Bifurcation of Travelling Waves and Quasiperiodic Behaviors of Dust Acoustic Waves in Strongly Coupled Dusty Plasma

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Abstract. Bifurcation and quasiperiodic behaviors of dust acoustic waves in strongly coupled dusty plasma have been investigated in the framework of perturbed Korteweg-de Vries (KdV) equation. Using bifurcation theory of planar dynamical systems, the condition of existence of solitary wave solution has been derived. Considering an external periodic perturbation, we have studied the quasiperiodic behavior of dust acoustic waves in strongly coupled dusty plasma.

Keywords: Bifurcation, Reduction Perturbation Technique (RPT), Quasiperiodicity, Strongly coupled dusty plasma.

I. INTRODUCTION

Dusty plasma is an ionized gas containing small particles of solid matter, each of which acquires a large electric charge by collecting electrons and ions from the plasma. It has been observed widely in planetary rings, asteroid zones, cemetery tails, the magnetosphere and also lower part of earth's atmosphere [1-2]. In a dusty plasma, beside the electrons and the most abundant ion species, there is an additional species with a different mass and charge. Due to this large amount of charge on a single dust particle, the effects of correlations become important. Strongly coupled plasmas (SCP) or non ideal plasmas are multi-component charged many-particle systems whose constituents are electrons, ions, atoms and dust. The fundamental characteristics of dusty (complex) plasma is the coupling parameter Γ defined as the ratio of the potential energy of interaction between neighboring particles to their kinetic energy. In strongly coupled dusty plasma (τ is greater than unity, high density and low temperature) the ion-sphere (IS) potential is the best suited model for accounting for plasma screening effects. The coupling parameter is defined as $\Gamma = \Gamma_c e^{(-k)}$ where $\Gamma_c = \frac{{q_d}^2}{k_B a T_d}$ is the coulomb coupling parameter, $k = \frac{a}{\lambda_d}$ .Here a is the lattice parameter, q_d is the charge of dust particle, T_d is the dust temperature, λ_d is the plasma debye length and k_B is the Boltzman constant. For $\Gamma \gg 1$ [3] Thomasand and Morfil demonstrated the existence of solid or liquid behavior of this complex plasma. When $1 \ll \Gamma \ll \Gamma_{cr}$ (Γ_{cr} is the critical value of coupling parameter beyond which the system becomes crystalline) then the dusty plasma behaves both solid and liquid character. And at this stage a new property viz. oblique character, visco-elastic effects appear. The dust grains are called quasi-crystalline state [4]. In the regime $\Gamma > \Gamma_{cr}$, viscosity disappears and only elasticity dominates over the system. Experimental evidence [5] showed that when Γ increase then dusty component becomes strongly coupled and attain a crystalline structure. This experimental phenomena known as plasma condensation which is useful to explain the phase transitions [5,11], low frequency wave propagation etc [6,7]. Ikeji (1986) theoretically predicted the formation of SCP in normal two-species plasma which was experimentally verified in different laboratory [8,9]. Thus dusty (complex) plasma is a major interdisciplinary research field to explain the fundamental physics of the strongly coupled coulomb and Yukawa systems [10,11]. The linear properties of Dust Acoustic (DA) waves SCP have been rigorously investigated by a number of authors [12,13].Now linear properties are now well understood from the theoretical and experimental points of view. A limited number of theoretical investigations has also made on nonlinear propagation of DA waves in SCP. Shukla and Mamun [14] studied the dust acoustics shock in a strongly coupled dusty plasma. Mamunet al.[15] investigated the dust acoustic shock waves due to strong correlation among arbitrary charged dust. Mamun and Shukla [16] also studied the formation of dust acoustic shock waves in a strongly coupled cryogenic dusty plasma. Theoretically Anowar [17] and Mamun and Shukla[18]also investigated the arbitrary amplitude dust acoustic shock waves in a strongly coupled plasma by applying pseudopotential approach. Rahaman et al. [19] studied the time dependent non-planar dust-acoustic solitary and shock waves in strongly coupled adiabatic dusty plasma. Garaiet al.[20]observed the velocity shear effect on the longitudinal wave in a strongly coupled dusty plasma. Chakrabarti and Ghosh [21] studied the longitudinal dust acoustic solitary waves in a strongly coupled dusty plasma. Non-linear wave is shown to be governed by KdV equation with a nonlocal nonlinear forcing and linear damping terms. Mamun and Shukla [22] have also studied the effects of cylindrical and spherical geometry on DA shock waves in a SCP system.

Most of the work done till today on nonlinear waves in SCP are mainly based on finding the evolution equation by RPT and study the soliton solution. But recently several authors have investigated the different type of waves by using the method of bifurcation of planar dynamical system. They also obtained the quasiperiodic and chaotic behavior of dusty plasma. Samanta et al.[23]studied the bifurcations of dust ion acoustic traveling waves in a magnetized quantum dusty plasma. Saha and Chatterjee [24] investigated the propagation and interaction of dust acoustic multisoliton in dusty plasmas with q-non extensive electrons and ions. Saha and Chatterjee [25] also studied the bifurcations of electron acoustic traveling waves in an unmagnetized quantum plasma with cold and hot electrons.

