{pn} e^{-i\omega_p t} + \hat{a}_{pn} e^{+i\omega_p t})\right] \tag{27}$$

After evaluating the Tzz (energy-momentum tensor) calculated at both regions, the Casimir force can be found using the following relationship obtains

$$F_c = \left\langle 0 \middle| \hat{T}_{zz}^{\Pi}(x = \frac{a}{2}) \middle| 0 \right\rangle - \left\langle 0 \middle| \hat{T}_{zz}^{I}(x = \frac{a}{2} + d) \middle| 0 \right\rangle \tag{28}$$

After doing a series of calculations and simplify the following relationship is obtained:

$$F_c = \frac{c}{2a\hbar}\sum_0^\infty \frac{p^2}{\sqrt{\frac{p^2c^2}{\hbar^2}+\frac{m_0^2c^4}{\hbar^2}}} - \frac{c^2}{2\pi\hbar^2}\int_0^\infty dp \frac{p^2}{\sqrt{\frac{p^2c^2}{\hbar^2}+\frac{m_0^2c^4}{\hbar^2}}} \tag{29}$$

Because the terms are quite ideal, we have considered the value is infinity fc. We need the introduction of cut off function (closer to the ideal conditions). If intended, the actual situation can be fix without ambiguity. Now, if the cut off function is:

$$X(\omega_p) = e^{-\lambda\omega_p\frac{\hbar}{c}} = e^{-\lambda\sqrt{\left(\frac{n\pi\hbar}{a}\right)^2+m_0^2c^2}} \tag{30}$$

$$X(\omega_p) = \begin{cases} 1 & \omega_p < \omega_c \\ 0 & \omega_p > \omega_c \end{cases}$$

Equation (29) is as follows:

$$F_c = \frac{c\hbar^2\pi^2}{2a^2}\left[\sum_{n=0}^\infty \frac{n^2 e^{-\lambda\sqrt{\left(\frac{n\pi\hbar}{a}\right)^2+m_0^2c^2}}}{\sqrt{\left(\frac{n\pi\hbar}{a}\right)^2+m_0^2c^2}} - \int_0^\infty dn \frac{n^2 e^{-\lambda\sqrt{\left(\frac{n\pi\hbar}{a}\right)^2+m_0^2c^2}}}{\sqrt{\left(\frac{n\pi\hbar}{a}\right)^2+m_0^2c^2}}\right] \tag{31}$$

To calculate the integral above consider the variable change in Casimir force [1]

$$b(n, m_0, \lambda) = \frac{n^2 e^{-\lambda\sqrt{\left(\frac{n\pi\hbar}{a}\right)^2+m_0^2c^2}}}{\sqrt{\left(\frac{n\pi\hbar}{a}\right)^2+m_0^2c^2}} \tag{32}$$

$$c(\alpha, m_0, \lambda) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dn e^{i\alpha n}\, b(n, m_0, \lambda) \tag{33}$$

And putting them in the relationship (31) we have Casimir force

$$F_c = -\frac{m_0^2c^3}{\pi\hbar}\left(\sum_{n=1}^\infty \left\{k_2\left(\frac{2anm_0c}{\hbar}\right) - \frac{k_1(\frac{2anm_0c}{\hbar})}{(\frac{2anm_0c}{\hbar})}\right\}\right) \tag{34}$$

The force Fc involves a converging series of Bessel functions. The force is computed numerically the surface as a function of the scalar square mass drawing.

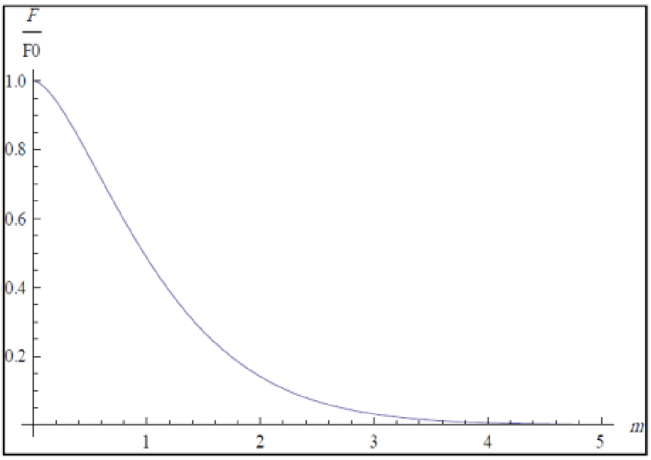

FIGURE 2. Casimir force versus mass.

Casimir force on the unit surface as a function of the mass between two parallel plates is

$$F_c = -\frac{m_0^2 c^3}{\pi\hbar}\sum_{n=1}^{\infty}\left\{2\left(\frac{\hbar}{2anm_0c}\right)^2 - \left(\frac{\hbar}{2anm_0c}\right)^2\right\} = -\frac{c\hbar}{4\pi a^2}\sum_{n=1}^{\infty}\frac{1}{n^2} \qquad (35)$$

The above series is Riemann Zeta function and equal to $\frac{\pi^2}{6}$ casimir resources for m = 0 between the two platesis: to get the follows.

$$F_c = -\frac{c\hbar\pi}{24a^2} \qquad (36)$$

Obviously, the classical limit $\hbar \longrightarrow 0$ is the desire Casimir force is zero.

VII. CONCLUSIONS

As can be seen from the Casimir force calculations for maximum value with zero mass particles capable of self and with the increase in the crime field Quantized, it finds the descending power of garlic In this case this article could be justified in the absence of a high-mass particles are very rare because the field by vacuum fluctuations themselves (or in other words, the creation and annihilation of particles) can be described and enough power to destroy such particles. In this article, to get square and drop skip Klein-Gordon and continue with the expansion of the field of Casimir force by using special functions. As was observed, a very long and difficult calculation in some special issues with different geometry to find the complex and extremely difficult task functions, so this is a good method for calculating methods, not force, Casimir but we can use the waste matter - the fluctuations to calculate the Casimir force has recommended. In this way, the quantized function instead of the Green field is calculated as-calculation of Green functions easier and a lot of geometry is known to be a very reasonable approach.

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