Exponential and Binomial Series Expansions in the Analysis of Temperature Profile of Solids Heated by Camera Flash

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Abstract. The solutions of time-dependent heat conduction problems for finite regions such as slabs and cylinders of finite radius are in the form of a series that converge rapidly for large values of diffusion time, *t* but, converge very slowly for small values of *t*, and hence are not suitable for numerical computations. Similarly, the solutions of many subsidiary equations resulting from the use of Laplace transform techniques are most often in hyperbolic forms. However, the Laplace transform conversion tables do not give inversion of such hyperbolic functions and, the integral inversion procedure is tedious and mathematically involved. A simple and straightforward procedure of expanding the resulting solution in negative exponentials and binomial series and the subsequent use of the Laplace conversion tables is hereby presented for inverting such solutions back to *t* domain. This method is supported by experimental determination of the thermal diffusivity of 50 μm thick aluminum, copper, nickel, silver and tungsten metals in one-layer solid configurations. By this analysis, the dual problem of finite pulse-time effects and heat loss from the surfaces of the samples, inherent in thermal diffusivity evaluation by the flash method are significantly minimized. When the experimental values obtained here are compared with other similar values found in literature, the agreement is within about ±3%

Keywords: Thermal diffusivity, Laplace transform, Hankel transforms, integral inversion technique, Binomial series expansion.

I. INTRODUCTION

Many scientific, engineering and technological problems involving heat transfer in nuclear reactor components (Fares, Nazar, Sha, & A, 2012), smelting of metals (Wang, Fang, Hang, & Gurrong, 2003), sintering of ceramics, (Zeng, Pal, & Stucker, 2012) and mass transport in underground water (Piechowski, 1997), etc, can be modeled by transient diffusion equations which are most often in second order partial differential equation (PDE) forms. Green's function approach, (Majumdar & Xia, 2007) and orthogonal expansion techniques provide much easier and straight forward methods for solving such problems but the solutions converge very slowly for small times and hence are not suitable for computations especially within small time limits. Laplace transform technique on the other hand provides solutions that are not only rapidly converging but also applicable for computations especially when small times are involved.

When one or more space variables are involved in the analysis of unsteady-state heat conduction problem or when two or more space variables are involved in steady-state heat conduction problem, partial differential equation (PDE) is obtained. Application of the Laplace transform technique changes the resulting equations from one governed by PDE to one governed by ordinary differential equation (ODE). Even though the application of Laplace transform in changing equations from PDE to ODE forms is generally simple and straight forward, the inversion of the resulting transform from the Laplace frequency domain *s*, to real time domain *t*, is mathematically tedious, unless the transform is available in the Laplace conversion tables. The Laplace transformation itself is not very satisfactory in treating problems involving more than one space variable, for such problems therefore, the best procedure is to rewrite the problem using integral transforms (Carslaw & Jaeger, 1959). This method like the Laplace transformation has the advantage of providing a routine procedure which is applied in the same way to all problems. The main disadvantage here is however its serious convergence difficulty which makes its application in solving heat diffusion problem purely formal.

By expanding the solution resulting from application of integral transform technique in negative exponentials and making use of the binomial series expansion, inversion to real time domain may be achieved using the Laplace conversion tables. Such solutions converge rapidly for small times (Carslaw & Jaeger, 1959; Ozisik, 1993), and hence the convergence difficulties associated with the method is reduced as presented in this analysis.

A. Mathematical Modeling

The first part of the mathematical analysis presented here is similar to ones by Kant in his analysis of laser induced heating of a multi-layered sample resting on a stationary half-space (Kant, 1988), and by Hui and Tan in their work on the thermal diffusivity of thin gold film, (Hui & Tan, 1993). However, unlike in their works where laser pulse is the source of heating, the heating here is initiated by a camera flash (NIKON SB 900) with flash width duration of about 10 milliseconds while a thermocouple at the back and center of the sample's rear face is used as temperature probe. The fact that a heating source with much longer width duration is used here instead of the shorter width in laser allows us the privilege of using an entirely different mathematical approach in arriving at the rear face temperature profile of the sample in real domain.

The physical problem may be described by a single-layer solid sample of thickness d, thermal diffusivity, α, density, ρ and thermal conductivity, k. This sample is instantaneously irradiated by the camera flash on the plane z = 0. Assuming the light from the camera flash is converted to heat on the sample's surface without causing any phase change; the energy propagates radially and axially through the sample and causes a temperature rise, T which is detected by the thermocouple at the rear face of the sample.