However, many basic problems of the DA waves in strongly coupled plasma are still the subject of intense experimental and theoretical studies. To the best of our knowledge, there is no investigation about the bifurcation of traveling waves and quasiperiodic behaviors of DA waves in SCP. This is the main motivation for following up this work. The organization of the present work is as follows: we introduce the basic equation in section 2. In section 3, we derive KdV equation and then we treat the KdV equation as a dynamical system in the framework of traveling waves. Bifurcation analysis are studied in section 4. In Section 5 the quasiperiodicity is discussed. Section 6 is kept for conclusion.

II. GOVERNING EQUATION

We consider the nonlinear Dust Acoustic (DA) waves propagation [26] in strongly coupled dusty plasma system consisting of arbitrary charged inertial cold dust fluid and inertia less maxwellian electron and ion fluids. The dynamics of the nonlinear DA waves in such strongly coupled dusty plasma is governed by the well-known generalized hydrodynamics (GH) equations [27,28]

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0 \tag{1}$$

$$D_{\tau n}[n(D_{tn}u+j\alpha\frac{\partial \phi}{\partial x})+\alpha_n\mu_n\sigma_n\frac{\partial n}{\partial x}]=\eta\frac{\partial^2 u}{\partial x^2} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n+\mu_e e^{\sigma\phi}-\mu_i e^{-\phi} \tag{3}$$

where n is the number density of negative dust fluid normalized to the equilibrium value n_0, u is the fluid speed normalized to $c_d=(Z_n k_B T_{i0}/m_n)^{1/2}$, T_s is the fluid temperature of the species s (with $s=n$ for negative dust fluid, $s=e$ for electron fluid, $s=i$ for ion fluid) normalized to the equilibrium value T_{s0}, ϕ is the wave potential normalized by $k_B T_{i0}/e$, $j=-1$ for negative dust fluid, $\alpha=1$, k_B is the Boltzmann constant, and $-e$ is the electric charge, $\sigma=T_i/T_e$, $\sigma_n=T_n/Z_nT_i$, and, $\mu_e=n_{e0}/Z_nn_0$, $\mu_i=n_{i0}/Z_nn_0$, where Z_n is the number of electrons residing on a negative dust grain surface. The time and space variables t and x are normalized to $\omega^{-1}[=(m_n/4\pi e^2Z_nn_{n0})^{1/2}$ and $\lambda_D[=(k_BT_i/4\pi e^2Z_nn_0)^2$ respectively. $D_{\tau n}=1+\tau_{mn}(\frac{\partial}{\partial t})$, $D_{tn}=\frac{\partial}{\partial t}+u_n(\frac{\partial}{\partial x})$, τ_{mn} is the viscoelastic relaxation time, and $\eta[=(\frac{4}{3}\eta_l+\varsigma)/m_nn_0\omega\lambda_D{}^2]$ is the normalized longitudinal viscosity coefficient, where η_l and ς are the transport coefficients of shear and bulk viscosities. To calculate these transport coefficients, there are various approaches discussed in the literature [29]. The viscoelastic relaxation time τ_{mn}, land consequently the compressibility μ_n are given by Ichimaru et al.[28] and Berkovsky et al.[29]

$$\tau_{mn}=\eta\frac{T_e}{T_n}[1-\mu_n+\frac{4}{15}u(\Gamma)]^{-1} \tag{4}$$

$$\mu_n=\frac{1}{T_n}\frac{\partial P_n}{\partial n_n}=1+\frac{1}{3}u(\Gamma)+\frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial \Gamma} \tag{5}$$

where $u(\Gamma)$ is a measure of excess internal energy of the system. It is calculated for weakly coupled plasmas $(\Gamma<1)$ $u(\Gamma)\approx-(\sqrt{3/2})\Gamma^{3/2}$ **[27].**

For $1<\Gamma<200$, $u(\Gamma)$ has analytically derived from the relation **[30]**

$$u(\Gamma)=-0.89\Gamma+0.95\Gamma^{1/4}+0.19\Gamma^{-1/4}-0.81 \tag{6}$$

where a small correction term due to finite number of particles is neglected. The dependence of the other transport coefficient η_l on Γ is somewhat more complex, and cannot be expressed in such a closed analytical form.

