Collisions of Ion Acoustic Multi-solitons in Electron-Positron-Ion Plasma with the Presence of Nonthermal Nonextensive Electrons

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Abstract. Face to face collision of ion acoustic multi-solitons is studied for the electron-positron-ion plasma with nonthermal non-extensive electrons using extended version of Poincare-Lighhill-Kuo (PLK) method. Three soliton expressions have been derived by Hirota's method. It is observed that there is a phase shift in each soliton due to collision. The change in phase shift is studied with nonthermal non-extensive parameter q in the range $0.6 < q \leq 1$. It has been seen that the nonthermal non-extensive parameter plays a significant role on phase shift.

Keywords: Multi-solitons interaction; extended Poincare-Lighthill-Kuo method; head-on collision; nonextensive distribution; nonthermal electrons

I. INTRODUCTION

The idea of an intrinsic analysis of nonlinear phenomena is now well understood, and leads to two new concepts, the strange attractor and the soliton. Both are related to amazing properties of nonlinear systems. The strange attractor is linked with the idea of chaos for a system of small number of degree of freedom while the soliton appears in the systems with the large number of degree of freedom. Soliton research is more important to physicists due to the balance between nonlinearity and dispersion. These nonlinearities have been found in propagation of some hydrodynamic waves, localized waves in astrophysical plasmas, the propagation of signals in optic fibers or, at the microscopic level, charge conducting in polymers localized modes in magnetic crystals and the dynamics of biological molecules such as DNA and proteins [1-4].

The propagation and interaction of solitons in plasma play a significant role in analysis of different features of waves. The basic characteristic of solitary wave is that it preserves the same structure even when it undergoes a collision. The term soliton as solitary wave was first proposed by Zabusky and Kruskal [5]. The fundamental behaviors of soliton interaction are (i) soliton-

soliton collisions are elastic, that is, the amplitude of solitons do not change (ii) after the interaction, each soliton gets an supplementary phase shift and (iii) the total phase shift of a soliton acquired during a certain time interval which can be determined as the sum of the elementary phase shifts in pair-wise collisions with other solitons. Multi-soliton interaction is considered very important as it plays an important role in the dynamics of a soliton. Two types of interaction usually occur viz. head-on collision and overtaking collision. Many researchers [6-11] have studied head-on collisions and phase shift of two opposite directional solitary waves by using the extended Poincare-Lighhill-Kuo (PLK) method. It is well known that the larger amplitude wave travels faster whereas lesser amplitude wave travels slower. When two or more different amplitude solitons propagate with different velocities but in the same direction i.e angle between them is zero, the larger amplitude wave first catches the smaller amplitude one and overtakes, this is known as overtaking collision. Thus, it can be written in a form such that its relationship to the solitary waves is explicitly displayed. The utility of this special formulation of the solution can be established by analyzing the structure during interaction of the multi soliton solutions of the KdV equation. Overtaking collisions of solitons, especially phase shifts after collision, have been studied by only a few researchers [7,8]. Sahu and his collaborators [12,13] have studied the nonplanar effect on the two solitons interaction when one soliton overtakes the other. They used Hirota's bilinear method [14] to find the multi-soliton solution. Roy *et al.* [14] have also studied overtaking collision of two soliton and obtained the phase shift during such collisions.

The study of head-on collision of solitary waves and their phase shift in electron-positron-ion plasma is one of the most rapidly growing areas of plasma physics and has been studied by several authors [11-16]. T. Maxworthy [15] first observed head-on collision between solitary waves. From the experiments it was found that the wave reached maximum amplitude which was greater than twice the first wave amplitude and these waves were affected by a time delay during their interaction. The results derived are then compared with the present theories and are found to be in qualitative agreement. Su and Mirie [6] theoretically investigated head-on collision between two solitary waves on the surface of an inviscid homogeneous fluid. Ghosh *et al.* [16] shows the effect of q-distributed electrons on the head-on collision of IAWs.

J. K. Xue [7] has investigated the head-on collision between two cylindrical/spherical dust ion-acoustic solitary waves (DIASW) in un-magnetized dusty plasmas and has shown how the non-planar geometry modifies analytical phase shifts. More recently, Ghosh *et al.* [16-19] have studied the interaction of solitary waves in one dimensional unmagnetized geometry, and also studied the head-on collision between ion acoustic solitary waves in cylindrical and spherical geometry.

