# Head on collision of dust acoustic multi-solitons in a nonextensive plasma

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**Abstract** The head-on collisions and overtaking collisions of dust acoustic multi-solitons in a dusty plasma with $q$-nonextensive velocity distributed ions are studied applying the Extended Poincare-Lighthill-Kuo(EPLK) method and using the results of Hirota's direct method. The EPLK method is used to derive two separate Korteweg-de Vries (KdV) equations where the solitons obtained from the KdV equation move along a direction opposite to that of solitons obtained from the other KdV equation. The Hirota's direct method is used to obtain multisolitons solutions for each KdV equation, and all of them move along the same direction where the fastest moving soliton eventually overtakes the others. Due to face to face interaction we obtain here respectively two solitons collision, four solitons collision, six solitons collision. Phase shifts acquired by each soliton due to both head-on collision and overtaking collision are derived analytically. It is seen that the parameter $q$ has a significant effect on the phase shift in each interactions.

**Keywords:** Dusty plasma, Head-on collision Multi-solitons.

## I. INTRODUCTION

A dusty plasma is an ionized gas containing dust particles with sizes varying from tens of nanometers to hundreds of microns. Nonlinear dust acoustic wave propagation in dusty plasma is one of the fast growing research topics of plasma physics because of its wide applications in different areas, such as, planetary rings, asteriod zones, magnetosphere, coating of thin films [1], plasma crystals [2], cometary tails and lower atmosphere of the earth [3]. It is important to note that the presence of different types of dust charged grains in a plasma introduces a number of different wave modes, such as, dust acoustic mode [4], dust ion acoustic mode [5], dust lattice mode [6], shukla-varma mode [7], dust Berstain-Green-Kruskal mode [8] and dust drift mode [9]. In 1990, Rao et al. [4] observed the existence of a new extremely low-phase velocity dust acoustic waves (DAW) in an unmagnetized dusty plasma. Shukla and Silin [10] studied the nonlinear dust-ion acoustic waves (DIAW) in a dusty plasma. Different experimental and theoretical observations performed by Angelo [11], Barkan et al. [12]-[13], Nakamuro et al. [14] and Duan et al. [15] have confirmed the linear and nonlinear features of both DAW and DIAW predicted by theory.

In most of the studies on dusty plasma system electrons were assumed to have Maxwell distribution. However Maxwell distribution is valid for the macroscopic ergodic equilibrium state and it may be inadequate to describe the long range interactions in unmagnetized collision less plasma containing the non-equilibrium stationary states. This type of stationary state may occur due to a number of physical mechanisms, for examples, external force field present in natural space plasma environments, wave-particle interaction, turbulence, etc. Space plasma observations clearly indicate the presence of ion and electron populations that are far away from their thermodynamic equilibrium [16]-[30]. Renyi proposed a new statistical approach [18], namely non-extensive statistics or Tsallis statistics depending on the derivation of Boltzmann-Gibbs-Shannon (BGS) entropic measure [19] to investigate the cases for which Maxwell distribution is not appropriate. This fact was first acknowledged by Renyi [18] and afterward it was proposed by Tsallis [19], where the entropic index $q$ that characterized the degree of non extensivity of the system. The Tsallis distribution which is also known as $q$-Gaussian, is a probability distribution arising from the optimization of the Tsallis entropy. Tsallis [19] modeled nonextensivity considering a composition law in the sense that the entropy of the composition $(A+B)$ of two independent systems $A$ and $B$ is equal to $S_q^{(A+B)} = S_q^{(A)} + S_q^{(B)} + (1-q)S_q^{(A)}S_q^{(B)}$, where $q$ which underpins the generalized entropy of Tsallis, is linked to the underlying dynamics of the system and gives a measure of the degree of its correlation. In statistical mechanics and thermodynamics, systems which are characterized by the property of nonextensivity, are systems for which the entropy of the whole system is different from the sum of the entropies of the respective parts. In other words, the generalized entropy of the whole is greater than the sum of the entropies of the parts if $q < 1$ which is known as superextensivity, whereas the generalized entropy of the system is smaller than the sum of the entropies of the parts if $q > 1$, which is known as subextensivity. The $q$-entropy may represent a suitable frame for the analysis of many astrophysical scenarios [20], [23], such as, stellar polytropes, solar neutrino problem, and peculiar velocity distribution of galaxy cluster. It is important to note that the $q$-distribution is not normalizable if $q < -1$. In extensive limiting case, if $q \to 1$, the $q$-distribution follows the Maxwell-Boltzmann velocity distribution.

It is well known that a delicate balance of nonlinearity and dispersion leads to the generation of solitons [24]. In particular, wave-wave interaction is one of the interesting and important nonlinear phenomenon in different types of plasma systems. It is important to note that Solitons [25]-[26] preserve the form asymptotically even when they undergo a collision among themselves. Zabusky and Kruskal [27] coined the term soliton for the first time in the literature. Interactions or collisions of solitons may occur in two different ways. First one is overtaking collision which was studied by inverse scattering method [28] and second one is head-on collision [29] in which the angle between two propagation directions of solitons is $\pi$. As solitons preserve the form asymptotically even when they undergo collision, we must search for the evolution of solitary waves propagating in the opposite directions, and require to employ a suitable asymptotic expansion that gives the interesting features of the trajectories of solitary waves after collision and phase shift. Many researchers [30] studied the variation of phase shift and the trajectories of a pair of solitary waves after collision applying extended version of PoincarÃ©-Lighthill-Kuo (PLK) method. El-Shamy [31] investigated the head-on collision between two ion thermal solitary waves in a pair-ion plasma using the same technique. Chatterjee et al. [32] studied head-on collision of ion acoustic solitary waves in a three-component unmagnetized collision less plasma comprising of cold ions, Boltzmann distributed positrons and superthermal electrons following the same way. They presented that two solitons are far apart at the initial stage and then they collide and after that they depart. Recently, Ferdousi and Mamun [33] studied the nonlinear dynamics of the dust-acoustic shock waves in a dusty plasma containing negatively charged mobile dust, nonextensive electrons with two distinct temperatures, and Maxwellian ions. Alam et al. [34] investigated the basic properties of nonplanar dust-ion-acoustic (DIA) shock waves in an unmagnetized dusty plasma system consisting of inertial ions, negatively charged immobile dust, and superthermal electrons with two distinct temperatures.

Recently, by using bifurcation theory of planar dynamical systems, a number of works[35]-[39] have been reported in the field of dusty plasmas. Sahu et al. [40] studied the propagation of twosolitons for ion acoustic waves (IAWs) in a plasma consisting of superthermal electrons and warm ions with nonplanar geometry. Sahu [41] also investigated the propagation of twosolitons for electron acoustic waves in an unmagnetized, collisionless plasma consisting of a cold-electron fluid and hot electrons with $\kappa$ velocity distribution, and stationary ions with non-planar geometry.

Recently, Roy et al. [42] studied the propagation of ion acoustic twosolitons interaction in a three components collision-less unmagnetized plasma in the framework of KdV equation using Hirota's direct method [43]. Saha and Chatterjee [44] studied the nonlinear propagations and interactions of dust acoustic multisolitons in a four components dusty plasma with negatively and positively charged cold dust fluids, q-nonextensive velocity distributed electrons and ions. Very recently, Roy et al. [45] studied head on collision of multisolitons in an electron-positron-ion plasma having superthermal electrons for the first time. The authors studied the head on collision of two solitons but did not discuss the collision of the three solitons.