III. KdV Equation and Traveling Wave System

To derive the KdV equation [31,32] for the DA solitary waves from basic equations (1)-(3), we use RPT [33,34] with the stretched coordinates [35,36]

$$\xi = -\varepsilon^{1/2}(x + Vt) \tag{7}$$

$$\tau = \varepsilon^{3/2} t \tag{8}$$

where V is the wave phase speed and ε is a smallness parameter which characterize the weakness of the dispersion($0 < \varepsilon < 1$). We can expand the quantities n, u and ϕ about the equilibrium values in power series of ε as

$$n = 1 + \varepsilon n^{(1)} + \varepsilon^2 n^{(2)} + \varepsilon^3 n^{(3)} + \ldots\ldots\ldots\ldots\ldots \tag{9}$$

$$u = 0 + \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \varepsilon^3 u^{(3)} + \ldots\ldots\ldots\ldots\ldots \tag{10}$$

$$\phi = 0 + \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \varepsilon^3 \phi^{(3)} + \ldots\ldots\ldots\ldots\ldots \tag{11}$$

Substituting the Eqs.(7)-(11) into the system of Eqs. (1)-(3) and equating the coefficient of lowest order of ε, we obtain the dispersion relation as

$$V^2 = \sigma_n \mu_n + \frac{1}{\mu_e \sigma + \mu_i} \tag{12}$$

Considering the coefficient of next higher order of ε , one can obtain the KdV equation as

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0 \tag{13}$$

where the nonlinear coefficient A and the dispersion coefficient B are given by

$$A = -B[g + \sigma^2 \mu_e - \mu_i] \tag{14}$$

$$B = \frac{(V^2 - \sigma_n \mu_n)^2}{2V} \tag{15}$$

$$g = \frac{3V^2 - \sigma_n \mu_n}{(V^2 - \sigma_n \mu_n)^3} \tag{16}$$

To discuss the traveling wave system we want to transform the KdV equation (13) to traveling wave system defining a variable given by

$$\chi = \xi - c\tau \tag{17}$$

where c denotes the velocity of traveling wave. Using $\psi(\chi)=\phi^{(1)}$ in the KdV equation (13), we obtain

$$-c\frac{d\psi}{d\chi}+A\psi\frac{d\psi}{d\chi}+B\frac{d^3\psi}{d\chi^3}=0 \tag{18}$$

Integrating Eq.(18) with respect to χ and using the conditions $\psi\to 0$, $\frac{d\psi}{d\chi}\to 0$,$\frac{d^2\psi}{d\chi^2}\to 0$, we obtain

$$-c\psi+A\frac{\psi^2}{2}+B\frac{d^2\psi}{d\chi^2}=0 \tag{19}$$

Then Eq. (19) can be expressed as the following dynamical system:

$$\begin{cases}\dfrac{d\psi}{d\chi}=z\\ \dfrac{dz}{d\chi}=\dfrac{1}{2B}(2c-A\psi)\psi\end{cases} \tag{20}$$

Equation (20) is a traveling wave system with Hamiltonian function given by

$$H(\psi,z)=\frac{z^2}{2}+\frac{1}{6B}(A\psi-3c)\psi^2=h,\text{say} \tag{21}$$

The system Eq. (20) is a planar dynamical system with parameters $\sigma,\sigma_n,\mu_e,\mu_i,\Gamma$. It is important to note that the phase orbits defined by the vector fields of Eq.(20) will determine all traveling wave solutions of the KdV Eq.(13). So, we investigate the bifurcations of phase portraits of Eq.(20) in the (ψ,z) phase plane depending on the parameter $\sigma,\sigma_n,\mu_e,\mu_i,\Gamma$. In this case, we want to mention that we are considering a physical system for which only bounded traveling wave solutions are meaningful. So, we only pay our attention to the bounded traveling are solutions of Eq.(13).It is well known that a solitary wave solution of Eq.(13)corresponds to a homoclinic orbit of Eq.(20) and a periodic orbit of Eq.(20) corresponds to a periodic traveling wave solution of Eq.(13).The bifurcation theory planar dynamical systems[36,37] plays a crucial role in the present work.

IV. Bifurcation Analysis

In this section, we study the bifurcation analysis of Eq.(20). When $B\neq 0$ and $Ac\neq 0$ then there are two equilibrium points at $E_0(\psi_0,0)$ and $E_1(\psi_1,0)$, where $\psi_0=0$, $\psi_1=\frac{2c}{A}$. Let $M(\psi_i,0)$ be the coefficient matrix of the linearized system of Eq.(20) at an equilibrium point $E_i(\psi_i,0)$. Then we have the Jacobian determinant

$$J=\det M(\psi_i,0)=-\frac{c}{B}+\frac{A}{B}\psi_i \tag{22}$$

Using the theory of dynamical systems [37,38], we know that the equilibrium point $E_i(\psi_i,0)$ of the planar dynamical system is a saddle point when $J<0$ and the equilibrium point $E_i(\psi_i,0)$ of the planar dynamical system is a center when $J>0$. Applying the above analysis of the physical parameters $\sigma,\sigma_n,\mu_e,\mu_i,\Gamma$ we have shown phase portraits of Eq.(20) depending on special values of the parameters, shown in Figs. (1-2).

