Dynamic features of dust acoustic waves in a four component dusty plasma with non-thermal ions

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Abstract. The dynamic features of dust acoustic waves in a four-component dusty plasma with non-thermal ions are investigated applying the bifurcation theory of planar dynamical systems. Employing the Galilean transformation, basic equations are reduced to a planar dynamical system. Existence of solitary and periodic waves is presented for the dynamical system through phase portrait analysis. Analytical forms of these waves are derived depending on physical parameters $\alpha, \beta, \beta_1, \mu_i, \mu_e, \sigma$ and v. Considering an external periodic perturbation, the quasiperiodic motion of the perturbed dynamical system for dust acoustic waves is shown. It is observed that the unperturbed dynamical system has the solitary and periodic wave solutions whereas the dynamical system with external periodic perturbation has quasiperiodic behavior with same values of parameters $\alpha, \beta, \beta_1, \mu_i, \mu_e, \sigma$ and v.

Keywords: Non-thermal Ions, Four-component Dusty Plasma.

I. INTRODUCTION

The study of nonlinear waves in dusty plasmas is one of the most rapidly growing areas of plasma physics and has been studied by several authors. Dust acoustic waves (DAW) are one such nonlinear wave which has been first predicted theoretically by Rao et al. [1]. Experimental observations have also been done [2,3]. Due to the dust grain dynamics a few new eigen modes like dust ion acoustic waves (DIAS), dust-Berstain-Greene-Kruskal (DBGK) mode, Shukla and Verma mode [4], dust-drift mode [5] are also introduced. In these works, most of the authors have considered the negatively charged dust only. The consideration of negatively charged dust is valid if dust charging process by collection of plasma particles (viz. electrons and ions) is much more important than other charging processes. But, it has been observed that the dust particles can be positively charged [6-9] by other important charging processes. There are three principal mechanisms by which a dust grain can be positively charged such as (i) Photo emission in the presence of a flux of ultraviolet photons, (ii) Thermionic emission induced by radiative heating, and (iii) Secondary emission of electrons from the surface of the dust grains.

In recent years, there have been considerable interests in understanding the different types of collective processes in plasmas containing electrons, ions and charged micron sized grain particles. Such plasmas occur frequently in many astrophysical systems including the interplanetary medium, planetary rings, asteroids, commentary tails, interstellar clouds, nebulae, aurora etc. and they are also produced in plasma discharges, optical fibers, dusty crystals, semiconductors as well as regions of hot fusion plasma and in devices for plasma-assisted materials processing [10-13].

A series of recent observations in space plasma [14-16] found that the ion distribution does not follow the Boltzmann distribution, and to model these situations, non-thermal distribution for ions has been suggested. Evidently, the Vela satellite [14] observed non-thermal ions from the Earth's bow shock. Furthermore, the Phobos2 satellite [15] also observed the loss of energetic ions from the upper ionosphere of Mars. The observations from the Nozomi satellite [16] indicate the occurrence of very large velocity protons near the Earth in the vicinity of the moon. Lundlin et al. [15] also showed that for the planet having not so strong magnetic field, the solar wind impacting with the planetary atmosphere results in non-thermal ion flux. Most of the studies in multi-species plasmas have focused on deriving Korteweg-de Vries (KdV) and Kadomstev-Petviashvili (KP) equation using reductive perturbation technique [17,18]. Sagdeev's pseudo-potential technique is used to investigate the existence of both solitary waves and double layers in four component dusty plasmas with the non-thermal ions [19,20]. Chatterjee et. al. [21] investigated the dressed soliton in four component dusty plasma in presence of non-thermal ions. Ghosh et. al. [22] have investigated the head-on collision of dust acoustic solitary waves in a four component dusty plasma with non-thermal ions.

