Pseudopotential Approach for Dust Ion Acoustic Solitary Waves in Quantum Dusty Plasmas

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Abstract. Sagdeev's pseudopotential method is used to study the propagation characteristics of dust ion acoustic solitary waves in an unmagnetized quantum dusty plasma by employing one dimensional quantum hydrodynamic model together with the Poisson equation. The asymptotic expansion has been used to obtain the pseudopotential function for small value of quantum diffraction parameter H. The existence domain of quantum dust ion acoustic wave has been investigated in terms of true Mach number limits. The numerical results show that the dust impurities can affect the amplitude, width and also the existence domain of the soliton. Effects of the quantum diffraction parameter on the width of the soliton is also discussed.

Keywords: Dust ion acoustic waves, Mach number, Sagdeev potential, Quantum diffraction parameter.

I.   INTRODUCTION

Quantum effects in plasmas may exhibit in different plasmas systems. It may play an important role when the de-Broglie wavelength of the plasma particles are comparable to the dimension of the system. In various natural and laboratory situations viz. superdense astrophysical bodies [1,2], nonlinear quantum optics [3], ultra small electronics devices [4], metallic nanostructures [5], etc, the existence of low temperature and high particle number density have been observed where the quantum effects can not be ignored. Recently it has been considered to study the quantum plasma echoes [6], expansion of quantum electron gas into vacuum [7], quantum Landau damping [8]. The quantum plasmas are studied mainly by two approaches, namely, quantum hydrodynamic (QHD) approach and quantum kinetic approach. The kinetic approach is needed to discuss the Landau damping of waves in a quantum plasma. In presence of electromagnetic fields, Wigner-Maxwell systems of equations are used for kinetic description of quantum plasmas. The mathematical derivation of QHD model was given by Madelung [9] long time back. Using the Schrödinger-Poisson system of equations the hydrodynamic equations can be derived [10]. A new force term in the form of gradient of Bohm potential [11] appears in the momentum equation which is due to the quantum tunnelling effects. As the plasma particles obey Fermi-Dirac distribution the pressure term in the momentum equation is described by the Fermi pressure law which includes the quantum statistical effects. Thus mathematical formulation for classical plasmas is suitably

modified by including these two quantum characteristics. Using QHD various plasma characteristics have been investigated by several authors.

Haas et al. [12] considered the one dimensional QHD model in the limit of small mass ratio of the charge carriers and briefly studied the effects of quantum diffraction parameter in linear as well as nonlinear regimes in an unmagnetized quantum plasma. Ali et al. [13] studied the linear and nonlinear properties of ion-acoustic waves in an unmagnetized electron-positron-ion quantum plasmas by employing one dimensional QHD model. Later, Haas [14] extended the QHD model for charged particle system to the case of non zero magnetic fields by deriving quantum magnetohydrodynamics model. Quantum electron acoustic [15-17] and quantum positron acoustic [18] solitary waves have been studied also by some authors. Ata-ur-Rahman et al. [19] studied the amplitude modulation of quantum-ion-acoustic wavepackets in electron-positron-ion plasmas. Using QHD model quantum ion acoustic wave has been investigated in carbon nanostructures [20] and metallic nanowires [21]. However, dust impurities may exist in quantum plasmas. For example, the microelectronic devices or metallic nanostructure are usually contaminated by highly charge dust impurities. The presence of dust clouds around white dwarfs has been reported [22]. In white dwarfs and neutron stars, the behaviour of the plasma particles can be approximated by treating them as a quantum dusty system. Recently, Sharma et al. studied the effect of Fermi pressure and Bohm potential on Jeans instability of quantum dusty plasma [23]. Ali and Shukla [24] studied the dust acoustic solitary wave propagation in a quantum plasmas. Misra and Roy Chowdhury [25] studied the amplitude modulation of dust-acoustic waves in a three-species quantum dusty plasma. Nonlinear quantum dust acoustic waves in nonuniform complex quantum dusty plasmas have been studied by El-Taibany et al. [26]. The quantum statistical as well as quantum diffraction effects on the quantum dusty magnetosonic wave in a quantum plasma has been studied by Wang et al. [27]. The dynamics of quantum dust-acoustic double layers [28] have been investigated in an unmagnetized quantum dusty plasma. The linear and nonlinear properties of dust ion acoustic waves have been studied by Masood et al. [29]. Khan and Mushtaq [30] also studied dust ion acoustic waves in a ultracold quantum plasmas. Treating the charged carbon nanotubes as the charged dust which are surrounded by electron and ion, dispersion properties of dust acoustic waves has been studied by Shukla [31]. A charged multiwalled carbon nanotube, which is surrounded by charged nanoparticles, is modeled as a cylindrical shell of electron-ion-dust plasma and dust ion acoustic waves oscillations is predicted theoretically by Fathalian and Shahram [32].