Let r be the radius of the circular heating ring, and let the layer be homogeneous and isotropic with heat propagating more along the z and r axes than it does along the φ axis in the cylindrical coordinate r, z, φ chosen. In other words, the heat transfer is two-dimensional along the r and z axes. As a consequence of Fourier's law and heat balance equation, the mathematical statement of this problem is expressed as, (Kant, 1988), (Hui & Tan, 1993)

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

there will be N such equations for N-layer solid composite. The corresponding initial condition for equation (1) is

$$T(r,\ z,\ t)=0\,;t=0. \tag{2}$$

The boundary conditions for perfect heat absorption at the sample's front surface is given as

$$-k\frac{\partial}{\partial z}T(r,z,t)=q(r,t);t=0 \tag{3}$$

and the adiabatic boundary condition at the rear face of the sample is given as

$$-k\frac{\partial}{\partial z}T(r,z,t)=0. \tag{4}$$

Application of integral transform technique transforms the problem from one governed by PDE to one governed by ODE. In the process however, the problem is converted from real time, t and spectral, r domains to frequency, s and spectral β domains in Laplace and Hankel spaces respectively

$$L[f(t)]=\bar{f}(s)=\int_0^\infty f(t)e^{-st}dt \tag{5}$$

Application of the integral transform in (5) transforms each of the four terms in equation (1) respectively to,

$$\int_0^\infty \frac{\partial^2 T_{(r,z,t)}}{\partial r^2}e^{-st}dt=\frac{\partial^2 \bar{T}_{(r,z,s)}}{\partial r^2} \tag{5a}$$

$$\int_0^\infty \frac{1}{r}\frac{\partial T_{(r,z,t)}}{\partial r}e^{-st}dt=\frac{\partial \bar{T}_{(r,z,s)}}{\partial r} \tag{5b}$$

$$\int_0^\infty \frac{\partial^2 T_{(r,z,t)}}{\partial z^2}e^{-st}dt=\frac{\partial^2 \bar{T}_{(r,z,s)}}{\partial z^2} \tag{5c}$$

$$\int_0^\infty \frac{1}{\alpha}\frac{\partial T_{(r,z,t)}}{\partial t}e^{-st}dt = -\frac{s}{\alpha}\bar{T}_{(r,z,s)} \tag{5d}$$

Hence equation (1) is rewritten as,

$$\frac{\partial^2\bar{T}}{\partial r^2} + \frac{1}{r}\frac{\partial\bar{T}}{\partial r} + \frac{\partial^2\bar{T}}{\partial z^2} = \frac{s}{\alpha}\bar{T}. \tag{6}$$

The Hankel integral transform on the other hand works on spatial variable to transform equation governed by r to one governed by β and is defined as,

$$H[g(r)] = \bar{g}(\beta) = \int_0^\infty f(r)J_0(\beta r)rdr. \tag{7}$$

Applying the transform in equation (7) to equation (6) taking note of the differential and integral properties of the Bessel function of the first kind and order zero, J_0 (βr) yields,

$$\frac{d^2\bar{\bar{T}}}{dz^2} - \gamma^2\bar{\bar{T}} = 0, \tag{8}$$

where

$$\gamma^2 = \alpha^{-1}s + \beta^2 \tag{9}$$

Here the first two terms in equation (6) have been evaluated using the transform in equation (7) as,

$$\int_0^\infty \left[\frac{\partial^2\bar{T}_{(r,z,s)}}{\partial r^2} + \frac{1}{r}\frac{\partial\bar{T}_{(r,z,s)}}{\partial r}\right] rJ_0(\beta r)dr = \int_0^\infty J_0(\beta r)\frac{d}{dr}\left(r\frac{\partial\bar{T}_{(r,z,s)}}{\partial r}\right)dr = \int_0^\infty udv \tag{9a}$$

by integrating the function above twice by parts and noting the differential properties of the Bessel function, $J_0(\beta r)$, namely,

$$\frac{d}{dr}[r^v J_v(\beta r)] = \begin{cases}\beta r^v J_{v-1}(\beta r)\\ -\beta r J_{v-1}(\beta r)\end{cases} \tag{9b}$$

$$\frac{d}{dr}[r^{-v} J_v(\beta r)] = \begin{cases}-\beta r^{-v} J_{v+1}(\beta r)\\ \beta r^{-v} J_{v+1}(\beta r)\end{cases} \tag{9c}$$

yields,

$$\int_0^\infty \left[\frac{\partial^2\bar{T}_{(r,z,s)}}{\partial r^2} + \frac{1}{r}\frac{\partial\bar{T}_{(r,z,s)}}{\partial r}\right] rJ_0(\beta r)dr = -\beta^2\bar{\bar{T}}(\beta,z,s). \tag{9d}$$