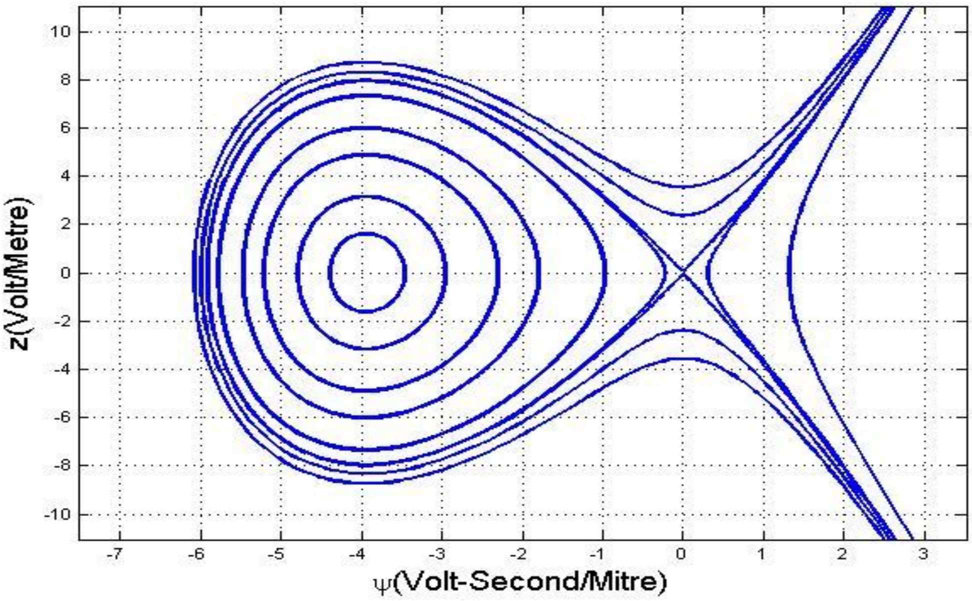


FIGURE 1: Phase portraits of the Eq:(20) for $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10, \sigma = 0.1, c = 3$

It is clear that the value of J at the equilibrium point $E_0(\psi_0,0)$ is negative and the value of J at the point $E_1(\psi_1,0)$ is positive. Thus the equilibrium point $E_0(\psi_0,0)$ of the system (21) is a saddle point and the equilibrium point $E_1(\psi_1,0)$ is a center. Then there exists a homoclinic orbit at equilibrium point $E_0(\psi_0,0)$ and a family of periodic orbits about the equilibrium point $E_1(\psi_1,0)$.

Corresponding to the homoclinic orbit(Fig.1) of the system (20) the solitary wave solution of the KdV Eq.(13) is given by,

$$\phi^{(1)} = \frac{3c}{A}\sec h^2\left(\frac{1}{2}\sqrt{\frac{c}{B}}\chi\right) \tag{23}$$

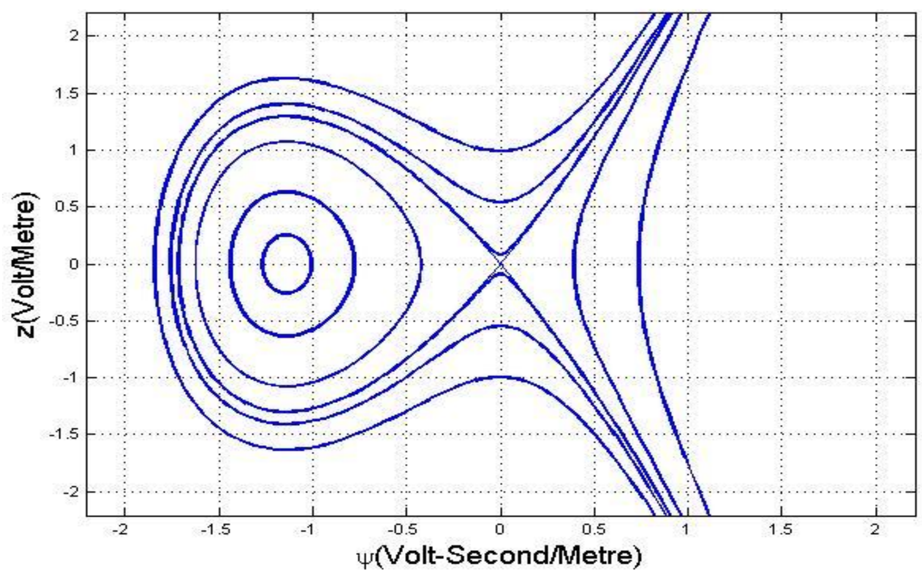


FIGURE 2 : Phase portraits of the Eq: (20) for $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10, \sigma = 0.1, c = 0.9$

V. Quasiperiodicity

The discovery of complex dynamics in the deterministic nonlinear system motivated the researchers to investigate simple mathematical models [39]. In the deterministic models, which are presented by mathematical equation may show different long term behavior such as stable focus, limit cycle oscillation, quasiperiodic oscillation and chaotic oscillation. The precise dynamics of the system are determined by model parameters and external forces. Periodic behavior is defined as occurrence of a phenomenon at regular intervals. Quasiperiodic oscillation is incommensurable periodic motions and the trajectory in the phase space densely cover the surface of the attractor. For a system, as the control parameter varies, successive Hopf-bifurcations occur and new fundamental frequency introduces into the system, which causes quasiperiodic oscillation of the system. In a quasiperiodic motion, the trajectory returns arbitrarily close to its starting point infinitely often. Quasiperiodicity can be exhibited in various branches of research fields, like, physics, chemistry, ecology etc.[40]-[41].