Recently, Hanif et al. [33] employed a numerical technique to study ion acoustic shock waves in a dense quantum plasmas. In order to study arbitrary amplitude solitary wave Sagdeev's pseudopotential method have been used by many authors. Inclusion of Bohm potential term in the momentum equation makes the task of finding the closed form analytical expression of pseudopotential difficult. However, Mahmood and Mushtaq [34] studied ion-acoustic wave propagation in an unmagnetized quantum plasmas by using Sagdeev's pseudopotential approach under quasineutrality condition. Later, Mahmood [35] employed the same method to study the dust ion acoustic waves in dense Fermi plasmas. In this present work Sagdeev's pseudopotential method have been employed without using the quasineutrality condition. In order to find pseudopotential function, asymptotic expansion is being used.

II.   BASICS EQUATIONS

Here an unmagnetized three component quantum dusty plasma composed of electrons, ions and negatively charged immobile dust particles is considered. In order to study dust ion acoustic (DIA) waves in a quantum dusty plasmas, the electrons are supposed to be inertialess and the phase velocity of the wave is assumed to be $v_{Fi} \ll \frac{\omega}{k} \ll v_{Fe}$ where v_{FS} is the Fermi velocity of the electrons($s = e$) and ions($s = i$). Then the one dimensional quantum hydrodynamic model for this system is governed by the following equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \tag{1a}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{e}{m_i}\frac{\partial \phi}{\partial x} = 0 \tag{1b}$$

$$0 = e\frac{\partial \phi}{\partial x} - \frac{1}{n_e}\frac{\partial p_e}{\partial x} + \frac{\hbar^2}{2m_e}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2}{\partial x^2}\left(\sqrt{n_e}\right)\right] \tag{1c}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_e + Z_{d0}n_{d0} - n_i) \tag{1d}$$

where n_s, v_s, m_s are the number density, fluid velocity and mass of the species electron ($s = e$) and ion ($s = i$) respectively, ϕ is the electrostatic potential, n_{d0} is the equilibrium dust number density, Z_{d0} is the number of electrons residing on the dust grains, $\hbar$ is the Planck's constant and $-e$ (e) is the electron (ion) charge. Here the electrons are assumed to follow the one dimensional zero-temperature Fermi gas pressure law [12]

$$p_e = \frac{m_e v_{Fe}^2}{3n_{e0}^2}n_e^3 \tag{2}$$

where the Fermi electron velocity is given by $v_{Fe} = \sqrt{2K_B T_{Fe}/m_e}$, K_B is the Boltzmann constant and T_{Fe} is the Fermi temperature. The charge neutrality condition at equilibrium is given by

$$n_{i0} = n_{e0} + Z_{d0}n_{d0} \tag{3}$$

Now, Eq.(1a)-(1d) can be written in the normalized form as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \tag{4a}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{4b}$$

$$0 = \frac{\partial \phi}{\partial x} - n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2\delta}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2}{\partial x^2}\left(\sqrt{n_e}\right)\right] \tag{4c}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \delta\, n_e + d - n_i \tag{4d}$$

where the wave potential ϕ is normalized by $\frac{2K_B T_{Fe}}{e}$, the ion fluid velocity v_i is normalized by quantum ion acoustic speed $C_0 = \left(\frac{2K_B T_{Fe}}{m_i}\right)^{\frac{1}{2}}$ and n_s is normalized by its unperturbed density n_{s0} ($s = e, i$). The space and time coordinates are normalized by the ion Fermi wave length in quantum plasma $\lambda = \left(\frac{2K_B T_{Fe}}{4\pi n_{i0}e^2}\right)^{\frac{1}{2}}$ and ion plasma period $\omega_{pi}^{-1} = \left(\frac{m_i}{4\pi n_{i0}e^2}\right)^{\frac{1}{2}}$, respectively. Here the dust density parameter $d = \frac{Z_{d0}n_{d0}}{n_{i0}}$, $\delta = \frac{n_{e0}}{n_{i0}}$, electron plasma period $\omega_{pe} = \left(\frac{4\pi n_{e0}e^2}{m_e}\right)^{\frac{1}{2}}$ and the nondimensional quantum parameter H is defined as $H = \frac{\hbar\omega_{pe}}{2K_B T_{Fe}}$. The charge neutrality condition (3) implies $\delta = 1 - d$.