Moreover, no work has been reported on the study of head on collision of multi-solitons in dusty plasmas for planar geometry. Also, there is no work on head on collision of three solitons and corresponding phase shifts in plasmas to the best our knowledge. In the present work, our aim is to study head on collision of two solitons and three solitons and phase shifts of dust acoustic two solitons and three solitons in a two component unmagnetized dusty plasma whose constituents are dust particles and $q$-nonextensive velocity distributed ions.

The remaining part of the paper is organized as follows: In section II, we consider basic equations and derivation of the KdV equations. The asymptotical solution of the KdV equation is given in section III. In Section IV, we present results and discussions. Finally, conclusions are given in section V.

## II. BASIC MODEL EQUATIONS AND DERIVATION OF THE KDV EQUATIONS

In this paper, we consider a two components unmagnetized dusty plasma whose constituents are dust particles and $q$-nonextensive velocity distributed ions. The normalized model equations for one-dimensional low velocity dust acoustic oscillations [46] are given by

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{\partial \psi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = n_d - n_i. \tag{3}$$

Here $n_d$ and $n_i$ are number densities of dust particles and ions, respectively, normalized by their unperturbed densities $n_{d0}$ and $n_{i0}$. In this case, $u_d$ and $\psi$ are dust fluid velocity and electrostatic wave potential, respectively, normalized by the dust acoustic speed $C_d = (Z_d T_i/m_d)^{1/2}$ and $T_i/e$, where $e$ is the electron charge, $m_d$ is mass of dust particles and $Z_d$ is number of the charge residing on the dust grains. The time $t$ and space variable $x$ are normalized by inverse of dust plasma frequency $\omega_{pd}^{-1} = (m_d/4\pi e^2 n_{d0} Z_d)^{1/2}$ and the Debye length $\lambda_d = (T_i/4\pi e^2 n_{d0} Z_d)^{1/2}$, respectively, by using the charge neutrality condition $n_{i0} = n_{d0} Z_d$.

In order to model ions, we use the following distribution function [47]:

$$f_i(v) = C_q\{1 + (q-1)[\tfrac{m_i v^2}{2T_i} - \tfrac{e\psi}{T_i}]\}^{\frac{1}{(q-1)}},$$

where $\psi$ denotes the electrostatic potential and the remaining variables or parameters obey their usual meaning. It is important to note that $f_i(v)$ is a special distribution that maximizes the Tsallis entropy and it conforms to the laws of thermodynamics. In this case, the constant of normalization is given by

$$C_q = n_{i0} \frac{\Gamma(\frac{1}{1-q})}{\Gamma(\frac{1}{1-q} - \frac{1}{2})} \sqrt{\frac{m_i(1-q)}{2\pi T_i}} \text{ for } -1 < q < 1, \tag{4}$$

and

$$C_q = n_{i0} \frac{1+q}{2} \frac{\Gamma(\frac{1}{q-1} + \frac{1}{2})}{\Gamma(\frac{1}{q-1})} \sqrt{\frac{m_i(q-1)}{2\pi T_i}} \text{ for } q > 1. \tag{5}$$

Integrating $f_i(v)$ over all velocity space, we have the following nonextensive ion number density:

$$n_i(\psi) = n_{i0}\{1 - (q-1)\frac{e\psi}{T_i}\}^{1/(q-1)+1/2}. \tag{6}$$

Therefore, the normalized ion number density [47] is given by

$$n_i(\psi) = \{1 - (q-1)\psi\}^{1/(q-1)+1/2} \tag{7}$$

where the parameter $q$ is a real number greater than $-1$, and it stands for the strength of nonextensivity. Figure 1 shows the function $f_i(v)$ vs $v$, for different values of $q = 0.5$ (dashed line), 1.0 (solid line), 1.5 (dotted line). Other parameter $T_i = 0.05$eV. It is clearly seen from this figure that the height and shape of the distribution changes remarkably with change of $q$. It also shows that the height is increasing along with the increasing values of the parameter $q$. It is seen that the function $f_i(v)$ reduces the Maxwell distribution when $q \rightarrow 1$.

Now we assume that two solitons $\alpha$ and $\beta$ in the plasma, which were asymptotically far apart in

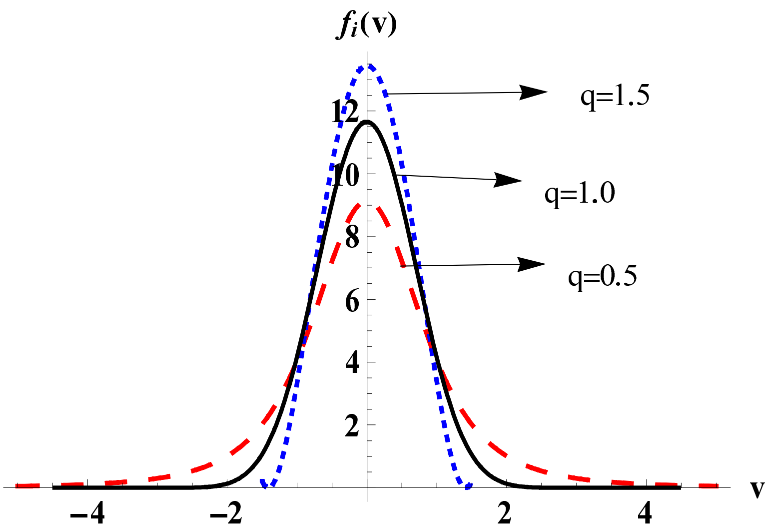


FIG. 1: Plot of $f_i(v)$ vs $v$ for $q$ =1.5 (dotted line), $q \rightarrow 1$(solid line), $q = 0.5$ (dashed line) and other parameter $T_i = 0.05$ ev.

the initial state travel toward each other. After some time they interact, collide, and then depart. We also assume that the solitons have small amplitudes proportional to $\epsilon$ (where $\epsilon$ is the small parameter characterizing the strength of non linearity ) and the interaction between two solitons is weak. Hence we expect that the collision will be quasi elastic, so it will only cause shifts of the post collision trajectories (phase shift). In order to analyze the effects of collision, we employ an extended PLK method. According to this method, the dependent variables are expanded as

$$n_d = 1 + \epsilon^2 n_1 + \epsilon^3 n_2 + \epsilon^4 n_3 + \cdots ; \quad (8)$$

$$u_d = u_0 + \epsilon^2 u_1 + \epsilon^3 u_2 + \epsilon^4 u_3 + \cdots ; \quad (9)$$

$$\psi = 0 + \epsilon^2 \psi_1 + \epsilon^3 \psi_2 + \epsilon^4 \psi_3 + \cdots ; \quad (10)$$

The independent variables are given by

$$\xi = \varepsilon(x - c_1 t) + \epsilon^2 P_0(\eta, \tau) + \epsilon^3 P_1(\eta, \xi, \tau) + \cdots ; \quad (11)$$

$$\eta = \epsilon(x + c_2 t) + \epsilon^2 Q_0(\xi, \tau) + \epsilon^3 Q_1(\eta, \xi, \tau) + \cdots ; \quad (12)$$

$$\tau = \epsilon^3 t; \quad (13)$$

where $\xi$ and $\eta$ denote the trajectories of two solitons travelling toward to each other, and $c_1$ and $c_2$ are the unknown phase velocity of DASWs. The variables $P_0(\eta,\tau)$ and $Q_0(\xi,\tau)$ are to be determined.