In this section, we present the quasiperiodic behavior of the perturbed system given by:

$$\begin{cases} \dfrac{d\psi}{d\chi} = z \\ \dfrac{dz}{d\chi} = \dfrac{1}{2B}(2c - A\psi)\psi + f_0 \cos(\omega\chi) \end{cases} \tag{24}$$

Where $f_0 \cos(\omega\chi)$ is the external periodic perturbation, f_0 is the strength of the periodic perturbation and ω is the frequency. In the Fig.3, we have presented the phase portraits of perturbed system (24) for the values of $\sigma = 0.1, \sigma_n = 0.0003, \mu_e = 0.55, \Gamma = 10, c = 3$, $f_0 = 0.1, \omega = 1$ and quasiperiodic motion of the system(24) is found with incommensurable periodic motion and the trajectory in the phase space winds around a torus filling its surface densely. In the Fig. 4(a)-4(b), we have plotted z vs. χ and ψ vs. χ respectively for the perturbed system (24) for above set of parameter values. In the Fig.5, we have presented the phase portraits of perturbed system (24) for the values of $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10$, $\sigma = 0.2, c = 0.9$, $f_0 = 0.1, \omega = 1$ and quasiperiodic motion of the system (24) is found with incommensurable periodic motion and the trajectory in the phase space winds around a torus filling its surface sparsely. In the Fig. 6(a)-6(b), we have plotted z vs. χ and ψ vs. χ respectively for the perturbed system (24) for above set of parameter values. In the Fig.7, we have presented the phase portraits of perturbed system (24)for the values of $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10$, $\sigma = 0.01, c = 1.2$, $f_0 = 0.1, \omega = 1$ and quasiperiodic motion of the system(24) is found with incommensurable periodic motion and the trajectory in the phase space winds around a torus filling its surface densely. In Fig. 8(a)-8(b), we have plotted z vs. χ and ψ vs. χ respectively for the perturbed system (24) for above set of parameter values. From these Figs. (3-8) it is clear that our perturbed system: (24) has quasiperiodic behavior in the presence of external forces but not chaotic. Sahu et al.[42] studied solitonic, quasi periodic and periodic patterns of nonlinear electron acoustic waves in quantum plasmas. Recently, Zhen et al. [43] investigated dynamic behavior of ion acoustic waves in dense quantum magnetoplasmas in presence of external perturbation.

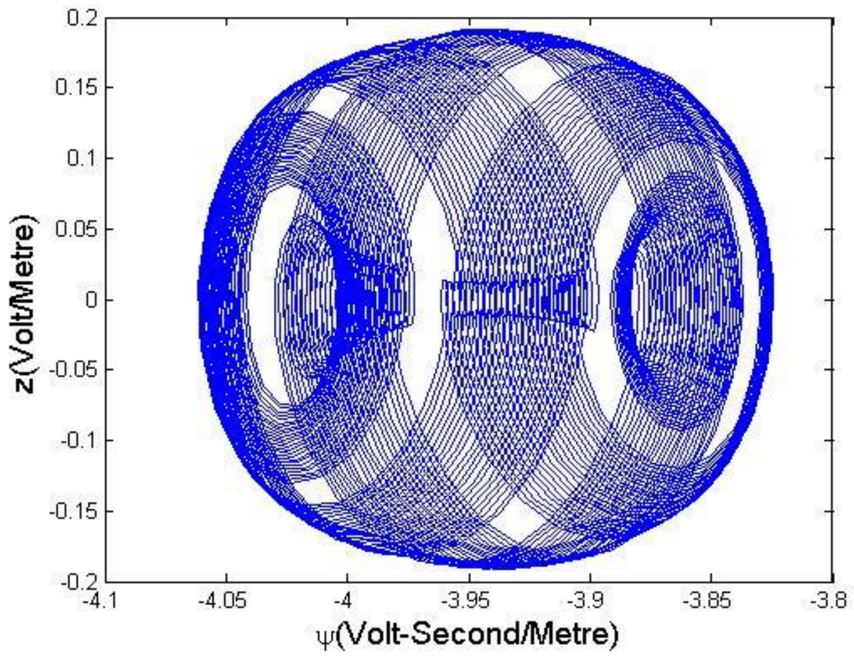
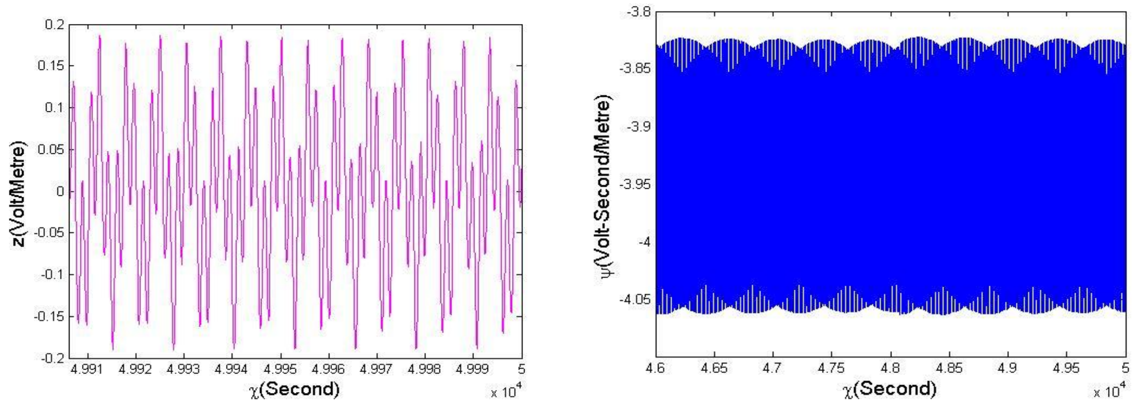