Recently, Samanta et al. [23] studied bifurcations of nonlinear traveling waves in a magnetized dusty plasma with q non-extensive electrons applying bifurcation theory of planar dynamical systems for the first time. A number works [24-27] on bifurcations of nonlinear waves in plasmas were reported through perturbative and nonperturbative approaches. Saha and chatterjee [28] investigated propagation and interaction of dust acoustic multi-soliton in dusty plasmas with q non-extensive electrons and ions. Roy et al. [29] studied theoretically the propagation of two ion acoustic soliton interaction in a three component collisionless unmagnetized plasma which consists of electrons, positrons and cold ions. Very recently, Saha et al. [30] investigated the dynamic behavior of ion acoustic waves in electron-positron-ion magnetoplasmas with superthermal electrons and positrons in the framework of perturbed and non-perturbed Kadomtsev-Petviashili (KP) equations. Sahu et al. [31] introduced nonlinear studies for the first time to find the quasiperiodic behavior of ion acoustic solitary waves in electron-ion quantum plasma. Sahu et al. [32] also studied the quasi periodic behavior in quantum plasmas due to the presence of Bohm potential.

In this work, our aim is to study dust acoustic solitary and periodic waves in a four-component dusty plasma with non-thermal ions using the bifurcation theory of planar dynamical systems. Two analytical solutions of the solitary and periodic waves are derived depending upon the parameters. Considering an external periodic perturbation, the quasiperiodic motion of the perturbed dynamical system is investigated.

The remaining part of the paper is organized as follows: In the next section, we consider basic equations. Following this, we consider corresponding dynamical system and phase portraits. Two analytical solutions are derived in the subsequent section. Before the concluding section, we consider quasiperiodic features of the perturbed dynamical system and the study is concluded in the final section.

II. BASIC EQUATIONS

In this section, we consider a four component dusty plasma whose constituents are Boltzmann distributed electrons, non-thermal ions, and also negatively and positively charged dust grains. The basic equations are given below:

$$\frac{\partial n_1}{\partial t}+\frac{\partial (n_1 u_1)}{\partial x}=0, \tag{1}$$

$$\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}=\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial n_2}{\partial t}+\frac{\partial (n_2 u_2)}{\partial x}=0, \tag{3}$$

$$\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}=-\alpha\beta\frac{\partial \phi}{\partial x}, \tag{4}$$

$$\frac{\partial^2 \psi}{\partial x^2}=n_1-n_2+n_e-n_i \tag{5}$$

with

$$n_e=\mu_e e^{\sigma\phi}, \tag{6}$$

$$n_i=\mu_i(1+\beta_1\phi+\beta_1\phi^2)e^{-\phi}. \tag{7}$$

where n_1 and n_2 are the negatively charged and positively charged dust number densities normalized by the equilibrium value n_{10}. u_1 and u_2 are the negatively and positively charged dust fluid speed normalized by $Z_1K_BT_i/m_1$. The electric potential ϕ is normalized by K_BT_i/e. Space variable x and time variable t are normalized by the Debye length $\lambda_D=\sqrt{(K_BT_i)/(4\pi n_{10}Z_1e^2)}$ and inverse of dust plasma frequency $\omega_{pl}^{-1}=\sqrt{m_1/(4\pi n_{10}Z_1^2e^2}$, respectively. We denote $\alpha=Z_1/Z_2, \beta=m_1/m_2, \mu_e=n_{e0}/(Z_1n_{10}),\ \ \mu_i=n_{i0}/(Z_1n_{10}), \sigma=T_i/T_e$. Z_1 and Z_2 are the number of electrons and protons residing on a negative and positive dust particles. Furthermore, $\beta_1=\dfrac{4\alpha_1}{1+3\alpha_1}$, where α_1 is the ion non-thermal parameter that determines the number of fast(energetic) ions. m_1 and m_2 are masses of negative and positive dust particles, respectively. T_i and T_e are ion and electron temperatures. K_B is the Boltzmann constant and e is the elementary charge.