Using extended PLK method and after some long but standard calculation, we get

$$\begin{aligned}
\lambda u_3 &= \int(\frac{\partial\psi_1^1}{\partial\tau} + A\psi_1^1\frac{\partial\psi_1^1}{\partial\xi} + B\frac{\partial^3\psi_1^1}{\partial\xi^3})\,d\eta \\
&+ \int(\frac{\partial\psi_1^2}{\partial\tau} - A\psi_1^2\frac{\partial\psi_1^2}{\partial\eta} - B\frac{\partial^3\psi_1^2}{\partial\eta^3})\,d\xi \\
&+ \int\int(C\frac{\partial P_0}{\partial\eta} + D\psi_1^2)\frac{\partial^2\psi_1^1}{\partial\xi^2}\,d\xi d\eta \\
&- \int\int(C\frac{\partial Q_0}{\partial\xi} + D\psi_1^1)\frac{\partial^2\psi_1^2}{\partial\eta^2}\,d\xi d\eta; \qquad (14)
\end{aligned}$$

where

$$\begin{aligned}
A &= \frac{1}{2}\Big[\Big(\frac{2}{q+1}\Big)^{3/2}\Big\{1-\Big(\frac{q-1}{2}\Big)^2\Big\} - 3\Big(\frac{q+1}{2}\Big)^{1/2}\Big]; \\
B &= \frac{1}{2}\Big(\frac{2}{q+1}\Big)^{3/2}, \\
C &= \frac{1}{2}\Big(\frac{2}{q+1}\Big)^{1/2}, \\
D &= \frac{1}{2}\Big[\Big(\frac{2}{q+1}\Big)^{3/2}\Big\{1-\Big(\frac{q-1}{2}\Big)^2\Big\} + \Big(\frac{q+1}{2}\Big)^{1/2}\Big], \\
\lambda &= \frac{2}{q+1}.
\end{aligned}$$

The first term in the right hand side of Eqs. (10) will be proportional to $\eta$ because the integrand is independent of $\eta$ and the second term in the right hand side of Eqs. (10) will be proportional to $\xi$ because the integrand is independent of $\xi$. These two terms of Eq. (10) are secular terms, which must be eliminated in order to avoid spurious resonances. Hence, we have

$$\frac{\partial\psi_1^1}{\partial\tau} + A\psi_1^1\frac{\partial\psi_1^1}{\partial\xi} + B\frac{\partial^3\psi_1^1}{\partial\xi^3} = 0; \qquad (15)$$

$$\frac{\partial\psi_1^2}{\partial\tau} - A\psi_1^2\frac{\partial\psi_1^2}{\partial\eta} - B\frac{\partial^3\psi_1^2}{\partial\eta^3} = 0. \qquad (16)$$

The third and fourth terms in Eqs. (10) are not secular terms at this order, they could be secular for the next order. Hence we have

$$C\frac{\partial P_0}{\partial\eta} = -D\psi_1^2; \qquad (17)$$

$$C\frac{\partial Q_0}{\partial\xi} = -D\psi_1^1. \qquad (18)$$

Equation (11) is a KdV equation. This wave is travelling in the $\xi$ direction. Eq. (12) is also a KdV equation. This wave is propagating in the $\eta$ direction which is opposite to $\xi$.

### (i) One Soliton solution and Phase Shift:

Using Hirota's method [43], one soliton solutions of the KdV equations (11) and (12) respectively

$$\psi_1^1 = \frac{12B}{A}\frac{\partial^2}{\partial\xi^2}(ln[f(\xi,\tau)]); \quad (19)$$

$$\psi_1^2 = \frac{12B}{A}\frac{\partial^2}{\partial\eta^2}(ln[f_1(\xi,\tau)]); \quad (20)$$

where $f = 1 + e^{\theta_1},\ f_1 = 1 + e^{\phi_1},\ \theta_1 = k_1 B^{-1/3}\xi - k_1^3\tau + \alpha_1,\ \phi_1 = -k_1 B^{-1/3}\eta - k_1^3\tau + \alpha_1$. The leading phase changes due to the collision can be calculated from Eqs. (13), and (14).
To obtain the phase shifts after a head-on collision of the two solitons, we assume that the solitons $\alpha$ and $\beta$ are, asymptotically, far from each other at the initial time ($t = -\infty$) i.e. soliton $\alpha$ is at $\xi = 0, \eta = -\infty$ and soliton $\beta$ is at $\eta = 0, \xi = +\infty$, respectively. After the collision ($t = +\infty$), soliton $\alpha$ is far to the right of soliton $\beta$, i.e.soliton $\alpha$ is at $\xi = 0, \eta = +\infty$ and soliton $\beta$ is at $\eta = 0, \xi = -\infty$. Thus from (14) we have

$$\frac{\partial Q_0(\xi,\tau)}{\partial\xi} = -\frac{12BD}{AC}\frac{\partial^2}{\partial\xi^2}(logf)$$
$$\Rightarrow \quad Q_0(\xi,\tau) = -\frac{12BD}{AC}\frac{\partial}{\partial\xi}(logf) = -\frac{12B^{2/3}D}{AC}\frac{k_1 e^{\theta_1}}{1+e^{\theta_1}} \quad (21)$$

and the corresponding phase shift

$$\begin{aligned}\triangle Q_0 &= \epsilon(x + c_2 t)|_{\xi=-\infty,\eta=0} - \epsilon(x + c_2 t)|_{\xi=\infty,\eta=0} \\ &= \epsilon^2 Q_0(\infty,\tau) - \epsilon^2 Q_0(-\infty,\tau) \\ &= -\frac{12\epsilon^2 DB^{2/3}}{AC}k_1. \quad (22)\end{aligned}$$

Similarly the other phase shift is

$$\triangle P_0 = \frac{12\epsilon^2 DB^{2/3}}{AC}k_1. \quad (23)$$

Phase shifts in Eq.(18) and Eq. (19) are similar to those obtained by several authors [48]-[49] but our approch is different.

### (ii) Two Solitons solution and Phase Shift:

Each of the KdV equations given by (11) and (12) has a number of soliton solutions, we consider here two solitons solutions of each of the KdV equations. The two solitons for a particular KdV equations move in the same directions, the fast moving soliton eventually overtakes the slower one and the two solitons solutions of (11) and (12) are propagated from the opposite directions, although they are far from each other initially, after sometimes they come together and the head-on collision will takes place and then depart. Using Hirota's method[43] two-soliton solution of the KdV Eq.(11) and Eq.(12) are given by

$$\psi_{1\xi} = \frac{12B}{A}\frac{\partial^2}{\partial\xi^2}(log\ g); \tag{24}$$

$$\psi_{1\eta} = \frac{12B}{A}\frac{\partial^2}{\partial\eta^2}(log\ g_1), \tag{25}$$

where $g = 1 + e^{\theta_1} + e^{\theta_2} + a_{12}e^{\theta_1+\theta_2}$, $g_1 = 1 + e^{\phi_1} + e^{\phi_2} + a_{12}e^{\phi_1+\phi_2}$, $\theta_i = k_iB^{-1/3}\xi - k_i^3\tau + \alpha_i$, $\phi_i = -k_iB^{-1/3}\eta - k_i^3\tau + \alpha_i, i = 1, 2$ and $a_{12} = (k_1 - k_2)^2/(k_1 + k_2)^2$.
As in the case of two solitons solution from Eq. (14) we have

$$\frac{\partial Q_0(\xi,\tau)}{\partial\xi} = -\frac{12BD}{AC}\frac{\partial^2}{\partial\xi^2}(log\ g)$$

$$\Rightarrow Q_0(\xi,\tau) = -\frac{12BD}{AC}\frac{\partial}{\partial\xi}(log\ g)$$

$$\Rightarrow Q_0(\xi,\tau) = -\frac{12B^{2/3}D}{AC}\frac{k_1e^{\theta_1} + k_2e^{\theta_2} + a_{12}(k_1+k_2)e^{\theta_1+\theta_2}}{1 + e^{\theta_1} + e^{\theta_2} + a_{12}e^{\theta_1+\theta_2}} \tag{26}$$

and the corresponding phase shift is

$$\begin{aligned}\triangle Q_0 &= \epsilon(x + c_2t)|_{\xi=-\infty,\eta=0} - \epsilon(x + c_2t)|_{\xi=\infty,\eta=0} \\ &= \epsilon^2Q_0(\infty,\tau) - \epsilon^2Q_0(-\infty,\tau) \\ &= -\frac{12\epsilon^2DB^{2/3}}{AC}\frac{a_{12}(k_1+k_2)}{a_{12}} \\ &= -\frac{12\epsilon^2DB^{2/3}}{AC}(k_1+k_2).\end{aligned} \tag{27}$$

