Leptonic Decays of Pseudoscalar Mesons: A New Approach

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Abstract. Because quarks in hadronic states are subjected to strong QCD forces, It is not straight-forward to apply calculation obtained at the quark level to physical processes involving hadrons because quarks in hadronic states are subjected to strong QCD forces. In this paper we consider the leptonic decays of pseudoscalar mesons of type M to lepton + antineutrino, where M is pseudoscalar meson with quark contents of . At the quark level, the corresponding process is q1 to q2 + lepton and antineutrino. However, when we fold the quark-level process to the meson decay process, the complication from QCD effects comes in. We make the fundamental assumption that the vertex of type can be approximated by an effective constant gamma-5 coupling. With this assumption, the hadronic process can then be related to the quark-level process. The model is applied to the leptonic decays of pi-meson, K-meson, D-meson, Ds-meson and B-meson.

Keywords: Electroweak Interactions, Leptonic Decays, Hadrons.

I. INTRODUCTION

The Standard Model (SM) has been successful in providing good agreement with experimental data. Many features of the SM have been well tested. However, although SM provides a general framework for quarks and leptons, it does not offer easy means for calculation when it comes to processes that involve strong QCD effects. Even for electroweak processes, although relatively simple at the quark level, the calculational details become involved and complicated when the processes are folded into hadronic states. This is because quarks are tightly bound inside hadrons. Because of the non-perturbative nature of such QCD forces, the description of the interactions of bound quarks in hadrons is still questionable.

In this paper, I shall focus on a specific class of processes namely the leptonic weak decays of the pseudoscalar mesons. Weak decay of quark, $q_1 \to q_2 \ell \nu$, is a simple process. How to relate the quark process to the corresponding hadronic process $M_1 \to M_2 \ell \nu$ is, however, not straightforward. A simple model is proposed here to describe how meson is coupled to quarks, a simple γ_5 coupling with a constant coupling constant.

The idea behind the model was first employed in the calculation of $\overline{K}^0 \to$ vacuum amplitude in relation to $\Delta I = 1/2$ rule [1,2]

II. LEPTONIC DECAY OF MESON

The model is applied to leptonic decay of pseudoscalar meson of type $M \to \ell \nu$, where the quark content of M is $q_1\overline{q}_2$. This process is shown diagrammatically in Fig. 1.

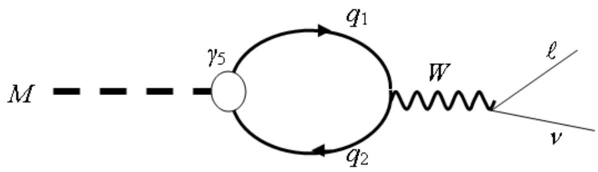

FIGURE 1. Diagram representing $M \to \ell \nu$, where M is $q_1\overline{q}_2$.

The amplitude for the process in Fig. 1 is given by

$$M = -i\frac{g^2}{2} g_M V_{12}\left[\int \frac{d^4k}{(2\pi)^4}\frac{N'^{\mu}}{D'}\right]\frac{-g_{\mu\nu} + q_\mu q_\nu / M_W^2}{q^2 - M_W^2}\overline{u}(p)\gamma^\nu L v(P-p) \tag{1}$$

where g_M is the meson-quark coupling, P, k, p are the momenta of the decaying meson M, the W-boson, and the charged lepton respectively. N'^{μ} and D' are respectively:

$$N'^{\mu} = Tr\left[\gamma_5(\not{P} - \not{k} - m_2)\gamma^\mu L(\not{k} - m_1)\right] \tag{2}$$

$$D' = [(P-k)^2 - m_2^2][k^2 - m_1^2] \tag{3}$$

The integration over the internal momentum k is logarithmically divergent. Introducing the cut-off momentum Λ yields

$$\int \frac{d^4k}{(2\pi)^4}\frac{N'^{\mu}}{D'} = \frac{i}{32\pi^2}(m_1 - m_2)P^\mu \ln\frac{\Lambda^2}{M^2} \tag{4}$$

For simplicity, we assume Λ/M to be sufficiently large. Putting Eq. (4) into Eq. (1), summing over the final states, and integrating over the phase space of the final states, the following expression is obtained for the decay rate:

$$\Gamma = \frac{G_F^2}{2(4\pi)^5}\left[g_M \ln(\Lambda^2/M^2)\right]^2 |V_{12}|^2 (m_1 - m_2)^2 M m_\ell^2 \left(1 - m_\ell^2/M^2\right)^2 \tag{5}$$