III. PLANAR DYNAMICAL SYSTEM AND PHASE PORTRAITS

In this section, we transform our model equations into a planar dynamical system and we obtain all possible phase portraits of the system. To do so, we introduce a new variable $\xi = x - vt$ where v is the speed of the ion acoustic traveling wave. Substituting the new variable ξ into Eqs. (1) and (2) and using the initial condition $u_1 = 0,\ n_1 = 1$ and $\phi = 0$, we can express the negatively charged dust number density as

$$n_1 = \frac{v}{\sqrt{v^2 + 2\phi}}. \tag{8}$$

Again, substituting ξ into Eqs. (3) and (4) and using the initial condition $u_2 = 0, n_2 = 1 - \mu_i + \mu_e$ and $\phi = 0$, one can express the positively charged dust number density as

$$n_2 = \frac{v(1 - \mu_i + \mu_e)}{\sqrt{v^2 - 2\alpha\beta\phi}}. \tag{9}$$

Using Eqs.(6)-(9) in Eq.(5) and considering the terms involving ϕ up to third degree, we have

$$\frac{d^2\phi}{d\xi^2} = a\phi + b\phi^2 + c\phi^3, \tag{10}$$

where

$$a = \mu_e - \mu_i(\beta_1 - 1) - \frac{1}{v^2}\{1 + (1 - \mu_i + \mu_e)\alpha\beta\},$$
$$b = \frac{1}{2}\left(\mu_e\sigma^2 - \mu_i\right) + \frac{3}{2v^4}\{1 - (1 - \mu_i + \mu_e)\alpha^2\beta^2\},$$
$$c = \frac{1}{6}\{\mu_e\sigma^3 + \mu_i(1 + 3\beta_1)\} - \frac{5}{2v^6}\{1 + (1 - \mu_i + \mu_e)\alpha^3\beta^3\}.$$

Then Eq. (10) is equivalent to the following planar dynamical system:

$$\begin{cases} \dfrac{d\phi}{d\xi} = z \\ \dfrac{dz}{d\xi} = a\phi + b\phi^2 + c\phi^3 \end{cases} \tag{11}$$

The system (11) is a planar dynamical system with Hamiltonian function:

$$H(\phi, z) = \frac{z^2}{2} - a\frac{\phi^2}{2} - b\frac{\phi^3}{3} - c\frac{\phi^4}{4}. \tag{12}$$

The system (11) is a planar dynamical system with parameters $\alpha, \beta, \beta_1, \mu_i, \mu_e, \sigma$ and v. It is clear that the phase orbits defined by the vector fields of Eq.(11) will determine all traveling wave solutions of Eq.(10). We will study the bifurcations of phase portraits of Eq.(11) in the (ϕ, z) phase plane depending on the parameters. A homoclinic orbit of Eq.(11) gives a solitary wave solution of Eq.(10). Similarly, a periodic orbit of Eq.(11) gives a periodic traveling wave solution of Eq.(10).

We study the bifurcation set and phase portraits of the planar Hamiltonian system (11). Clearly, on the (ϕ, z) phase plane, the abscissas of equilibrium points of system (11) are the zeros of $f(\phi) = \phi(\phi^2 + \frac{b}{c}\phi + \frac{a}{c})$. Let $E_i(\phi_i, 0)$ be an equilibrium point of the dynamical system (11) where $f(\phi_i) = 0$. When $b^2 - 4ac > 0$, there exist three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$, where $\phi_0 = 0$, $\phi_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2c}$, and $\phi_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2c}$.

If $M(\phi_i, 0)$ is the coefficient matrix of the linearized system of the traveling system (11) at an equilibrium point $E_i(\phi_i, 0)$, then we get

$$J = \det M(\phi_i, 0) = -cf'(\phi_i) \qquad (13)$$

By the theory of planar dynamical systems [33-34], it is clear that the equilibrium point $E_i(\phi_i, 0)$ of the planar dynamical system (11) is a saddle point when $J < 0$ and the equilibrium point $E_i(\phi_i, 0)$ of the planar dynamical system (11) is a center when $J > 0$.

Applying the systematic analysis of the physical parameters $\alpha, \beta, \beta_1, \mu_i, \mu_e, \sigma$ and v, we have presented the phase portrait of the system (11) in the figures 1-2.