Similarly the other phase shift is

$$\triangle P_0 = \frac{12\epsilon^2DB_1^{2/3}}{A_1C}(k_1+k_2). \tag{28}$$

### (iii) Three Solitons solution and Phase Shift:

Finally the three solitons solutions of (11) and (12) have the form [Hirota's method[43]]

$$\psi_{1\xi} = \frac{12B}{A}\frac{\partial^2}{\partial\xi^2}(log\ h); \tag{29}$$

$$\psi_{1\eta} = \frac{12B}{A}\frac{\partial^2}{\partial\eta^2}(log\ h_1), \tag{30}$$

where

$$\begin{aligned}
h &= 1 + e^{\theta_1} + e^{\theta_2} + e^{\theta_3} + a_{12}^2 e^{\theta_1+\theta_2} + a_{13}^2 e^{\theta_3+\theta_1} + a_{23}^2 e^{\theta_2+\theta_3} + a^2 e^{\theta_1+\theta_2+\theta_3} \\
h_1 &= 1 + e^{\phi_1} + e^{\phi_2} + e^{\phi_3} + a_{12}^2 e^{\phi_1+\phi_2} + a_{13}^2 e^{\phi_3+\phi_1} + a_{23}^2 e^{\phi_2+\phi_3} + a^2 e^{\phi_1+\phi_2+\phi_3} \\
\theta_i &= k_i B^{-1/3}\xi - k_i^3\tau + \alpha_i; \\
\phi_i &= -k_i B^{-1/3}\eta - k_i^3\tau + \alpha_i; \quad i = 1, 2, 3; \\
a_{lm}^2 &= (\frac{k_l - k_m}{k_l + k_m})^2; l,\ m = 1, 2, 3,\ l < m, \quad a^2 = \prod_{l,m=1,l<m}^{3} a_{lm}^2,
\end{aligned}$$

$\alpha_i$ is the initial phase of the $i^{th}$ soliton in a three-soliton, and the corresponding phase shifts are given by

$$\triangle P_0 = \frac{12\epsilon^2 DB^{2/3}}{AC}(k_1 + k_2 + k_3), \tag{31}$$

$$\triangle Q_0 = -\frac{12\epsilon^2 DB^{2/3}}{AC}(k_1 + k_2 + k_3). \tag{32}$$

## III. ASYMPTOTIC SOLUTION

**Two Solitons:**

Now we want to find the solution of equations (11) and (12) with asymptotic approximation. We get from Eq. (15) and (16)

$$\psi_1^1 = \frac{12B^{1/3}}{A}\frac{k_1^2 e^{\theta_1} + k_2^2 e^{\theta_2} + a_{12}e^{\theta_1+\theta_2}(k_2^2 e^{\theta_1} + k_1^2 e^{\theta_2}) + 2(k_1 - k_2)^2 e^{\theta_1+\theta_2}}{(1 + e^{\theta_1} + e^{\theta_2} + a_{12}e^{\theta_1+\theta_2})^2}, \tag{33}$$

$$\psi_1^2 = \frac{12B^{1/3}}{A}\frac{k_1^2 e^{\phi_1} + k_2^2 e^{\phi_2} + a_{12}e^{\phi_1+\phi_2}(k_2^2 e^{\phi_1} + k_1^2 e^{\phi_2}) + 2(k_1 - k_2)^2 e^{\phi_1+\phi_2}}{(1 + e^{\phi_1} + e^{\phi_2} + a_{12}e^{\phi_1+\phi_2})^2}, \tag{34}$$

with $\theta_i = \frac{k_i}{B^1/3}\xi - k_i^3\tau + \alpha_i,\ \phi_i = -\frac{k_i}{B^1/3}\eta - k_i^3\tau + \alpha_i,\ (i = 1, 2),\ a_{12} = (k_1 - k_2)^2/(k_1 + k_2)^2$. For $\tau >> 1$, $e^{-(\theta_1+\theta_2)}$, $e^{-(2\theta_1+\theta_2)}$, $e^{-(\theta_1+2\theta_2)}$ and $e^{-(\phi_1+\phi_2)}$, $e^{-(2\phi_1+\phi_2)}$, $e^{-(\phi_1+2\phi_2)}$ are non dominant terms. Neglecting the non dominant terms and after some rearrangement, we get

$$\psi_1^1 = \frac{12B^{1/3}}{A}\left[\frac{a_{12}k_1^2 e^{-\theta_1}}{(e^{-\theta_1} + a_{12})^2} + \frac{a_{12}k_2^2 e^{-\theta_2}}{(e^{-\theta_2} + a_{12})^2}\right] \tag{35}$$

$$\psi_1^2 = \frac{12B^{1/3}}{A}\left[\frac{a_{12}k_1^2 e^{-\phi_1}}{(e^{-\phi_1} + a_{12})^2} + \frac{a_{12}k_2^2 e^{-\phi_2}}{(e^{-\phi_2} + a_{12})^2}\right]. \tag{36}$$

Using the result $e^{-x}/(1+e^{-x})^2 = sech^2(x/2)/4$ and writing $a_{12} = e^{in|a_{12}|}$, we get

$$\begin{aligned}\psi_1^1 \approx & \frac{6B^{1/3}}{A}\Bigg[\frac{k_1^2}{2}sech^2\Bigg\{\frac{k_1}{2B^{1/3}}\Bigg(\xi - B^{1/3}k_1^2\tau - \Delta_1^{'}\Bigg)\Bigg\} \\ & +\frac{k_2^2}{2}sech^2\Bigg\{\frac{k_2}{2B^{1/3}}\Bigg(\xi - B^{1/3}k_2^2\tau - \Delta_2^{'}\Bigg)\Bigg\}\Bigg], \end{aligned} \quad (37)$$

$$\begin{aligned}\psi_1^2 \approx & \frac{6B^{1/3}}{A}\Bigg[\frac{k_1^2}{2}sech^2\Bigg\{\frac{k_1}{2B^{1/3}}\Bigg(-\eta - B^{1/3}k_1^2\tau - \Delta_1\Bigg)\Bigg\} \\ & +\frac{k_2^2}{2}sech^2\Bigg\{\frac{k_2}{2B^{1/3}}\Bigg(-\eta - B^{1/3}k_2^2\tau - \Delta_2\Bigg)\Bigg\}\Bigg]. \end{aligned} \quad (38)$$

Eqs. (33) and (34) , it is clearly seen that for $\tau >> 1$ $sech^2$ type shape is preserved. It is also seen from (33) and (34) that if we are dealing with four solitons, two in each pair moving in the same direction with the phase shift $\Delta_i^{'},\ \Delta_i = \pm\frac{2B^{1/3}}{k_i}ln|\sqrt{a_{12}}|,\ (i = 1, 2)$.