3

III.   LINEAR WAVES

To study the linear properties of quantum DIA waves, Eq.(4a)-(4d) have been linearized by writing the dependent variables as a sum of equilibrium and perturbed parts. Assuming that the all the perturbed quantities are varying as $exp\, i(kx - \omega t)$, where k is the wave number and ω is the wave frequency, the dispersion relation for quantum dust ion acoustic waves is obtained as

$$\omega^2 = \frac{k^2\left(1+\frac{k^2 H^2}{4\delta}\right)}{\delta + k^2\left(1+\frac{k^2 H^2}{4\delta}\right)} \qquad (5)$$
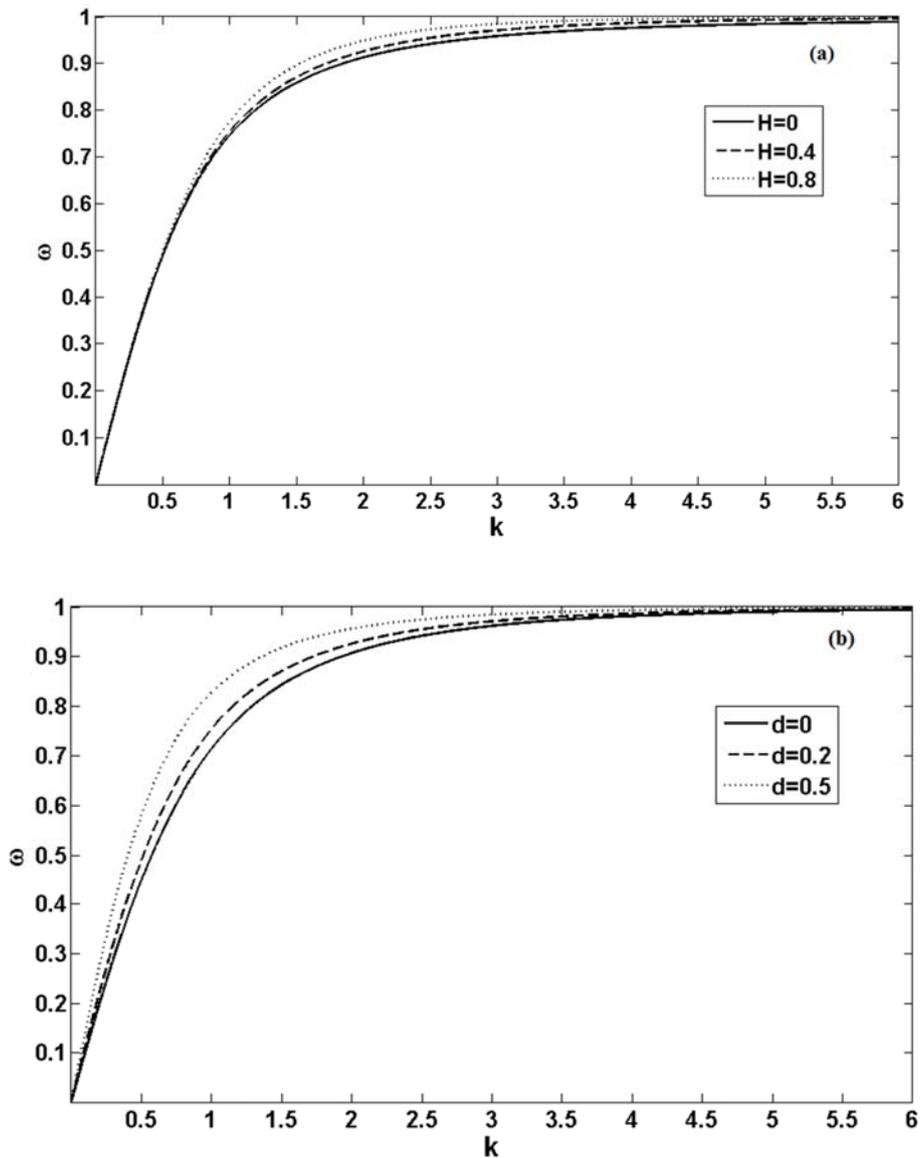
where $\delta = 1 - d$.

FIGURE 1. Plot of ω vs k for (a) different values of H at $d = 0.2$ (b) different values of d at $H = 0.4$.

The above relation (5) shows that the phase velocity is affected by quantum correction together with the dust concentration. In long wavelength limits ($k^2 \ll 1$) the dispersion relation will be $\omega \approx k$. But the QHD model is not applicable for small wavelengths [12]. In the absence of dust components the dispersion relation (5) reduces to the dispersion relation for quantum ion acoustic waves as obtained in Ref. 12. The linear variation of wave frequency ω with the wave number k is plotted in in Fig. 1(a) and 1(b) for different values of H and d, respectively. It is observed that the asymptotic value $\omega = 1$ reached faster as we increase the value of H and d, respectively. Also the phase velocity increases due to the presence of dust components.