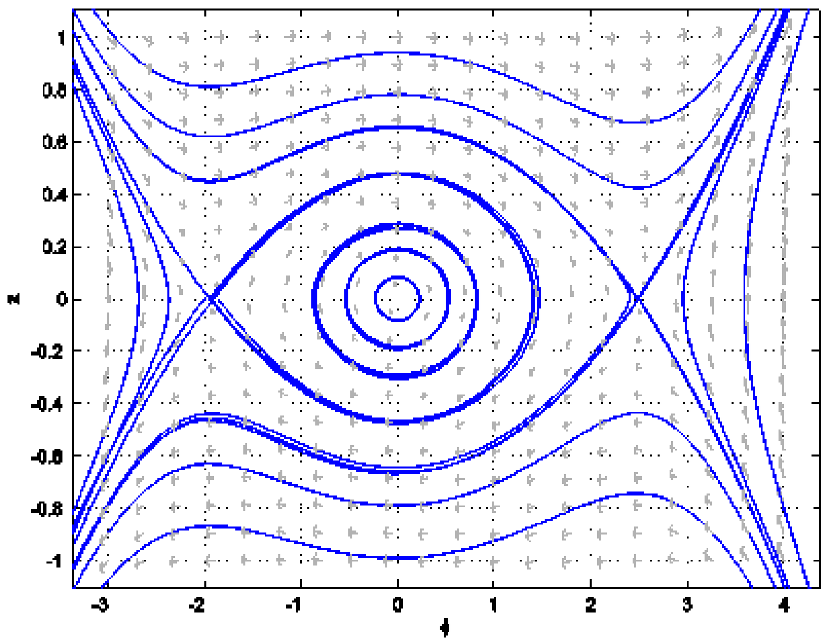


FIGURE 1. Phase portrait of Eq.(11) for $\alpha = 0.5, \beta = 0.55, \beta_1 = 0.3, \mu_i = 0.21, \mu_e = 0.21, \sigma = 0.2$ and $v = 2$.

In figure 1, we have shown the phase portrait of the system (11) for $\alpha = 0.5, \beta = 0.55, \beta_1 = 0.3$, $\mu_i = 0.21, \mu_e = 0.21, \sigma = 0.2$ and $v = 2$. The system (11) has three equilibrium points at $E_0(\phi_0,0)$, $E_1(\phi_1,0)$ and $E_2(\phi_2,0)$ with $\phi_2 < 0 < \phi_1$, where $E_1(\phi_1,0)$ and $E_2(\phi_2,0)$ are saddle points, and $E_0(\phi_0,0)$ is a center. There is a homoclinic orbit at $E_2(\phi_2,0)$ enclosing the center at $E_0(\phi_0,0)$.

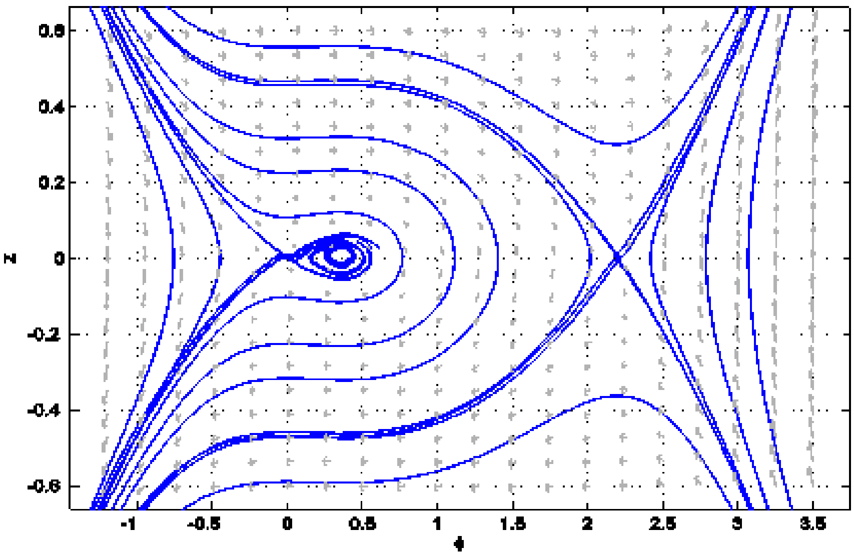


FIGURE 2. Phase portrait of Eq.(11) for $\alpha = 1, \beta = 1, \beta_1 = 0.3, \mu_i = 0.5, \mu_e = 0.6, \sigma = 0.4$ and $v = 2$.