**Three Solitons:**

Similarly, for $\tau >> 1$ the solutions of Eq. (11) and (12) respectively transform into a superposition of three single-soliton as

$$\psi_1^1 \sim \sum_{i=1}^{3} A_i sech^2\Bigg[\frac{k_i}{2B^{1/3}}(\xi - k_i^2B^{1/3}\tau + \delta_i^{'})\Bigg], \quad (39)$$

$$\psi_1^2 \sim \sum_{i=1}^{3} A_i sech^2\Bigg[\frac{k_i}{2B^{1/3}}(-\eta - k_i^2B^{1/3}\tau + \delta_i)\Bigg], \quad (40)$$

where $A_i = \frac{3B^{1/3}k_i^2}{A}, (i = 1, 2, 3)$ are the amplitudes , $\delta_1^{'} = \delta_1 = \pm\frac{2B^{1/3}}{k_1}\log|\frac{\alpha_{123}}{\alpha_{23}}|$, $\delta_2^{'} = \delta_2 = \pm\frac{2B^{1/3}}{k_2}\log|\frac{\alpha_{123}}{\alpha_{31}}|$, $\delta_3^{'} = \delta_3 = \pm\frac{2B^{1/3}}{k_1}\log|\frac{\alpha_{123}}{\alpha_{12}}|$ are the phase shifts of the solitons.

## IV. RESULTS AND DISCUSSIONS

Overtaking collision is realized by applying two or more consecutive voltage pulses between the two plasmas with amplitudes such that the first pulse generates a small amplitude soliton and the second pulse produces a soliton of large amplitude compared to the previous one and so on. since Since the larger amplitude soliton propagate faster, it will overtake the smaller one. This has been studied by Hirota's method. Now we want to study if same number of independent discharges are put at the opposite end of the plasma with same amplitude respectively in which the waves are

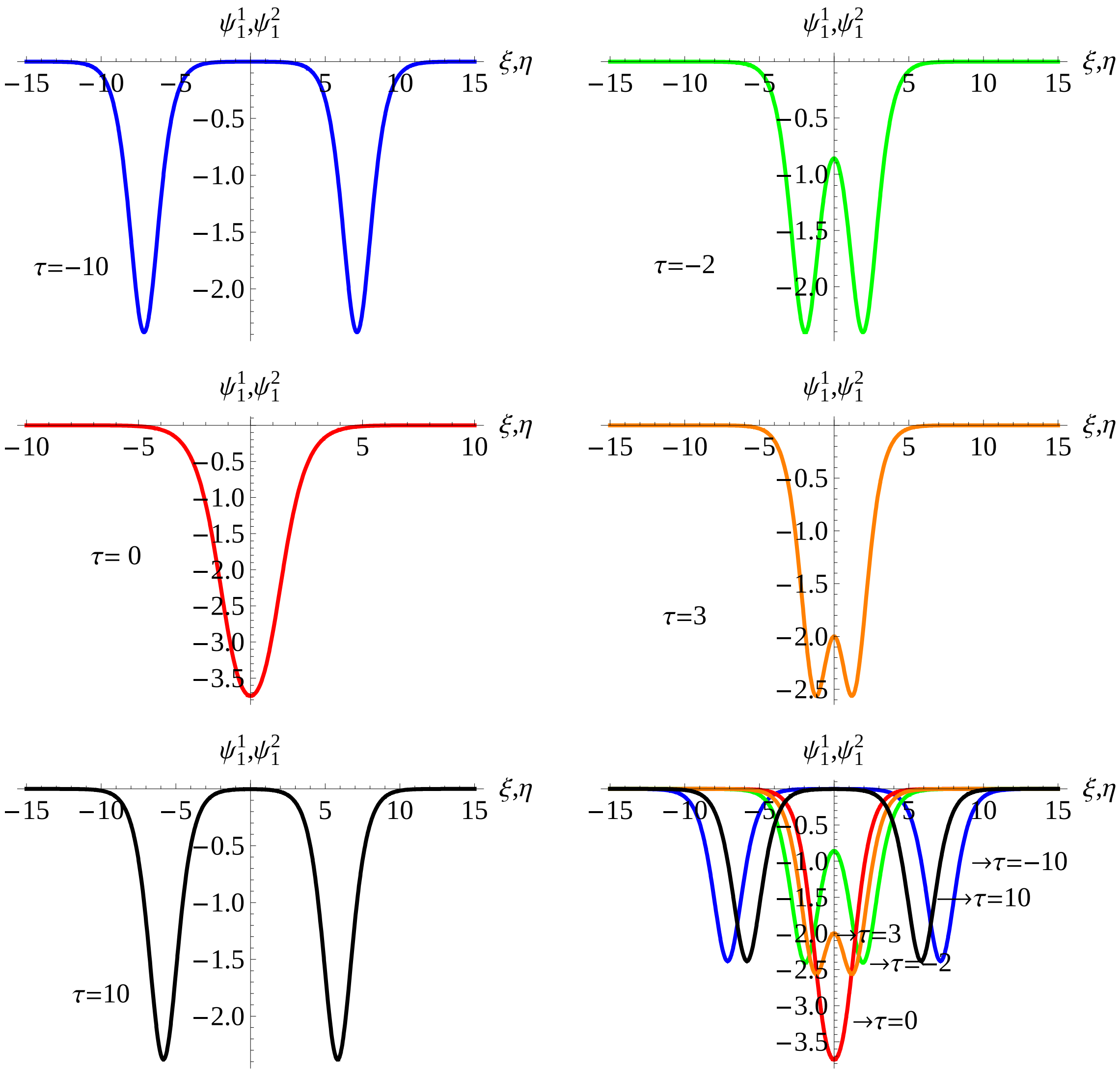


FIG. 2: Variation of the rarefactive one-soliton profiles $\psi_1^1$ and $\psi_1^2$ for different values of $\tau$ with $k_1 = 1, \alpha_1 = 1$ and $q = 2$.

detected, then a soliton is excited at each end of the main plasma at the same time. The solitons propagate toward each other and interact near the center of the main plasma.

In Figure 2, we have plotted a pair of rarefactive one-solitons $\psi_1^1$ and $\psi_1^2$ for several values of $\tau$ for time evolution of head on collision. It is seen that the soliton $\psi_1^1$ shifted towards right as time increases where as the soliton $\psi_1^2$ shifted towards left as time increases. It is seen that the two solitons preserve their shapes after collision which agrees with our analytical result.

In Figure 3, we have plotted a pair of compressive single solitons $\psi_1^1$ and $\psi_1^2$ for several values of $\tau$ for time evolution of head on collision. It is seen that the soliton $\psi_1^1$ shifted towards right as