FIGURE 3: Phase portraits of the perturbed system: (24) for $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10$, $f_0 = 0.1, \omega = 1, \sigma = 0.1, c = 3$.



(a) **(b)**

FIGURE 4: Plot of z vs. χ and ψ vs. χ of the perturbed system: (24) $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10,\ \sigma = 0.1, c = 3,\ f_0 = 0.1, \omega = 1$.

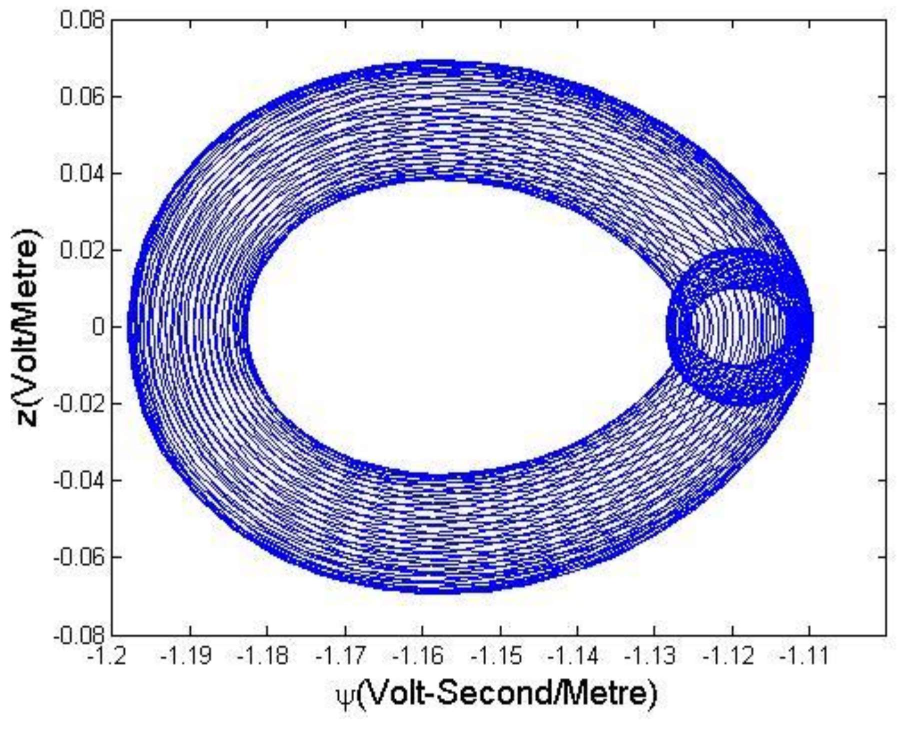
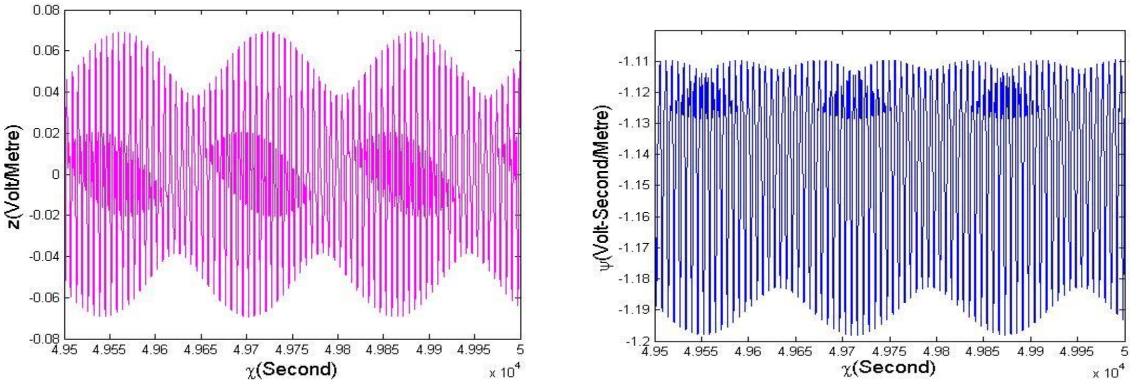


FIGURE 5: Phase portraits of the perturbed system :(24) for $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10,$ and $\sigma = 0.2, c = 0.9,\ f_0 = 0.1, \omega = 1$.