In figure 2, we have shown the phase portrait of the system (11) for $\alpha = 1, \beta = 1, \beta_1 = 0.3$, $\mu_i = 0.5, \mu_e = 0.6, \sigma = 0.4$ and $v = 2$. The system (11) has three equilibrium points at $E_0(\phi_0,0)$, $E_1(\phi_1,0)$ and $E_2(\phi_2,0)$ with $0 < \phi_1 < \phi_2$, where $E_0(\phi_0,0)$ and $E_2(\phi_2,0)$ are saddle points, and $E_1(\phi_1,0)$ is a center. There is a homoclinic orbit at $E_0(\phi_0,0)$ enclosing the center at $E_1(\phi_1,0)$.

IV. ANALYTICAL DUST ACOUSTIC WAVE SOLUTIONS

In this section, we derive two analytical dust acoustic traveling wave solutions which are solitary wave solutions and periodic wave solutions using the dynamical system (11) and the Hamiltonian function (12). It is important to note that $sn(\Omega\xi, k)$ is the Jacobean elliptic function [35] with modulo k.

(i) Corresponding to family of periodic orbits about $E_0(\phi_0,0)$ in figure 1, the system has a family of periodic wave solutions:

$$\phi(\xi)=\frac{(\beta_1-\gamma_1)\delta_1 sn^2(\Omega\xi,k)-\gamma_1(\beta_1-\delta_1)}{(\beta_1-\gamma_1)sn^2(\Omega\xi,k)-(\beta_1-\delta_1)}, \tag{14}$$

with $\Omega=\sqrt{-\frac{c(\beta_1-\delta_1)(\gamma_1-\alpha_1)}{8}}$, $k=\sqrt{\frac{(\alpha_1-\delta_1)(\beta_1-\gamma_1)}{(\alpha_1-\gamma_1)(\beta_1-\delta_1)}}$, where $\alpha_1,\beta_1,\gamma_1$ and δ_1 are roots of the equation $h+\frac{c}{4}\phi^4+\frac{b}{3}\phi^3+\frac{a}{2}\phi^2=0$ with $\alpha_1>\beta_1>\gamma_1>\delta_1, h\in(h_2,0)$.

(ii) Corresponding to homoclinic orbit at $E_0(\phi_0,0)$ in figure 2, the system has solitary wave solution:

$$\phi(\xi)=\frac{\left(\frac{4b}{c}+6A\sqrt{\frac{2a}{c}}\right)}{3(A^2-1)}, \tag{15}$$

where

$$A=\exp(\mp\sqrt{a}\xi)-\frac{b}{3}\sqrt{\frac{2}{ac}}.$$

V. QUASIPERIODIC FEATURES

In this section, we will discuss the quasiperiodic motion of the following perturbed system:

$$\begin{cases}\frac{d\phi}{d\xi}=z,\\ \frac{dz}{d\xi}=a\phi+b\phi^2+c\phi^3+f_0\cos(\omega\xi),\end{cases} \tag{16}$$

where $f_0\cos(\omega\xi)$ is the external periodic perturbation, f_0 is strength of the external perturbation and ω is the frequency. The difference between the system (11) and the system (16) is that only external periodic perturbation is added with the system (11).