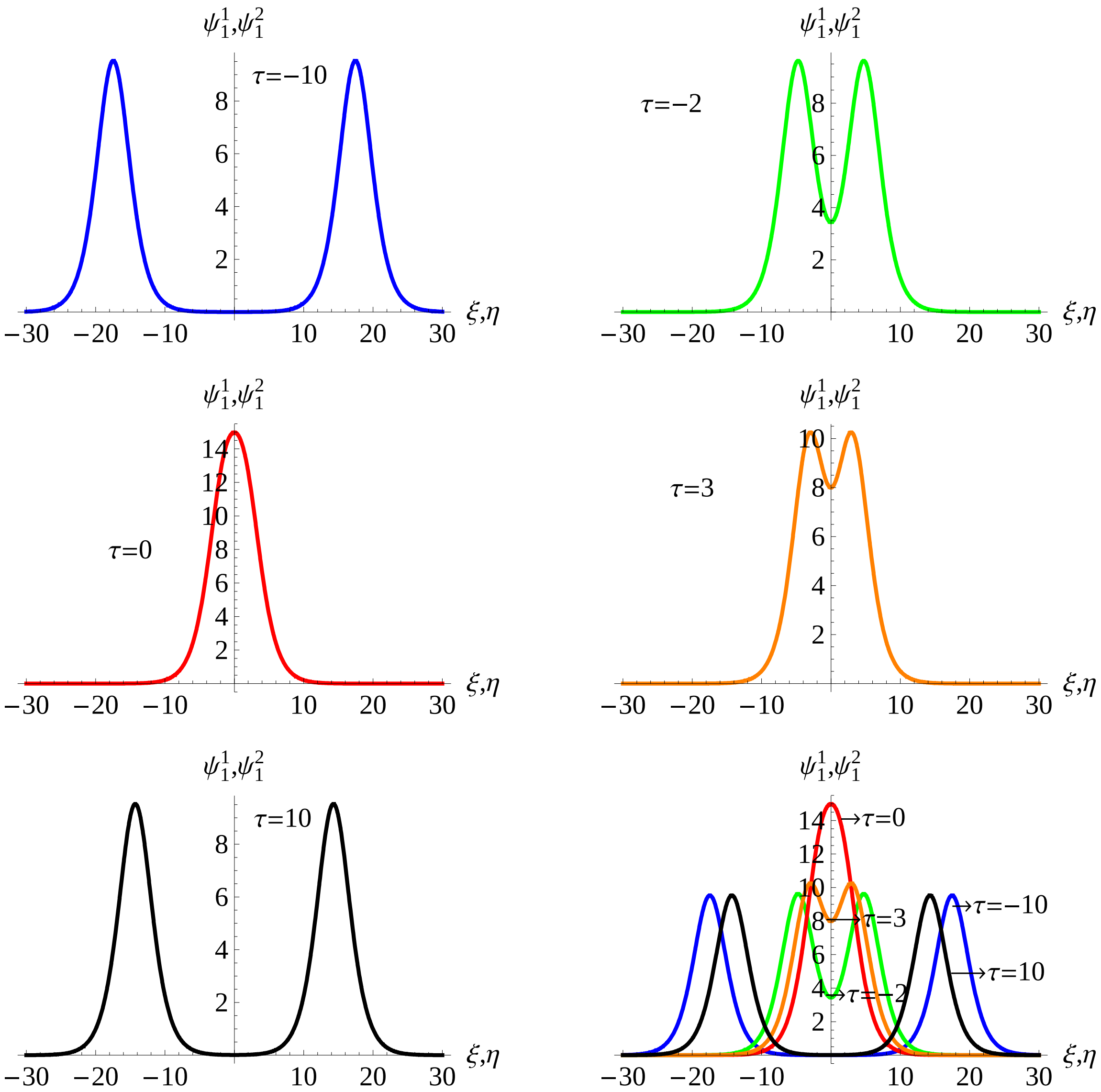


FIG. 3: Variation of the compressive one-soliton profiles $\psi_1^1$ and $\psi_1^2$ for different values of $\tau$ with $k_1 = 1, \alpha_1 = 1$ and $q = -0.5$.

time increases where as the soliton $\psi_1^2$ shifted towards left as time increases, which agrees with our analytical result.

The scattering of four rarefactive solitons will be clear from Figure 4. In Figure 4, we see the positions of the solitons when $\tau = -10$ the two solitons seen on the left hand side are moving to the right side and the two solitons on the right hand side moving towards the left side as we approach $\tau = 0$, the fast solitons on each side overtake their slower partners. The overtaking can be seen when $\tau = -2$ and $\tau = -1$. Again when $\tau = 0$, it shows the merge of four solitons.

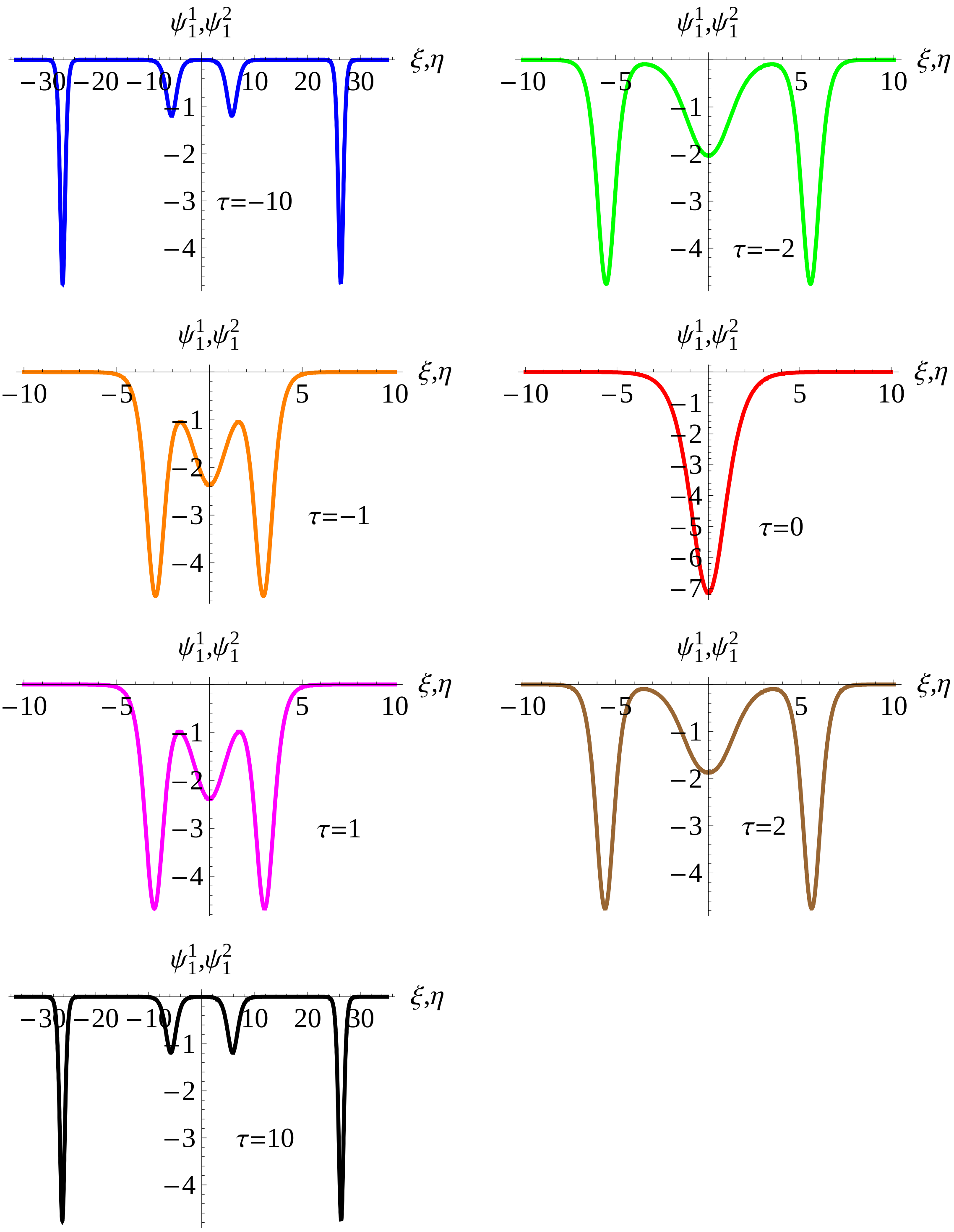


FIG. 4: Variation of the rarefactive two-soliton profiles $\psi_1^1$ and $\psi_1^2$ for different values of $\tau$ with $k_1 = 1, k_2 = 2, \alpha_1 = 1, \alpha_2 = 1$ and $q = 2$.