(a) **(b)**

FIGURE 6: Plot of z vs. χ and ψ vs. χ of the perturbed system :(24) $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10, \sigma = 0.2, c = 0.9$, $f_0 = 0.1, \omega = 1$.

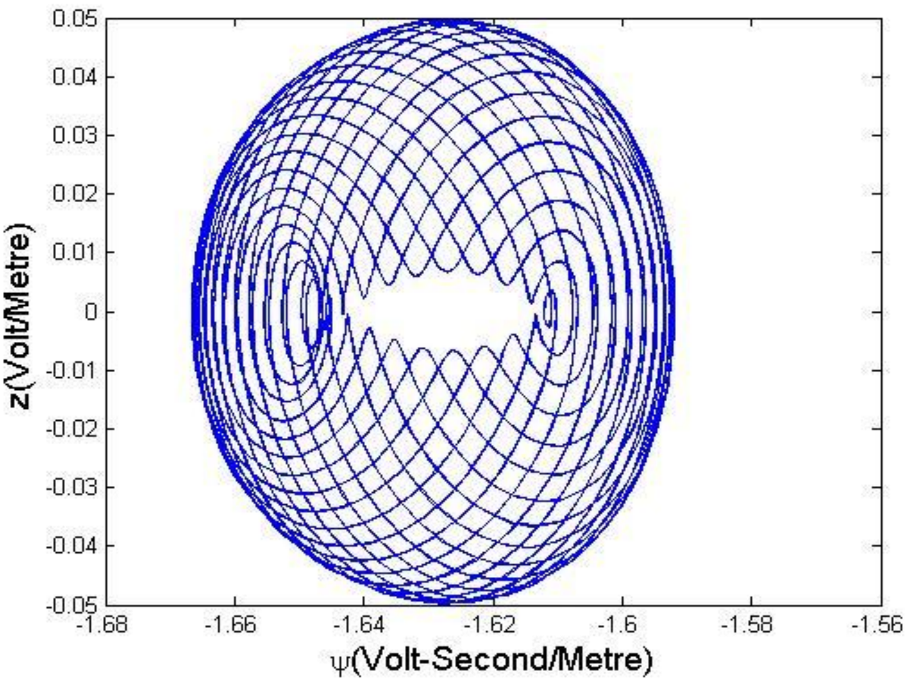
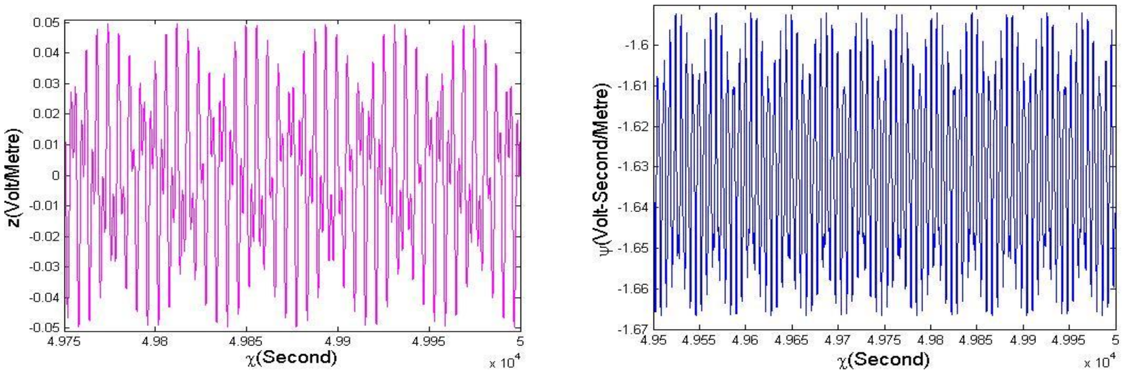


FIGURE 7: Phase portraits of the perturbed system :(24) for $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10,$ and $\sigma = 0.01, c = 1.2$, $f_0 = 0.1, \omega = 1$.



(a) **(b)**

FIGURE 8: Plot of z vs. χ and ψ vs. χ of the perturbed system : (24) $\sigma_n = 0.00003; \mu_e = 0.55, \Gamma = 10, \sigma = 0.01, c = 1.2, f_0 = 0.1, \omega = 1$.

VI. Conclusions

In this paper, we have derived KdV equation in electron-ion strongly coupled dusty plasma with Maxwellian electron and ion. Using bifurcation theory of planar dynamical systems to the KdV equation, we have presented the existence of solitary wave solution and periodic traveling wave solutions. Two exact solutions of these waves are obtained depending on system parameters $\sigma, \sigma_n, \mu_e, \mu_i, \Gamma$. Considering the external perturbation, the quasi periodic behavior of dust acoustic waves has been studied in detail.