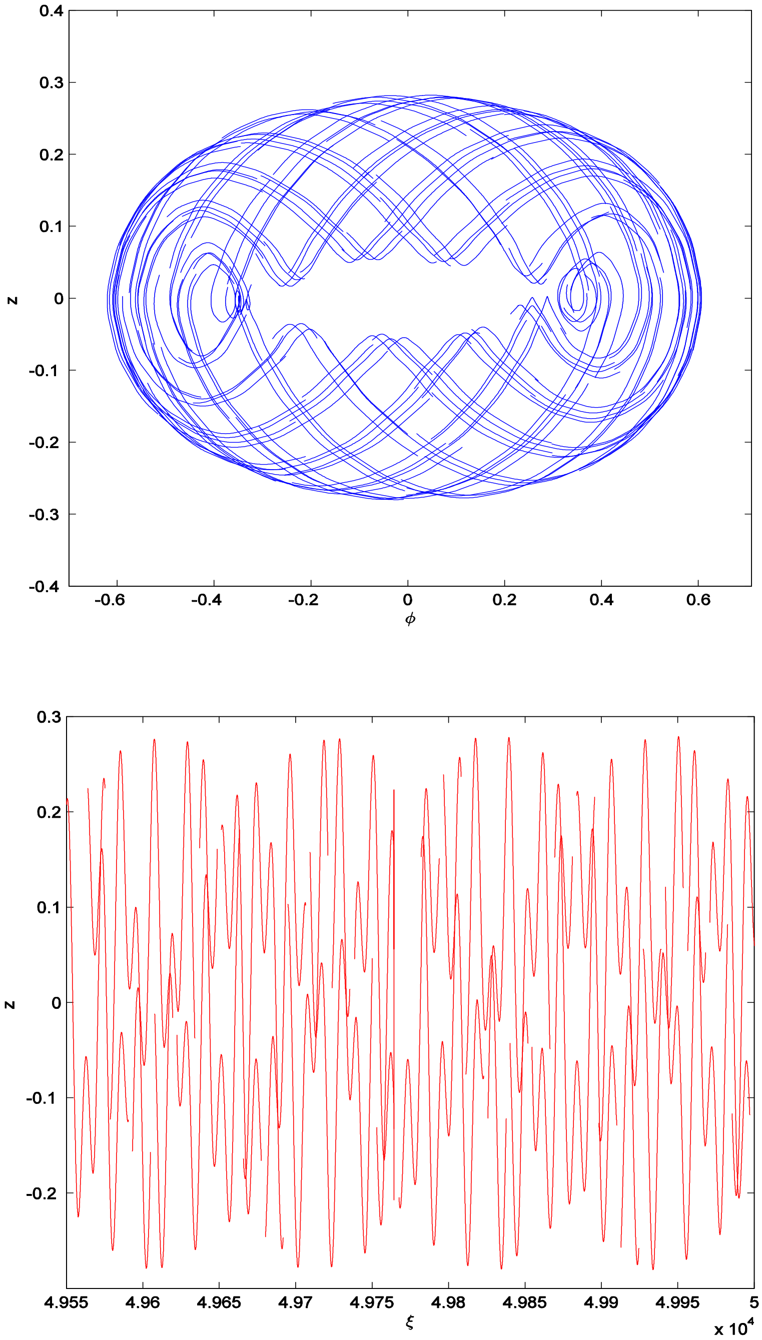
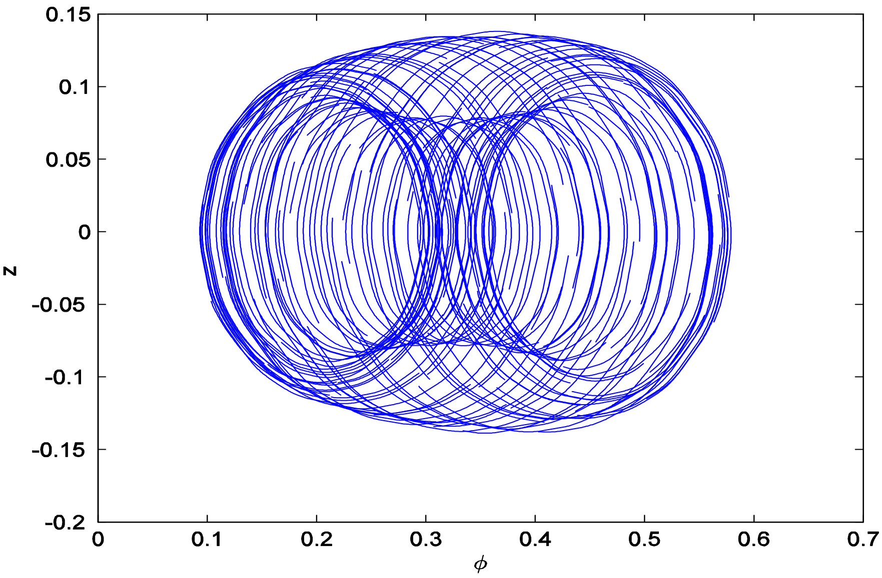


FIGURE 3. Phase portrait (left) and plot of z vs. ξ (right) of Eq. (16) for $f_0 = 0.1,\ \omega = 1$ with same values of other parameters as Fig. 1.