Using the transform in equation (7), the third term in equation (6) is also evaluated as,

$$\int_0^\infty \frac{\partial^2 \bar{T}}{\partial z^2} r J_0(\beta r) dr = \frac{d^2}{d^2 z} \bar{\bar{T}}(\beta, z, s) \tag{9e}$$

while the last term in equation (6) is evaluated as,

$$\int_0^\infty -\frac{s}{\alpha} \bar{T}_{(r,z,s)} r J_0(\beta r) dr = -\frac{s}{\alpha} \bar{\bar{T}}(\beta, z, s). \tag{9f}$$

This formally completes the procedure for the transformation of equation (1) using the Laplace and Hankel integral transforms to obtain equation (8) in ODE form.

Using the same procedure as above, equations (2 – 4) for the initial and boundary conditions are transformed respectively as;

$$\bar{\bar{T}}(\beta, z, s) = 0\,; s \to \infty \tag{10a}$$

$$-k \frac{d}{dz} \bar{\bar{T}}(\beta, z, s) = q(\beta, s); z = 0 \tag{10b}$$

$$-k \frac{d}{dz} \bar{\bar{T}}(\beta, z, s) = 0;\ z = d. \tag{10c}$$

We note that equation (8) is a second order ODE with characteristic roots given as $\pm\gamma_i$ and therefore has a general solution of the form, (Hui & Tan, 1993)

$$\bar{\bar{T}}(\beta, z, s) = A(\beta, s) e^{\gamma z} + B(\beta, s) e^{-\gamma z} \tag{11}$$

The coefficients of the constants A and B in equation (11) are arbitrary constants which are determined using the initial and boundary conditions to obtain the particular solution to equation (8).

Integral inversion technique using residue theorem is used by various workers in this field, for inverting the solution back to real time and space domains. Solutions resulting from such inversions are however not only mathematically difficult but their slow convergence at small values of t exposes the solution to finite pulse-time effect and heat loss problems from the surfaces of the sample. In this work therefore, we introduce a new method of inversion process using the tables of Laplace transform. This method has the advantages of being easy and straightforward, minimizes the problems posed by finite pulse effect and non-ideal situation at the surfaces of the experimental sample as well as providing solutions that have little or no convergence difficulties at small values of t.

B. Solution to the Problem

Considering the simplest case in which the heat pulse is deposited on the surface of a single-layer solid sample with adiabatic boundary conditions on the upper and lower surfaces of the sample as in this work; the solution to equation (11) is obtained as follows.

Adiabatic boundary at the rear surface of the sample implies;

$$B = Ae^{-2\gamma d}, \tag{12}$$

adiabatic condition at the top surface of the sample implies

$$A = B - \frac{q(\beta,s)}{k\gamma}. \tag{13}$$

From equations (12) and (13), we obtained the constants as;

$$A = \frac{q(\beta,s)}{k\gamma(1+e^{-2\gamma d})}, \tag{14}$$

$$B = \frac{e^{-2\gamma d}q(\beta,s)}{k\gamma(1+e^{-2\gamma d})}. \tag{15}$$

Hence equation (11) becomes,

$$\bar{\bar{T}}(\beta,z,s) = q(\beta,s)\frac{e^{\gamma(d+z)}+e^{-\gamma(d+z)}}{k\gamma(e^{\gamma z}+e^{-\gamma z})} \tag{16a}$$

with $q(\beta,s)$ defined as $q/_{2\pi}$, (Hui & Tan, 1993, 1994a, 1994b). Equation (16a) can easily be converted to real time domain term by term using the Laplace transform conversion tables, (Carslaw & Jaeger, 1959; Ozisik, 1993) as illustrated below.

Since the inversion of equation (16a) is not available in the Laplace transform conversion tables, we introduced a method of inversion via the same tables by first expanding the function represented by equation (16a) in negative exponentials and binomial series.

This however requires that the radial component, β of the heat diffusion equation is set to zero to allow for expansion in negative exponentials and the subsequent use of the tables of Laplace transform for inversion. We note that this procedure does not pose any problem in situations where the radial diffusion of heat is totally not considered. In diffusion problems where the radial diffusion of heat is as important as the axial diffusion of heat, however, the radial diffusion part must be regained back after the inversion. Moreover, heat diffusion problems can be broken into two independent components; axial and radial diffusion parts, (Kehoe et al., 2009; Murphy et al., 2005), (Kant, 1988), (Hui & Tan, 1993), (Carslaw & Jaeger, 1959; Ozisik, 1993).