IV. ARBITRARY AMPLITUDE SOLITONS

To obtain travelling wave solutions of the Eqs. (4a)-(4d) that are stationary in a frame moving with a velocity M, we suppose that all the dependent variables depend on $\xi = x - Mt$; M being the Mach number normalized to the quantum ion acoustic speed C_s. Then Eq. (4a) and (4b) reduces to

$$n_i = \frac{M}{M - v_i} \tag{6a}$$

$$(v_i - M)^2 = M^2 - 2\phi \tag{6b}$$

where we have imposed the boundary conditions as $\xi \to \pm\infty$, $n_i \to 1$, $v_i \to 0$ and $\phi \to 0$. Then Eqs. (6a) and (6b) implies that

$$n_i = \frac{1}{\sqrt{1 - \frac{2\phi}{M^2}}} \tag{7a}$$

Eq.(4c) reduces to

$$n_e^2 = 1 + 2\phi + \frac{H^2}{\delta}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2}{\partial\xi^2}\left(\sqrt{n_e}\right)\right] \tag{7b}$$

where we have imposed the boundary conditions as $\xi \to \pm\infty$, $\phi \to 0$, $n_e \to 1$ and $\frac{\partial^2}{\partial\xi^2}\left(\sqrt{n_e}\right) \to 0$. When the quantum diffraction effect is negligibly small (*i.e.* $H = 0$) integrating Eq.(7b) we get $n_e = (1 + 2\phi)^{\frac{1}{2}}$ which when plugged again in the Bohm potential terms of Eq.(7b), the following density expression for electron is obtained [23]

$$n_e^2 = 1 + 2\phi + \frac{H^2}{\delta}\left[(1 + 2\phi)^{-\frac{1}{4}}\frac{\partial^2}{\partial\xi^2}(1 + 2\phi)^{\frac{1}{4}}\right] \tag{8}$$

Eq.(8) which expresses the electron density as a function of the electrostatic potential is derived on the basis of the semiclassical limit where H is small up to the second order of magnitude [23]. Substituting $1 + 2\phi = z$, Eqs. (8), (7a) and (4d) reduces to

$$n_e = \left[z + \frac{H^2}{\delta}\left\{-\frac{3}{16z^2}\left(\frac{dz}{d\xi}\right)^2 + \frac{1}{4z}\frac{d^2z}{d\xi^2}\right\}\right]^{\frac{1}{2}} \tag{9a}$$

$$n_i = \frac{1}{\sqrt{1 - \frac{z-1}{M^2}}} \tag{9b}$$

$$\frac{1}{2}\frac{d^2z}{d\xi^2} = \delta\, n_e + d - n_i \tag{9c}$$

Now substituting the density expressions from Eqs. (9a) and (9b) in Eq. (9c), we obtain

$$\frac{1}{2}\frac{d^2 z}{d\xi^2} = \delta\left[z + \frac{H^2}{\delta}\left\{-\frac{3}{16z^2}\left(\frac{dz}{d\xi}\right)^2 + \frac{1}{4z}\frac{d^2 z}{d\xi^2}\right\}\right]^{\frac{1}{2}} + d - \frac{1}{\sqrt{1-\frac{z-1}{M^2}}} \tag{10}$$

Using the boundary condition $\phi \to 0$, $\frac{d\phi}{d\xi} \to 0$ and $\frac{d^2\phi}{d\xi^2} \to 0$ as $\xi \to \pm\infty$ we obtain the energy integral form

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + U(\phi) = 0 \tag{11a}$$

where

$$U(\phi) = -\frac{1}{4}V(1 + 2\phi) \tag{11b}$$

by considering

$$\frac{1}{2}\left(\frac{dz}{d\xi}\right)^2 = V(z) \tag{12a}$$

and so

$$\frac{d^2 z}{d\xi^2} = V'(z) \tag{12b}$$

Eq. (11a) can be considered as an 'energy integral' form for an oscillating particle of unit mass with velocity $\frac{d\phi}{d\xi}$ and position ϕ in a potential $U(\phi)$. Using the charge neutrality condition we have $U(0) = U'(0) = 0$ which implies

$$V(1) = V'(1) = 0. \tag{12c}$$

Using Eqs. (12a) and (12b), Eq. (10) reduces to

$$\frac{1}{2}V'(z) = \delta\left[z + \frac{H^2}{\delta}\left\{-\frac{3}{8z^2}V(z) + \frac{1}{4z}V'(z)\right\}\right]^{\frac{1}{2}} + d - \frac{1}{\sqrt{1-\frac{z-1}{M^2}}} \tag{13}$$