Here, G_F is the Fermi coupling constant given by

$$G_F = \frac{g^2}{4\sqrt{2}M_W^2} = 1.1663787 \times 10^{-11} MeV^{-2} \tag{6}$$

A. Comparison with Experimental Values

It is observed from Eq. (5) that the decay rate depends on: (i) the overall coupling G_F, (ii) combined meson-quark coupling $g_1 g_2 \ln(\Lambda^2/M^2)$, and (iii) CKM mixing matrix element V_{12}. The CKM mixing matrix is given by [8]

$$|\mathbb{V}| = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97425 & 0.2252 & 0.00393 \\ 0.230 & 0.953075 & 0.0406 \\ 0.0084 & 0.0387 & 0.9992 \end{pmatrix} \quad (7)$$

The decay rate, Eq. (5), also involves the kinematic factor $Mm_\ell^2\left(1-m_\ell^2/M^2\right)^2$, which is the same as obtained from conventional analysis. It also has an additional dynamical factor $(m_1 - m_2)^2$, which is the square of the quark mass difference. We use the following set of values for the quark mass:

$$m_u = 2.3 \text{ MeV},\ m_d = 4.8 \text{ MeV},\ m_s = 95 \text{ MeV},\ m_c = 1.275 \text{ GeV},\ m_b = 4.18 \text{ GeV}. \quad (8)$$

Out of the various modes of $M \to \ell\nu$, the mode with higher mass of the charged lepton is selected for analysis to estimate $g_M \ln(\Lambda^2/M^2)$. The result is listed in Table 1.

TABLE 1. Values of $g_M \ln(\Lambda^2/M^2)$ estimated from leptonic decay rates.

Decay Modes	Decay Rate (MeV)	$g_M \ln(\Lambda^2/M^2)$	$m_1 - m_2$ (GeV)
$\pi^+ \to \mu^+\nu_\mu$	2.528×10^{-14}	8484.4	0.0025
$K^+ \to \mu^+\nu_\mu$	3.379×10^{-14}	912.96	0.0927
$D^+ \to \mu^+\nu_\mu$	2.418×10^{-13}	69.602	1.2702
$D_s^+ \to \tau^+\nu_\tau$	7.372×10^{-11}	18.177	1.180
$B^+ \to \tau^+\nu_\tau$	7.233×10^{-14}	1243.2	4.1777

TABLE 2. Calculated decay rates against measured rates.

Decay Modes	Calculated Rate (MeV)	Measured Rate (MeV)
$\pi^+ \to e^+\nu_e$	3.2443×10^{-18}	3.110×10^{-18}
$K^+ \to e^+\nu_e$	8.6805×10^{-19}	8.422×10^{-19}
$D^+ \to e^+\nu_e$	5.6920×10^{-18}	$< 5.57 \times 10^{-15}$
$D^+ \to \tau^+\nu_\tau$	6.4471×10^{-13}	$< 7.59 \times 10^{-13}$
$D_s^+ \to \mu^+\nu_\mu$	7.5531×10^{-12}	7.635×10^{-12}
$D_s^+ \to e^+\nu_e$	1.7769×10^{-16}	$< 1.58 \times 10^{-13}$
$B^+ \to \mu^+\nu_\mu$	3.2503×10^{-16}	$< 4.02 \times 10^{-16}$
$B^+ \to e^+\nu_e$	7.6085×10^{-21}	$< 7.63 \times 10^{-16}$

The value of $g_M \ln(\Lambda^2 / M^2)$ so calculated is then used to estimate the decay rates of the other decay modes of the same meson, the results of which are listed in Table 2.

III. CONCLUSIONS

I have presented a simple model on the coupling of meson to quark, which is represented by a simple γ_5 coupling with a constant coupling constant. The model is applied to leptonic weak decays of the pseudoscalar mesons of type $M_1 \to M_2 \ell \nu$. The calculation involves an evaluation over a quark loop. The integration over the internal momentum around the quark loop is logarithmically divergent. A cut-off momentum Λ is introduced which enters as an additional parameter. Table 2 shows that the calculated decay rates for a number of decay processes agree well with the measured decay rates.

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