Hence, with $\beta = 0$, equation (16a) becomes,

$$\bar{\bar{T}}(0,z,s) = \frac{q}{2\pi}\frac{\left(e^{\sqrt{s/\alpha}(z-d)}+e^{-\sqrt{s/\alpha}(z-d)}\right)}{k\sqrt{s/\alpha}\left(e^{\sqrt{s/\alpha}d}-e^{-\sqrt{s/\alpha}d}\right)}. \tag{16b}$$

This is rewritten as,

$$\bar{\bar{T}}(z,s) = \frac{q}{2\pi}\frac{1}{k}\frac{\left(e^{(z-2d)\sqrt{s/\alpha}}+e^{-z\sqrt{s/\alpha}}\right)}{\sqrt{s/\alpha}\left(1-e^{-2d\sqrt{s/\alpha}}\right)}, \quad (17)$$

or,

$$\bar{\bar{T}}(z,s) = \frac{q}{2\pi}\frac{1}{k}\frac{\left(e^{(z-2d)\sqrt{s/\alpha}}+e^{-z\sqrt{s/\alpha}}\right)}{\sqrt{s/\alpha}}\left(1-e^{-2d\sqrt{s/\alpha}}\right)^{-1}. \quad (18)$$

The last term in equation (18) is expanded in binomial series as,

$$\left(1-e^{-2d\sqrt{s/\alpha}}\right)^{-1} = \sum_{n=0}^{\infty} e^{-2nd\sqrt{s/\alpha}} \quad (18a)$$

Hence, equation (18) finally becomes,

$$\bar{\bar{T}}(z,s) = \frac{q}{2\pi}\frac{1}{k}\frac{\sum_{n=0}^{\infty} e^{-K_1\sqrt{s}}}{\sqrt{s/\alpha}} + \frac{q}{2\pi}\frac{1}{k}\frac{\sum_{n=0}^{\infty} e^{-K_2\sqrt{s}}}{\sqrt{s/\alpha}} \quad (19)$$

Here, the constants k_1 and k_2 are defined as

$$K_1 = \frac{(z-2d-2nd)}{\sqrt{\alpha}}, \quad (19a)$$

and

$$K_2 = \frac{(z+2nd)}{\sqrt{\alpha}}. \quad (19b)$$

The inversion of the solution in equation (19) is readily available from the table of Laplace transforms, for example, (Ozisik, 1993), Table 7-1, case 43 or, (Carslaw & Jaeger, 1959), Appendix V, case number 7. With this inversion, equation (19) finally becomes,

$$T(z,t) = \frac{q}{2\pi}\frac{\sqrt{\alpha}}{k}\frac{1}{\sqrt{\pi t}}\sum_{n=0}^{\infty}\left(e^{-\frac{(z-2d-2nd)^2}{4\alpha t}} + e^{-\frac{(z+2nd)^2}{4\alpha t}}\right) \quad (20)$$

for converging thermal wave technique adopted in this work, the radial diffusion of heat is as important as the axial diffusion of heat hence, the radial component is regained by multiplying each of the two terms in equation (20) by the independently transformed radial component, (Hui & Tan, 1993), (Kant, 1988), to get,

$$T(z,r,t) = \frac{q}{2\pi}\frac{\sqrt{\alpha}}{k}\frac{1}{\sqrt{\pi t}}\sum_{n=0}^{\infty}\left(e^{-\frac{(z-2d-2nd)^2}{4\alpha t}} + e^{-\frac{(z+2nd)^2}{4\alpha t}}\right)e^{-\frac{r^2/4\alpha t}{2\alpha t}}. \quad (21)$$

Equation (21) is used as the mathematical model for the experimental work in this analysis.

We note that the first four terms (n = 0, 1, 2, 3) of the series in equation (21) above are enough to give the temporal temperature profile of the camera-flash irradiated single-layer metal sample with thickness $\leq 200\ \mu m$.

We also note that equation (21) is very similar to that obtained by Hui and Tan in their analysis of temperature profile for a single-layer solid using the Laplace transform technique, (Hui & Tan, 1993) and therefore, the two are compared to verify the validity of the method adopted in this work.

C. Thermal Diffusivity Experiment

The experimental set up shown in figure (1) which is used for this work is the Camera Thermal Flash set up for the Applied Optics Laboratory of the Department of Physics, Faculty of Science, University Putra, Malaysia.