In figure 3, we have presented phase portrait (left) and plot of z vs. ξ (right) of the perturbed system (16) for α=0.5, β=0.55, β_1=0.3, μ_i=0.21, μ_e=0.21, σ=0.2, ν=2, f_0=0.1, and ω=1, with initial condition ϕ=0.2, z=0.01.



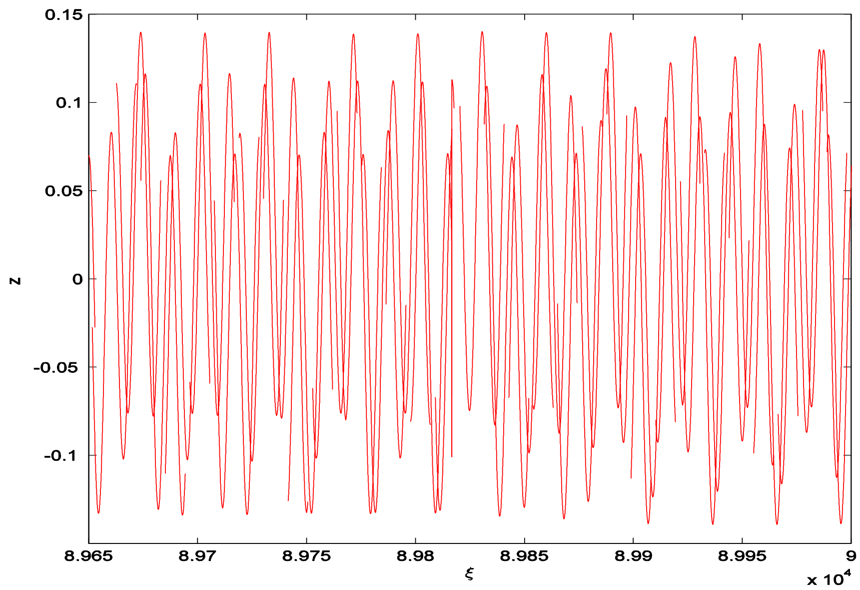


FIGURE 4. Phase portrait (left) and plot of z vs. ξ (right) of Eq. (16) for $f_0 = 0.1,\ \omega = 1$ with same values of other parameters as Fig. 2.

In figure 4, we have presented phase portrait (left) and plot of z vs. ξ (right) of the perturbed system (16) for α=1, β=1, β_1=0.3, μ_i=0.5, μ_e=0.6, σ=0.4, v=2, f_0=0.1, and ω=1, with initial condition ϕ=0.01, z=0.01.