Next for small value of H, expanding $V(z)$ in an asymptotic series expansion in H we suppose that

$$V(z) = g_1(z) + H^2 g_2(z) + O(H^4) \tag{14}$$

Putting this expression of $V(z)$ in Eq. (13) and equating the constant term and coefficient of H^2 from both side we have

$$\frac{1}{2}\frac{dg_1}{dz} = \delta\sqrt{z} + d - \frac{1}{\sqrt{1-\frac{z-1}{M^2}}} \tag{15a}$$

$$\frac{1}{2}\frac{dg_2}{dz} = \sqrt{z}\left(-\frac{3g_1}{16z^3} + \frac{1}{8z^2}\frac{dg_1}{dz}\right) \tag{15b}$$

and then integrating and using the condition (12c) we get

$$g_1(z) = 2\left[2M^2\left(\sqrt{1-\frac{z-1}{M^2}} - 1\right) + d(z-1) + \frac{2\delta}{3}\left(z^{\frac{3}{2}} - 1\right)\right] \tag{16a}$$

$$g_2(z) = z^{-\frac{3}{2}}\left[M^2\left(\sqrt{1-\frac{z-1}{M^2}} - 1\right) + \frac{d}{2}(z-1) + \frac{\delta}{3}\left(z^{\frac{3}{2}} - 1\right)\right] \tag{16b}$$

Then putting the expression of $g_1(z)$ and $g_2(z)$ in Eq. (14) and making the substitution $z = 1 + 2\phi$, Eq. (11b) reduces to

$$U(\phi) = \left(1 + \frac{H^2}{4(1+2\phi)^{\frac{3}{2}}}\right)\left[M^2\left(1 - \sqrt{1 - \frac{2\phi}{M^2}}\right) - \phi d + \frac{\delta}{3}\left\{1 - (1 + 2\phi)^{\frac{3}{2}}\right\}\right] \tag{17}$$

Here we study the properties of solitary waves by analyzing the Sagdeev potential $U(\phi)$. For existence of solitary waves we need to satisfy the following conditions:

(i) $U''(\phi) < 0$ at $\phi = 0$, so that the fixed point at the origin is unstable.

(ii) $\exists$ a nonzero ϕ_m, the maximum (or minimum) value of ϕ, at which $V(\phi) = 0$.

(iii) $U(\phi) < 0$, for $0 < |\phi| < |\phi_m|$.

Condition (i) gives the lower limit of M for existence of solitary waves as $M > M_c$ where $M_c = \frac{1}{\sqrt{1-d}}$. Now here for $d \to 0$, $M_c \to 1$ which is same as obtained for simple electron-ion plasmas. Next we turn to find upper limit of Mach number. It is observed from Eq.(7a) and (8) that in order to prevent wave braking we require $\phi < \frac{M^2}{2}$ and $\phi > -\frac{1}{2}$. At this limiting potential $\phi = \frac{M^2}{2}$, we require $U\left(\frac{M^2}{2}\right) > 0$ which is necessary to obtain the upper limit of the Mach number for existence of positive potential solitons. Hence the upper limit of M, say M_u, for existence of positive potential soliton is obtained from the condition $U\left(\frac{M^2}{2}\right) = 0$ which implies

$$\left(1 + \frac{H^2}{4(1+M^2)^{\frac{3}{2}}}\right)\left[M^2 - \frac{M^2 d}{2} + \frac{\delta}{3}\left\{1 - (1 + M^2)^{\frac{3}{2}}\right\}\right] = 0 \tag{18}$$

V.   RESULTS AND DISCUSSIONS

In this section the obtained numerical results have been discussed. The asymptotic expansion for $U(\phi)$ has been obtained for small value of H. Using the given set of parameters here $g_2(z)$ is found to be small for $-\frac{1}{2} < \phi < \frac{M^2}{2}$. The values of the parameter are taken as [25, 29] $n_{e0} \sim 5 \times 10^{29}$, $n_{i0} \sim 2 \times 10^{30}$, $Z_{d0} \sim 10^3$ and $T_{Fe} \sim 10^2 K$.
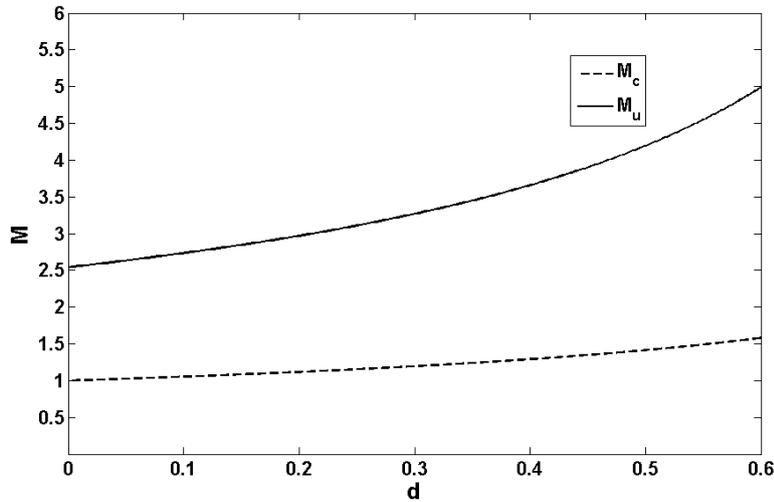