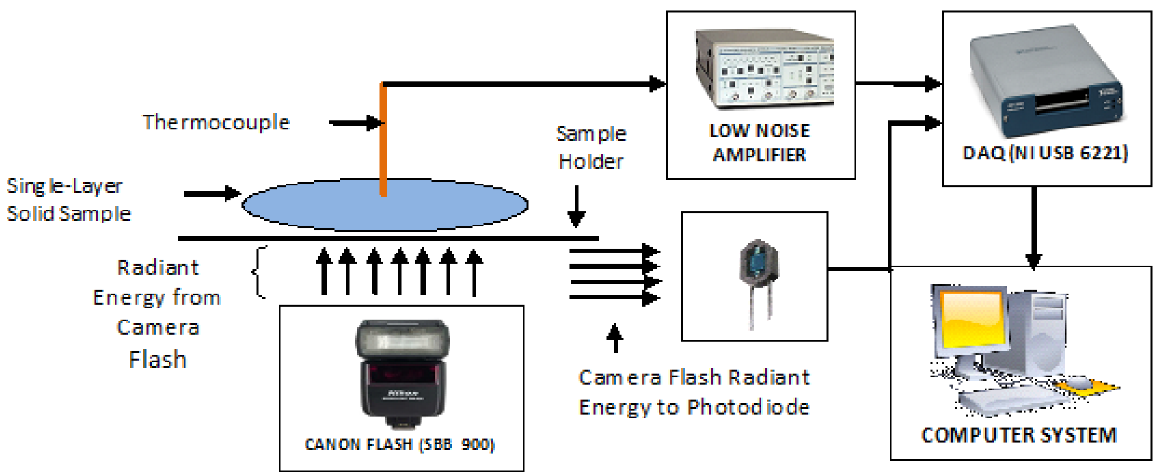

FIGURE 1: Schematic diagram of experimental set

The camera flash irradiates the sample of radius R on the sample's front surface. The resulting heat subsequent to the beam absorption is detected as temperature signal at the sample's rear surface directly below the center of the sample holder by a system of thermocouple and photodiode. The temperature signal is then fed to a low-noise amplifier and data acquisition (DAQ) and processing system.

In each sixty (60) seconds of the thermal flash experiment, about 300,000 data points are recorded. Each set of data is therefore exported to Origin software for noise reduction, data normalization, and interpolation to reduce the data to manageable size. The data is then exported to Mathematica Software for simulated fitting session (with equation (21)). Finally, each set of data point is exported to QtiPLOT software for final fitting, error estimation and residual plotting.

The fittings of the single-layer theoretical model to the experimental data to find the thermal diffusivity values for the aluminum, copper, nickel, silver and tungsten foils respectively are done using a Scaled Levenberg-Marquardt algorithm with 10 iterations and tolerance of 0.001 with no weighting.

II. RESULTS AND DISCUSSIONS

To show the appropriateness of our derived mathematical model, represented by equation (21), we compare with a model derived by similar work found in literature, (Hui & Tan, 1993), and the two plots are as shown in figure 2.

We note that both curves shown in figure 2 have good delay times as a characteristic feature of curves derived using the Converging Thermal Wave Method. They also rise steadily to attain their corresponding maximum temperatures and finally decay exponentially to zero at later times. The delay time recorded for curve (a) is however around 0.8s while that for curve (b) is around 0.2s. As properties of tungsten foil are chosen for plotting the two curves, the delay time for curve (a) is more appropriate since the radial diffusion of heat takes appreciable time before converging at the center of the sample for detection, (Kehoe, Murphy, & Kelly, 2009; Murphy, Kehoe, Pietralla, Winfield, & Floyd, 2005), (Stehl, Schreck, Fischer, Gsell, & Stritzker, 2010).

The time for the transient temperature curves to reach their corresponding maximum temperature is also different for the two curves. Whereas, the time for curve (a) is obtained to be around 4.7 s that for curve (b) is recorded to be around 2.0s.