Positions of the solitons for $\tau = 2$ and $\tau = 1$ are just the mirror images of positions $\tau = -2$ and $\tau = -1$ respectively. So eventually each soliton acquires two phase shifts, one due to head on

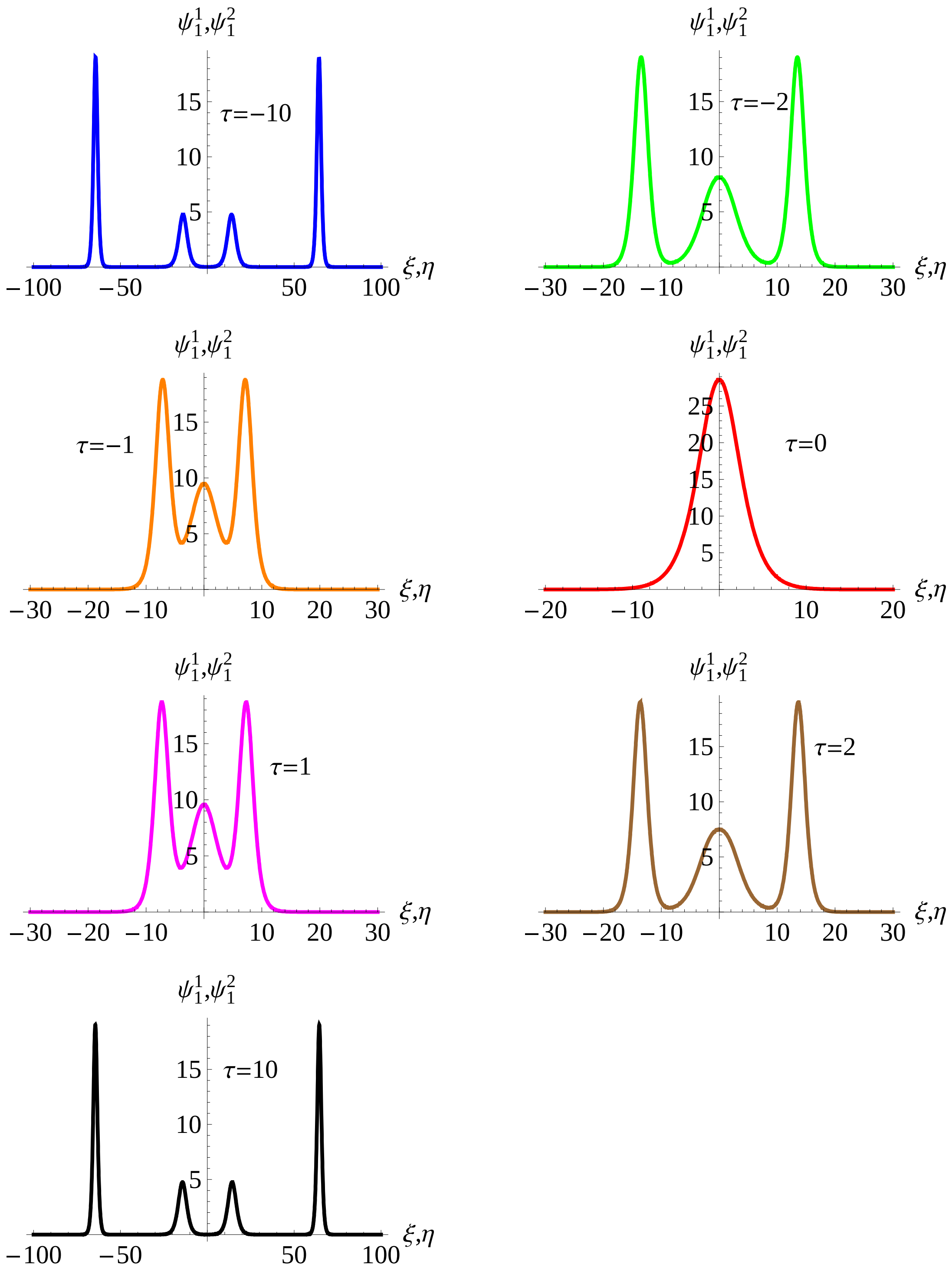


FIG. 5: Variation of the compressive two-soliton profiles $\psi_1^1$ and $\psi_1^2$ for different values of $\tau$ with $k_1 = 1, k_2 = 2, \alpha_1 = 1, \alpha_2 = 1$ and $q = -0.5$.

collision and the other because of over taking of one soliton by another.

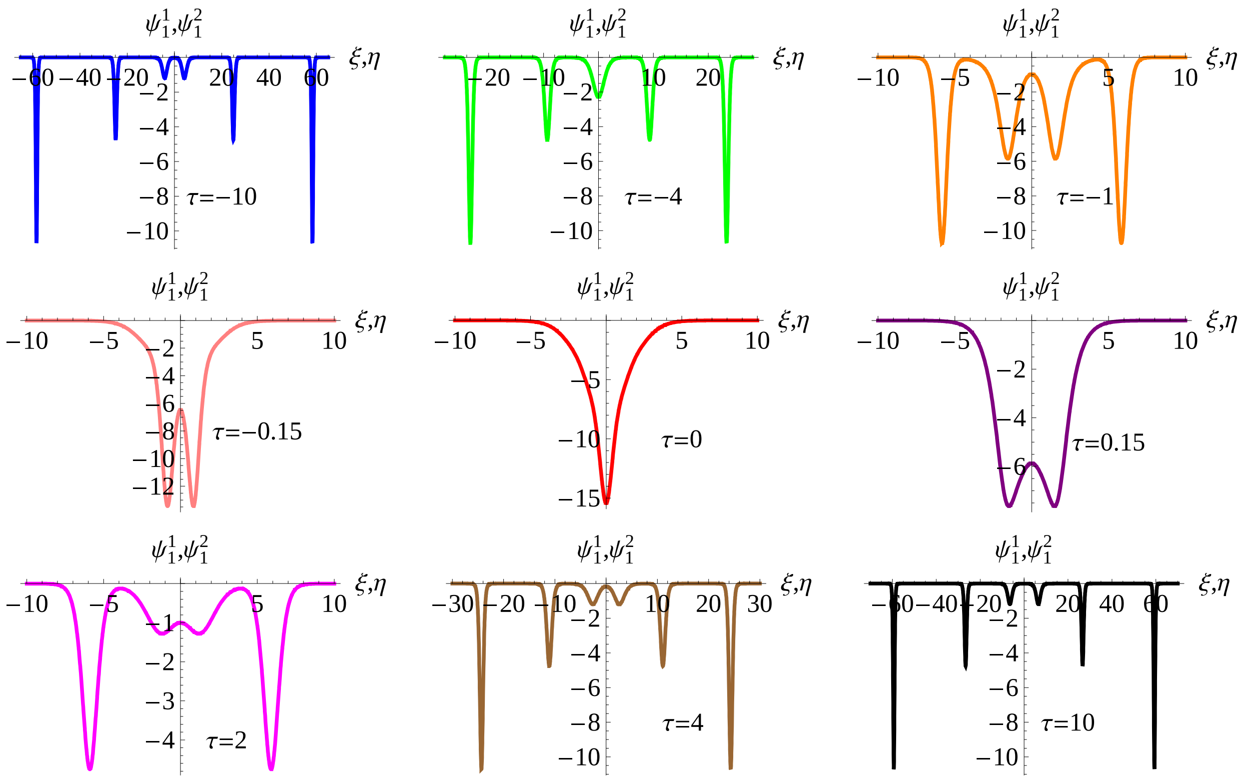


FIG. 6: Variation of the rarefactive three-soliton profiles $\psi_1^1$ and $\psi_1^2$ for different values of $\tau$ with $k_1 = 1, k_2 = 2, k_3 = 3, \alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 1$ and $q = 2$.