FIGURE 2. Lower and Upper Mach number limits are plotted against d.

From previous section it is found that the solitary wave can exist for Mach number limits $M_c < M < M_u$. The variation of lower and upper Mach number limits with d are plotted in Fig. 2. It is found that consideration of quantum effects increases the range of Mach number limits in comparison with DIA solitary wave in a simple classical dusty plasma [36]. Particularly at $d = 0$, from the Fig.2 (*i.e.* in absence of dust and so the ion and electron number densities are equal) it is observed that the Mach number lies in $1 < M < 2.55$ whereas for classical electron-ion plasmas with Boltzmann distribution of electron, the Mach number lies in $1 < M < 1.6$ [37]. It should be noted that as the value of M depends upon a specific normalization, some care should be taken to interpret the results physically. The true Mach number is defined by the ratio M/M_c as the reference speed C_0, used in normalization of M disappears from this ratio [38, 39]. Thus the existence condition, $M > M_c$ for solitary waves, follows that the true Mach number $M/M_c > 1$ and so the soliton structures are inherently superacoustic.
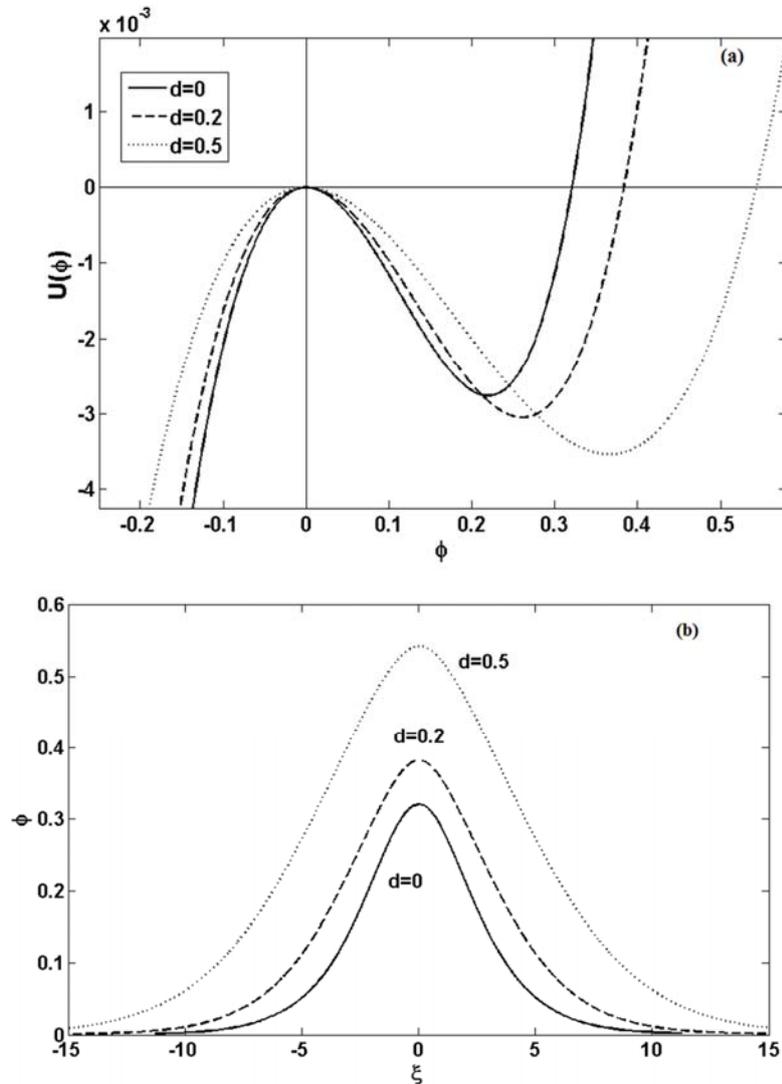

FIGURE 3. (a) Plot of $U(\phi)$ for different values of d. (b) Plot of corresponding potential profiles. Here, $H = 0.5$, $M/M_c = 1.2$.