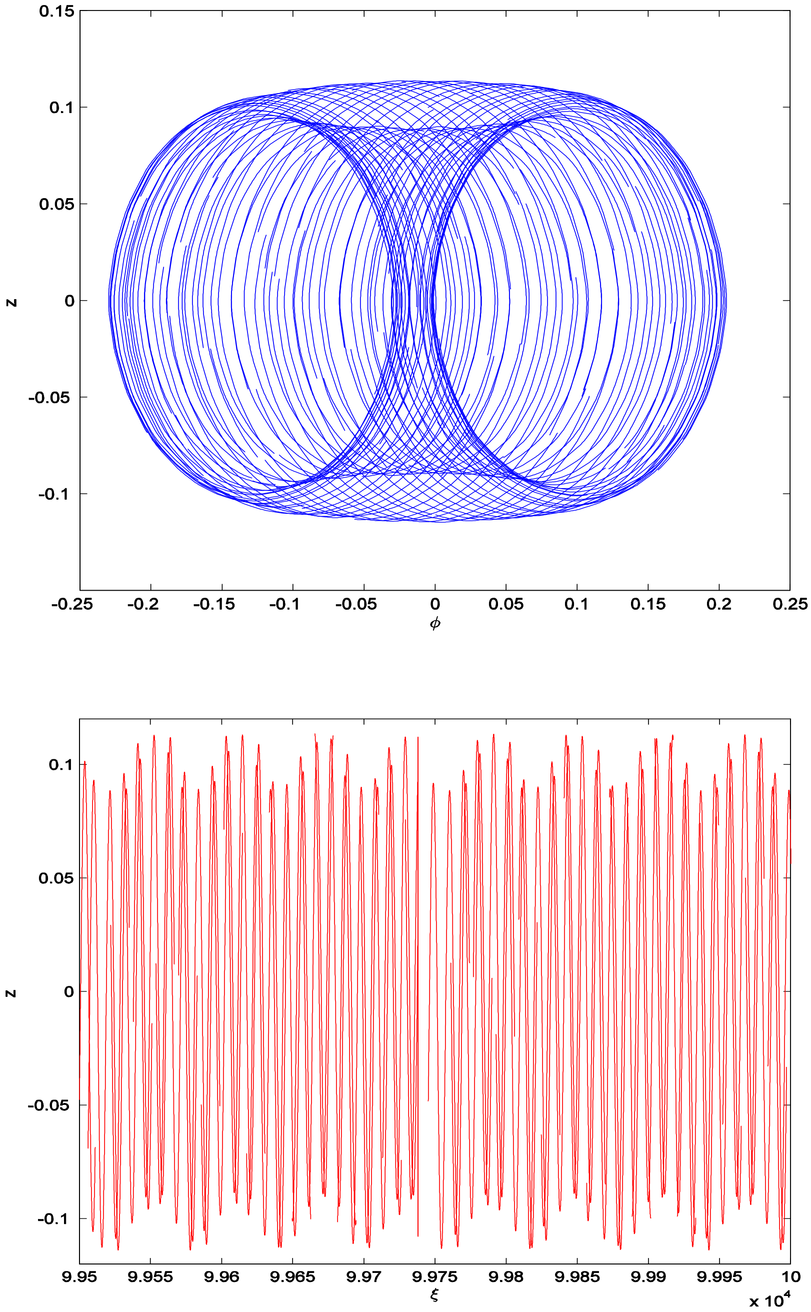


FIGURE 5. Phase portrait (left) and plot of z vs. ξ (right) of Eq. (16) for $\alpha = 0.5, \beta = 0.5, \beta_1 = 0.2, \mu_i = 0.25,$ $\mu_e = 0.25, \sigma = 0.4\ v = 2,\ \ f_0 = 0.1,\ \omega = 1.$

In figure 5, we have presented phase portrait (left) and plot of z vs. ξ (right) of the perturbed system (16) for α=0.5, β=0.5, β_1=0.2, μ_i=0.25, μ_e=0.25, σ=0.4, v=2, f_0=0.1, and ω=1, with initial condition ϕ=0.001, z=0.001.

A quasiperiodic motion of the system (16) is observed with incommensurable periodic motions and the trajectory in the phase space winds around torus filling its surface densely. It is easily seen that stable oscillatory behavior is possible in the system (16) depending on different parameters. The presence of slow and fast frequency components is visible. From figures 3-5, it is found that the perturbed system (16) has the quasiperiodic behavior but not chaotic in presence of an external periodic perturbation.

VI. CONCLUSIONS

Applying bifurcation theory of planar dynamical systems, we have investigated dust acoustic solitary and periodic waves in a four component dusty plasma consisting of Boltzmann distributed electrons, non-thermal ions, negatively and positively charged dust grains. Using the Galilean transformation, basic equations are reduced to a planar dynamical system. Existence of solitary and periodic waves is presented through phase portraits analysis. Analytical forms of these waves are derived depending upon parameters $\alpha, \beta, \beta_1, \mu_i, \mu_e, \sigma$ and v. The parameters $\alpha, \beta, \beta_1, \mu_i, \mu_e, \sigma$ and v significantly influence the characteristics of nonlinear dust acoustic wave solutions. Considering an external periodic perturbation, the quasiperiodic behavior of the perturbed dynamical system is presented using numerical simulations. It is to be noted that the dynamical system has the solitary and periodic wave solutions whereas the perturbed dynamical system shows quasiperiodic motion with same values of parameters $\alpha, \beta, \beta_1, \mu_i, \mu_e, \sigma$ and v. Our present study could be applied in understanding the dynamic features for dust acoustic waves in laboratory plasmas as well as space plasmas [36], where negatively and positively charged dust particles are present.

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