The presence of energetic particles in plasma can change astrophysical plasma environment and the distribution functions are to be considered as highly nonthermal. Nonthermal electron distribution turned out to be a very general characteristic feature of space plasma. Cairns *et al.* [20] showed that the nonthermal distribution of electrons may change the nature of ion solitary structure. Cairns *et al.* [20] first introduce a distribution for the high-energy particles which are observed in space plasma. This distribution is represented in terms of a parameter α, which measures the deviation from Maxwellian distribution. In 1988 Tsallis [21] introduced a general Boltzmann-Gibbs-Shannon (BGS) entropic measure in which a parameter q is characterized as the degree of nonextensivity of the system. The generalized entropy of whole is greater than the sum of the entropies of the parts if $q<1$ which is known as superextensivity, whereas the generalized entropy of the system is smaller than the sum of the entropies of the parts if q>1, which is known as subextensivity. This Tsallis (1988) model is commonly used for investigation of ion- acoustic dynamics in plasmas. In our present work, we consider the Cairns-Tsallis distribution of electron. It is accepted that the Tsallis distribution behaves very differently in the two ranges, -1 < q < 1 and q > 1 [22]. It is important to note that the q-distribution is not normalizable if q<1. In extensive limiting case, if $q \to 1$, the q-distribution follows the Maxwell-Boltzmann velocity distribution. When the value of q is more than 1 it is not allowed value in this work as this physical region is not consistent with a long-tailed distribution function associated with an excess of energetic particles and is physically not relevant to our purposes. However, expressions for electron number density are different for the values of q > 1 and -1 < q < 1. It is surprising that in the previous analysis [14,15], [16-19], involving the pure Tsallis type distribution (i.e., for α=0), use the same electron number density distribution for both the regions. In this study, we have restricted the values of q in the very limited range, $0.6 < q \le 1$ (arguments for selecting this range are discussed in [22]) .

Most of the researchers investigated only head-on collision between two opposite directional solitary waves of two different KdV equations. Our work is unique in the way that it studies the collisions of multi-solitons obtained from each KdV equation by using PLK method, an area that is not yet explored by previous researchers.

II. BASIC EQUATIONS AND DERIVATION OF KDV EQUATIONS

Let us consider a three component unmagnetized, collisionless plasma with cold ions, Boltzmann distributed positrons, and nonthermal nonextensive distributed electrons. The basic equations are

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial (u_i)}{\partial x} = -\frac{\partial \psi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{n_e}{1-p} - \frac{p\, n_p}{1-p} - n_i. \tag{3}$$

$$n_e = (1+(q-1)\psi)^{(q+1)/2(q-1)} \times (1 + A\psi + B\psi^2),$$
$$n_p = \exp(-\sigma\psi),\ \sigma = \mathrm{T}_e / T_p,$$
$$p = n_{p0} / n_{e0} = 1 - n_{i0} / n_{e0},$$
$$A = -16q\alpha / (3 - 14\mathrm{q} + 15\mathrm{q}^2 + 12\alpha),$$
$$B = 16(2\mathrm{q}-1)\,\mathrm{q}\,\alpha / (3 - 14q + 15q^2 + 12\alpha).$$

where n_e n_p and n_i are the electron, positron and ion number densities respectively, normalized to the perturbed ion number density, u_i is the ion fluid velocity normalized to the ion acoustic speed $C_s = (K_B T_e / m_i)^{1/2}$, where K_B is the Boltzmann constant. The space and time coordinates are normalized to the Debye length $\lambda_D = (K_B T_e / 4\pi n_{i0} e^2)^{1/2}$ and ion plasma period $\omega_{pi}^{-1} = (m_i / 4\pi n_{i0} e^2)^{1/2}$, respectively. The electrostatic wave potential ψ is normalized to $K_B T_e / e$, where e is the electron charge. T_e, T_p, α and q are the electron temperature, positron temperature, nonthermal parameter and the strength of nonextensivity, respectively.

A physically meaningful nonextensive nonthermal velocity distribution is followed due to the presence of energetic electrons in a variety of astrophysical plasma environments and measurements of their distribution functions [23] - [25]. As the nonextensive character of the nonthermal electrons increases, the distribution may become less or more prominent and high-energy states are less or more probable than in the extensive nonthermal case. Relying on the most recent work Tribeche et al. [26] the electrons are assumed to follow a nonextensive nonthermal velocity distribution function given by

$$f_e(v_x) = \mathrm{C}_{q,\alpha}\left(1+\alpha\frac{v^4{}_x}{v^4{}_{te}}\right)\left\{1-(q-1)\frac{v^2{}_x}{2v^2{}_{te}}\right\}^{1/(q-1)} \tag{4}$$

where $v_{te} = (T_e / m_e)^{1/2}$, T_e and m_e are the electron thermal velocity, the electron velocity, and electron mass respectively.