In Fig. 3(a), the Sagdeev potential $U(\phi)$ is plotted for different values of dust density parameter d and corresponding potential profiles are plotted in Fig. 3(b). It is observed that both the amplitude and width of the quantum dust ion acoustic waves increases as d increases which is in agreement with what has been reported by S.A. Khan and A. Mushtaq [29].
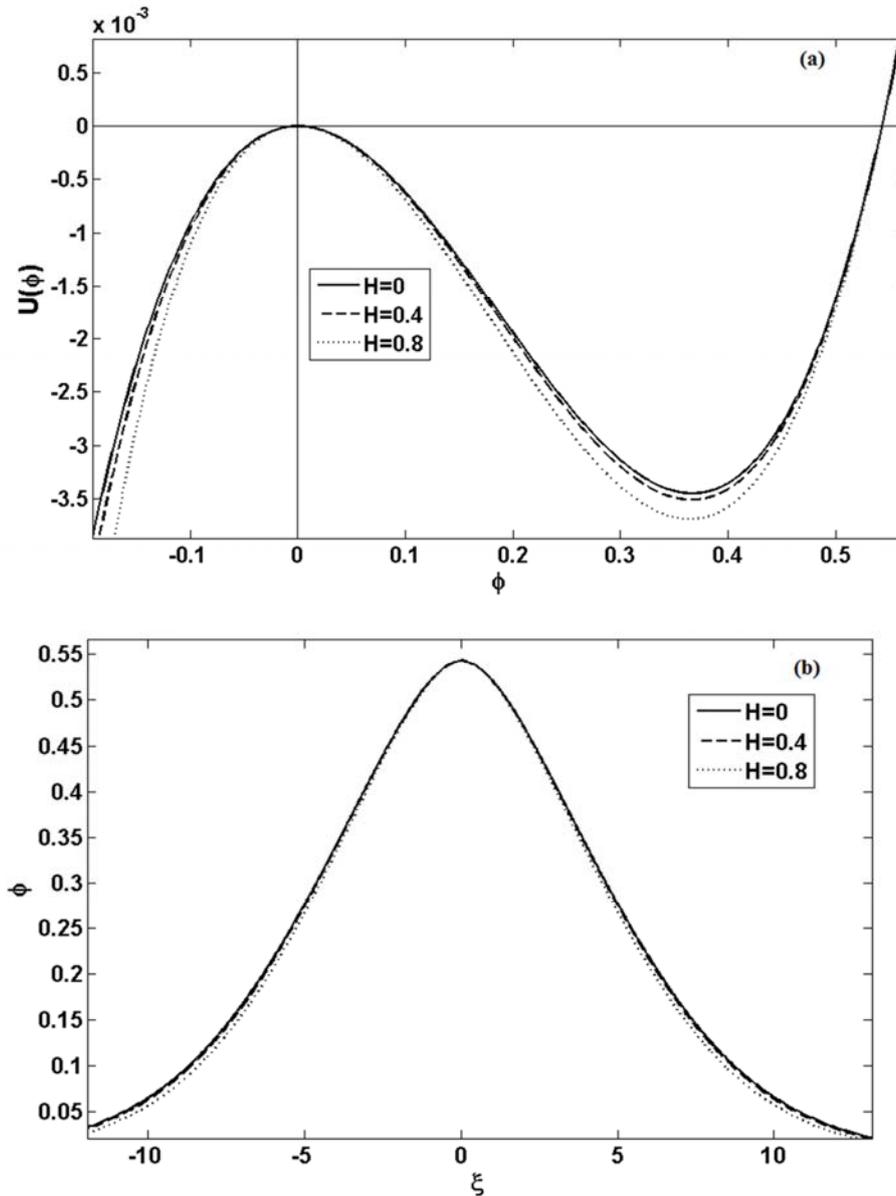

FIGURE 4. (a) Plot of $U(\phi)$ for different values of H. (b) Plot of corresponding potential profiles. Here, $d = 0.5$ and $M/M_c = 1.2$.