The scattering of four compressive solitons will be clear from Figure 5. In Figure 5, we see the positions of the solitons when $\tau = -10$ the two solitons seen on the left hand side are moving to the right side and the two solitons on the right hand side moving towards the left side as we approach $\tau = 0$, the fast solitons on each side overtake their slower partners. The overtaking can be seen when $\tau = -2$ and $\tau = -1$. Again when $\tau = 0$, it shows the merge of four solitons. Positions of the solitons for $\tau = 2$ and $\tau = 1$ are just the mirror images of positions $\tau = -2$ and $\tau = -1$ respectively. So eventually each soliton acquires two phase shifts, one due to head on collision and the other because of over taking of one soliton by another.

The scattering of six rarefactive solitons is presented in Figure 6. In Figure 6, we see the positions of the solitons when $\tau = -10$ the three solitons seen on the left hand side are moving to the right side and the three solitons on the right hand side moving towards the left side as we approach $\tau = 0$, the fast solitons on each side overtake their slower partners. The overtaking can be seen when $\tau = -4$ and $\tau = -1$. Again when $\tau = 0$, it shows the merge of six solitons. So eventually each soliton acquires two phase shifts, one due to head on collision and the other

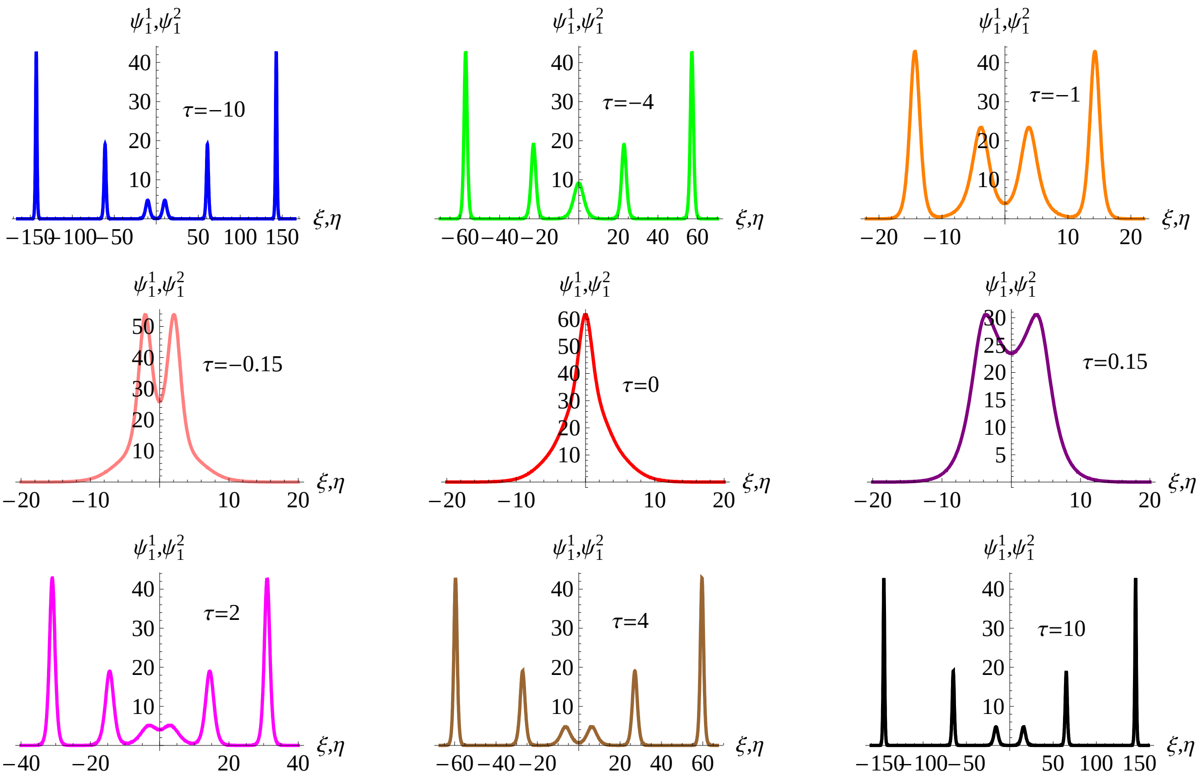


FIG. 7: Variation of the compressive three-soliton profiles $\psi_1^1$ and $\psi_1^2$ for different values of $\tau$ with $k_1 = 1, k_2 = 2, k_3 = 3, \alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 1$ and $q = -0.5$.

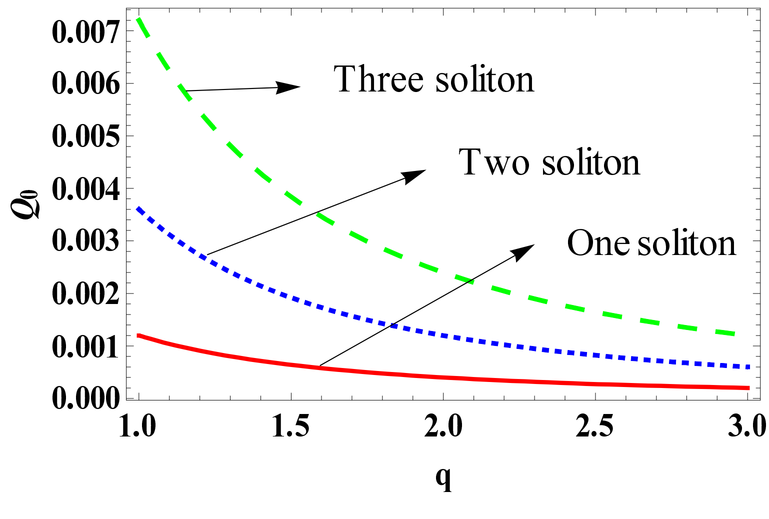


FIG. 8: Plots of the Phase shifts $Q_0$ against $q$ for One, Two and Three soliton.

because of over taking of one soliton by another.

The scattering of six compressive solitons is presented in Figure 7. In this figure, one can see the positions of the solitons when $\tau = -10$ and the three solitons seen on the left hand side, are moving to the right side and the three solitons on the right hand side moving towards the left side as we approach $\tau = 0$, the fast solitons on every side overtake their slower partners. The overtaking can be seen when $\tau = -4$ and $\tau = -1$. Again when $\tau = 0$, it shows the merge of four solitons. So eventually each soliton acquires two phase shifts, one due to head on collision and the other because of over taking of one soliton by another.

Figure 8 plots the phase shifts $Q_0$ against $q$ for one soliton, two Soliton and three soliton. Solid line, dotted line and dashed line represent the phase shifts of the one soliton, two soliton and three soliton, respectively. Phase shifts of the one soliton, two soliton and three soliton monotonically decrease with increase of the value of $q$ from 1 to 3. The phase shift $Q_0$ attains its maximum value when the value of $q$ is nearer to 1 for all one soliton, two soliton and three soliton.

## V. CONCLUSIONS

We have considered essentially head-on collisions and overtaking collisions of dust acoustic multi-solitons using a two step method, where we first derived two different KdV equations using the PLK method and then extracted one-soliton, twosolitons and threesolitons solutions for each KdV equation using the Hirota's direct method. In this work, we have studied head-on collisions of dust acoustic multi-solitons and phase shifts of dust acoustic multi-solitons in a two components unmagnetized dusty plasma for the first time to the best of our knowledge.

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