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1. P.K.Shukla, Phys.Plasmas 11,3676(2004).
2.T.V.Losseva,S.I.Popel ,M.U.Yu,J.X.Ma , Phys.Rev.E 75,046403(2007).
3.H.ThomasandG.Morfil, Nature (London)379, 806(1996).
4.P.K.Kaw and A.Sen, Phys.Plasmas 5,3552(1996).
5.G.E.Morfill, H.M.Thomas,U.Konopka,M.Zuzic,Phys.Plasmas 6,1769 (1999).
6.J.Pieper, J.Goree, Phys. Rev. Lett. 77, 3137 (1996)
7.A.Melzer, A.Homann and A.Piel, Phys. Rev. E 53, 2757 (1997)
8.J.Chu and H.Lee, Phys. Rev. Lett. 72, 4009 (1994).
9.A.Melzer, T.Trottenberg and A.Piel, Phys. Lett. A 191, 301(1994).
10.Y.Hayashi and K.Tachibana, Jpn. J. Appl. Phys. 33, L804(1994).
11.H.Thomas,G.E.Morfill,V.Demmel,J.Goree,B.Feuerbacher and D.Mhlmann, Phys.Rev.Lett.73,652(1994).

12.A.A.Mamun,Astrophys. Space Sci. 268, 443(1999).
13.P.K.Shuklaand A.A.Mamun, Introduction to Dusty Plasma. Physics. Bristol, UK,IoP(2002).
14.P.KShukla and A.A. Mamun, IEEETrans.Plasma Sci.29,221(2001).
15.A.A.Mamun, B.P.Eliasson and P.K.Shukla, Phys.Lett. A 332,412(2004)
16.A.AMamun and P.K.Shukla, Phys.Lett A 373,3161(2009a).
17.M.G.M.Anowar, M.S.Rahaman and A.A.Mamun, Phys.Plasmas 16,053704 (2009).
18.A.A.Mamun and P.K.Shukla, Euro.Phys.Lett.87,55001(2009b).
19.M.S.Rahaman,B.Shikha and A.A.Mamun, J.Plasma Phys.79,3,249-255(2013).
20.S.Garai, D.Banerjee, M.S.Janaki and N.Chakrabarti,Astrophys.SpaceSci 349,789-798(2014).
21.N.Chakrabarti and S.Ghosh, J Plasma Physics.81,905810310 (2015).
22.A.A.Mamun and P.K.Shukla, New J.Phys.11,103022(2009c)
23.U.K.Samanta, A.Saha, and P.Chatterjee, Astrophys.Space Sci.347,293-298(2013).
24.A.Saha,P.Chatterjee, Astrophys.Space Sci,353,169-177(2014).
25.A.Saha and P.Chatterjee, Astrophys.Space Sci.349:239-244(2014).
26.N.N.Rao, P.K.Shukla and M.Y.Yu, Planet.Space Sci. 38,543(1990).
27.R.Abe, Prog.Theor.Phys,21,475(1959).
28.A.A.Mamun, P.K.Shukla and T.Farid, Physics of Plasmas,7,2329 (2000).
29.M.A.Berkovsky, Phys.Lett.A 166,365(1992).
30.W.L.Slattery and G.D.Doolen, Physical Review A,21(6),2087(1980).
31.H.Alinejad, Astrophys.Space Sci.327,131(2010).
32.H.R.Pakzad, Astrophys.Space Sci.330,30(2010)
33.H.Washimi and T.Taniuti, Phys.Rev.Lett. 75,996(1996).
34.M.S.Zobaer, N.Roy and A.A.Mamun, Astrophys.Space Sci.343,675(2013).
35.S.Maxon and J.Viecelli, Phys.Rev.Lett.32,4(1974).
36.A.A.Mamun, Phys.Lett. A 372,686(2008).
37.S.N.Chow, J.K.Hale, Method of Bifurcation Theory.Springer,New York(1981).
38.J.Guckenhemer, P.J.Holmes, Nonlinear Oscillation,Dynamical System and Bifurcation of Vectorfields. Springer, New york(1983).
39. E.N.Lorenz, Deterministic nonperiodic flow, Journal of the Atmospheric Science,20,130-141 (1963).
40. S. Ostlund, D. Rand, J. Sethna, E. Siggira ,Universal properties of the transition from quasi-periodicity to chaos in dissipative systems, Physica D: Nonlinear Phenomena, 8, 3(1983).
41. G.S. Skinner, H.L. Swinney, Periodic to quasiperiodic transition of chemical spiral rotation, Physica D: Nonlinear Phenomena, 48, 1(1991).
42.B.Sahu, S.Poria and R.Roychoudhury, Astrophys. Space Sci. 341,567(2012).
43. H.Zhen, B.Tian, Y. Wang, H.Zhong and W.Sun, Phys. Plasma 21,012304(2014).