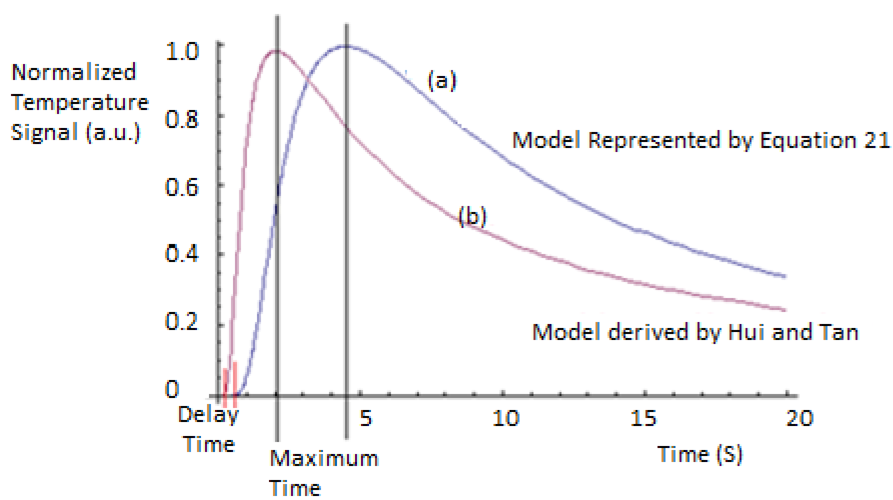

FIGURE 2: Temperature transients for the two curves

Both axial and radial parts are then plotted together with total components of the two curves and are as shown in figures 3 and 4 for the model developed in this analysis and that of Hui and Tan, respectively.

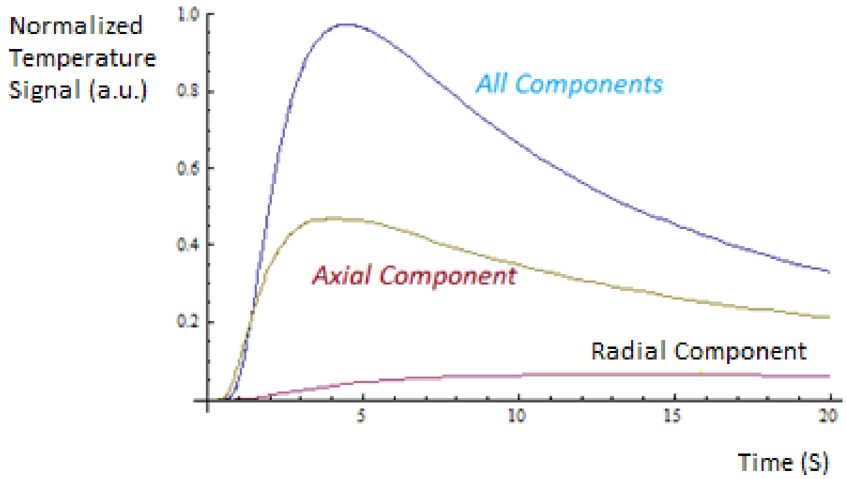

FIGURE 3: Plotted parts of components of the Theoretical Model Developed

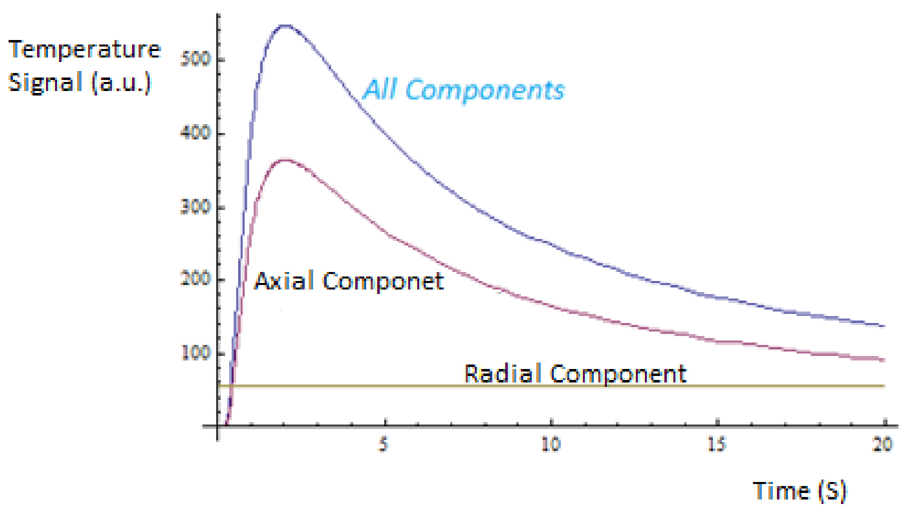

FIGURE 4: Plotted parts of components of the Theoretical Model Derived by Hui and Tan