Fig. 4(a) shows the Sagdeev potential $U(\phi)$ for different values of the quantum diffraction parameter H and the corresponding potential profiles are shown in Fig. 4(b). In the present model the small change of the width of the soliton is observed due to the change in quantum diffraction parameter H. Here it is found that for small increase in the parameter H, the amplitude does not differ but the width increases by small amount. However, Haas [14] pointed out, for moderate H quantum diffraction effects can be negligible if the density is slowly varying in comparison with

9

typical length scale due to the presence of a third order derivative of the Bhom potential. Here also small effect of the quantum diffraction parameter is found. In Fig. 5(a), $U(\phi)$ is plotted for different values of true Mach number and corresponding potential profiles are plotted in Fig. 5(b). It is observed that both amplitude and width of the soliton increases as the true Mach number increases.
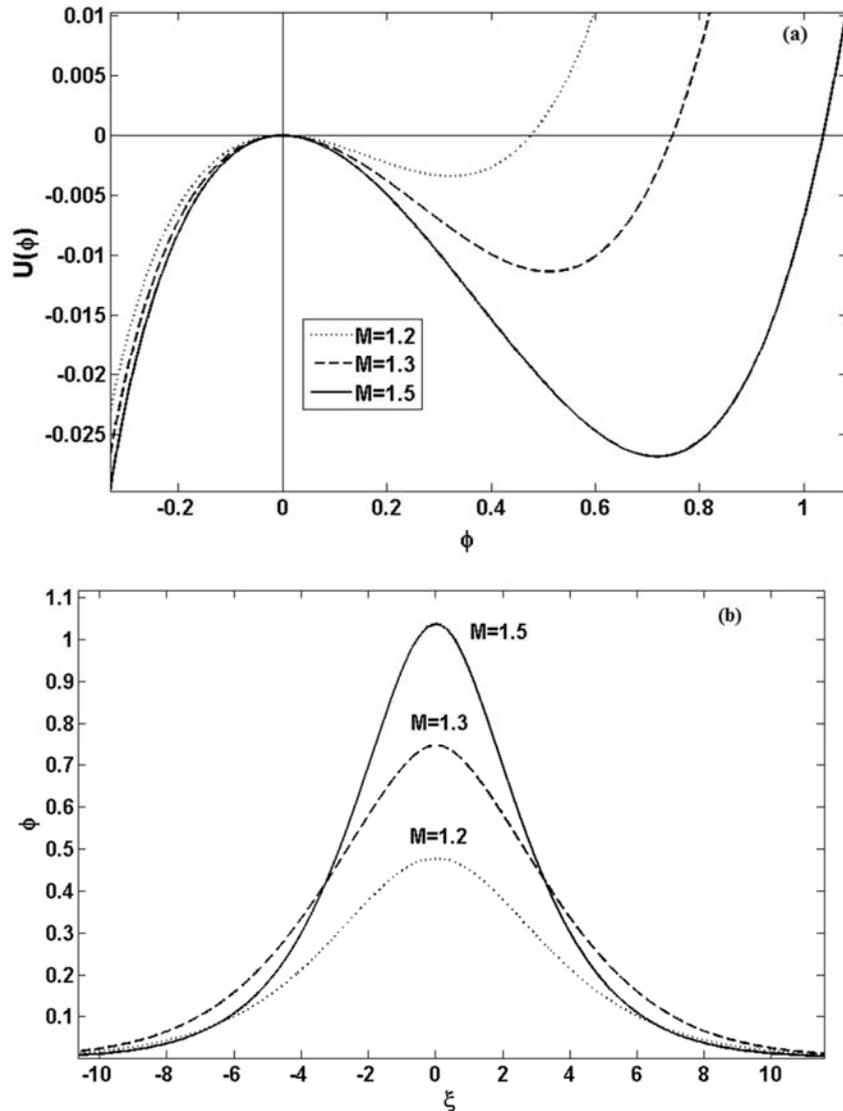

FIGURE 5. (a) Plot of $U(\phi)$ for different values of M/M_c. (b) Plot of corresponding potential profiles. Here, $d = 0.5$ and $H = 0.5$.

VI.   CONCLUSIONS

In this work, the existence domain as well as the propagation characteristics of dust ion acoustic solitary wave have been studied by using Sagdeev's psuedopotential approach in an unmagnetized quantum dusty plasma together with the Poisson equation. The psuedopotential function is obtained by employing asymptotic expansion for small values of the quantum diffraction

parameter H. The dispersion relation is obtained by linear wave analysis and the earlier results in Ref. 12 have been retrieved in absence of dust components. It is found that the range of the Mach number for existence of DIA solitary waves increases in presence of quantum statistical effects and also the presence of dust particles increases the range of the Mach number limits. It is also studied that as the true Mach number increases both amplitude and width of the solitons increase. The amplitude as well as the width of the electrostatic potential structure are also increased due to the presence of dust particle in quantum plasmas. Small effects of quantum diffraction parameter on the width of the soliton is also observed.

Acknowledgements

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11

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