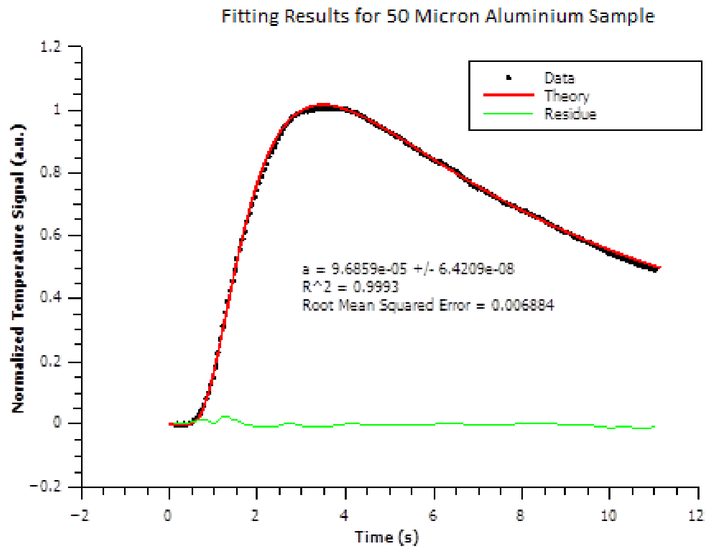

FIGURE 5: Fitting Results for Al (50 µm Thick) showing thermal diffusivity value

The mathematical theory developed in this work (equation 21) is applied to the experimental determination of the thermal diffusivity values for the five metals studied in this analysis. This is achieved by fitting experimental data with the theoretical model using QtiPlot software and the results are as shown in figures 5 – 9. These results are tabulated and compared to similar results from standard laser flash experiment and are as shown in Table 1 below.

The agreement of the results obtained in this analysis with values available in literature indicates the feasibility of using the analysis presented in this work to obtain the temperature-time history of a camera flash irradiated solid sample and possibility of using the Laplace transform conversion table in place of the tedious conversion by contour integral method.

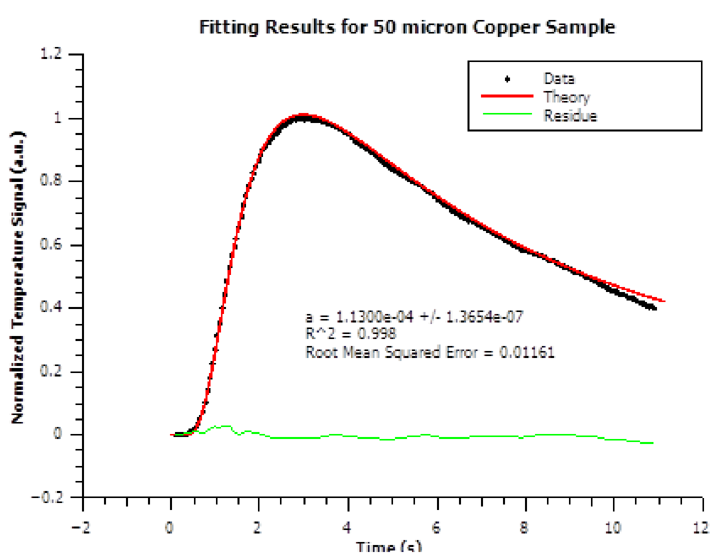

FIGURE 6: Fitting Results for Cu (50 um Thick) showing thermal diffusivity value

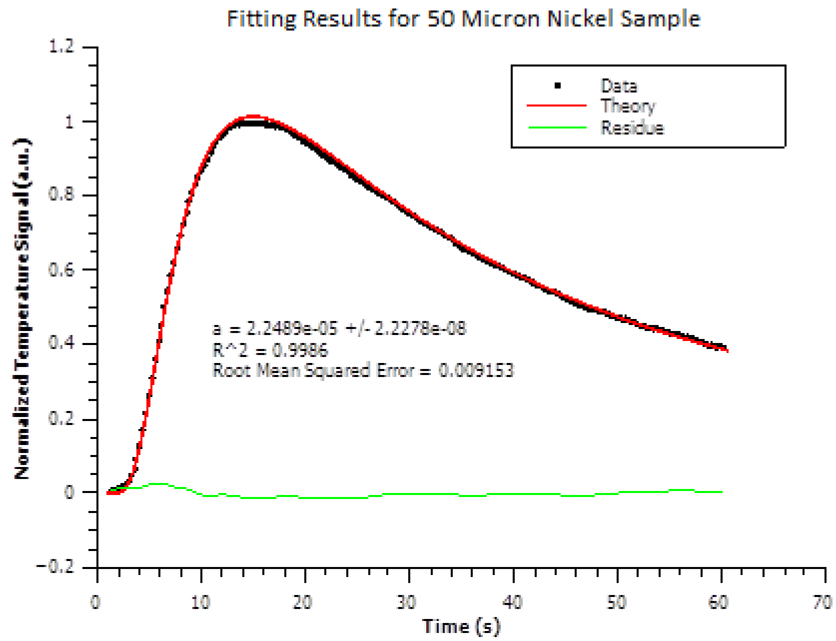

FIGURE 7: Fitting Results for Ni (50 µm Thick) showing thermal diffusivity value

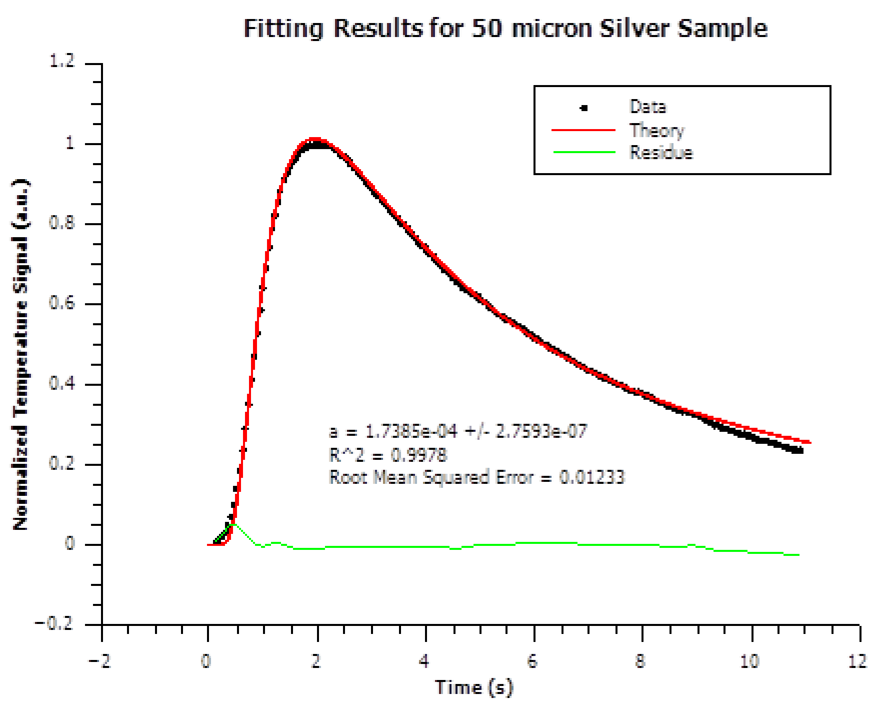

FIGURE 8: Fitting Results for Ag (50 µm Thick) showing thermal diffusivity value

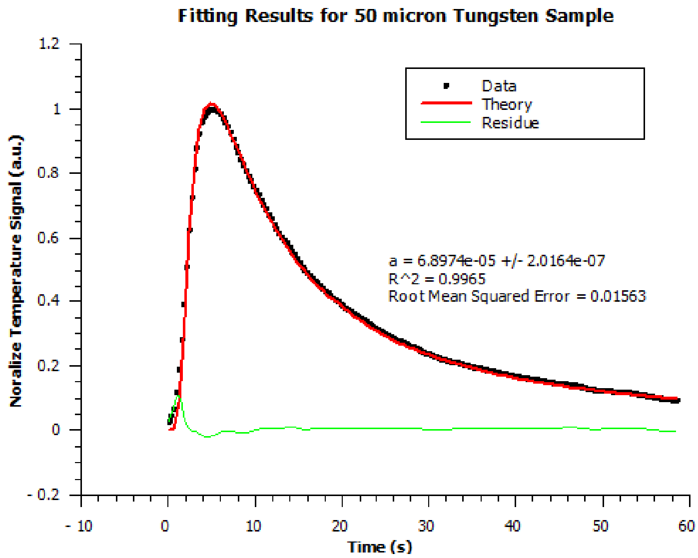

FIGURE 9: Fitting Results for W (50 μm Thick) showing thermal diffusivity value

TABLE 1: Comparison of Thermal Diffusivity Values with Literature

Metal	Purity (%)	Thickness (μm)	Thermal Diffusivity Value $\times 10^{-5} m^2 s^{-1}$ Present Work	Thermal Diffusivity Value $\times 10^{-5} m^2 s^{-1}$ Literature value
Aluminum	99.95	50.00	9.69	9.54
Copper	99.95	50.00	11.30	11.62
Nickel	99.95	50.00	2.25	2.31
Silver	99.97	50.00	17.39	-
Tungsten	99.96	50.00	6.90	-

III. CONCLUSIONS

For small diffusion time, expansions in negative exponentials and binomial series may be used to evaluate the temperature profile for a camera flash irradiated sample. This simplifies the mathematical theory as Laplace conversion table is used in place of the difficult inversion by integral method in obtaining the full-field temperature profile for the irradiated sample. The mathematical theory is verified by the experimental determination of the thermal diffusivity values of five different metal foils of equal thickness and the agreement of the evaluated thermal diffusivities with literature values is found to be within about 3% error.

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