$$C_{q,\alpha} = \begin{cases} n_{e0}\sqrt{\dfrac{m_e}{2\pi T_e}}\dfrac{\Gamma(\frac{1}{1-q})(1-\mathrm{q})^{5/2}}{\Gamma(\frac{1}{1-q}-\frac{5}{2})[3\alpha+(\frac{1}{1-q}-\frac{3}{2})(\frac{1}{1-q}-\frac{5}{2})(1-\mathrm{q})^2]} & \text{for } -1<q<1 \\ n_{e0}\sqrt{\dfrac{m_e}{2\pi T_e}}\dfrac{\Gamma(\frac{1}{1-q}+\frac{3}{2})(\mathrm{q}-1)^{5/2}(\frac{1}{q-1}+\frac{3}{2})(\frac{1}{q-1}+\frac{5}{2})}{\Gamma(\frac{1}{1-q}+1)[3\alpha+(\mathrm{q}-1)^2(\frac{1}{q-1}+\frac{3}{2})(\frac{1}{q-1}+\frac{5}{2})]} & \text{for } q>1 \end{cases} \tag{5}$$

is the constant of normalization. Here α , q , Γ are the number of nonthermal electrons, the strength of nonextensivity, and Gamma function respectively. For $q > 1$, the distribution function (4) represents a thermal cutoff on the maximum value of the velocity of the electrons, given by

$$v_{\max} = \sqrt{\frac{2T_e}{m_e(\mathrm{q}-1)}}, \tag{6}$$

beyond which no probable states exist. For $q = 1$, the nonthermal distribution is obtained. High energy states are more probable than in the extensive case when $q < 1$. Integrating Eq. (4) over all velocity space, we get the electron density as

$$n_e(\phi) = \begin{cases} \int_{-\infty}^{\infty} f_e(v_x)\,\mathrm{d}\,v_x & \text{for } -1<q<1 \\ \int_{-v_{\max}}^{v_{\max}} f_e(v_x)\,\mathrm{d}\,v_x & \text{for } q>1 \end{cases}$$

$$= n_{e0}\left[1+(q-1)\frac{e\phi}{T_e}\right]^{(q+1)/2(q-1)}\left[1+A\left(\frac{e\phi}{T_e}\right)+B\left(\frac{e\phi}{T_e}\right)^2\right] \quad (7)$$

where $A=-16q\alpha/(3-14q+15q^2+12\alpha)$ and $B=16(2q-1)q\alpha/(3-14q+15q^2+12\alpha)$. We used this expression only for the range 0.6<q<1. In the extensive limiting case ($q\to 1$), the density (7) reduces to the well-known nonthermal electron density,

$$n_e(\phi)=n_{e0}\left(1-\frac{4\alpha}{1+3\alpha}\left(\frac{e\phi}{T_e}\right)+\frac{4\alpha}{1+3\alpha}\left(\frac{e\phi}{T_e}\right)^2\right)\exp\left(\frac{e\phi}{T_e}\right)$$

Now it is assumed that two solitons in the plasma, which are asymptotically far apart initially but they travel toward each other. After a certain time both solitons interact, collide, and then depart. We also assume that the solitons having small amplitudes which are proportional to ε (where ε is the smallness parameter characterizing the strength of non linearity) and the interaction between two solitons is not strong enough. Hence it is expected that the collision is quasi elastic, so it will only cause shifts of the post collision trajectories (phase shift). In order to analyze the effects of collision, we employ an extended PLK method. Here, the dependent variables are expanded as

$$n_i = 1+\varepsilon^2 n_1+\varepsilon^3 n_2+\varepsilon^4 n_3+..., \quad (8)$$

$$u_i = 0+\varepsilon^2 u_1+\varepsilon^3 u_2+\varepsilon^4 u_3+..., \quad (9)$$

$$\psi = 0+\varepsilon^2 \psi_1+\varepsilon^3 \psi_2+\varepsilon^4 \psi_3+..., \quad (10)$$

The independent variables are given by

$$\xi=\varepsilon(x-c_1t)+\varepsilon^2P_0(\eta,\tau)+\varepsilon^3P_1(\eta,\xi,\tau)+..., \quad (11)$$

$$\eta=\varepsilon(x+c_2t)+\varepsilon^2Q_0(\xi,\tau)+\varepsilon^3Q_1(\eta,\xi,\tau)+..., \quad (12)$$

$$\tau=\varepsilon^3 t. \quad (13)$$

where ξ and η denote the trajectories of two solitons traveling toward to each other, where as c_1 and c_2 are two unknown phase velocities of IASWs. Two more functions $P_0(\eta,\tau)$ and $Q_0(\xi,\tau)$ are to be determined.

Using extended PLK method and after some long but standard calculation, we get

$$-\frac{2}{a_1}u_3=\int\left(\frac{\partial\psi_{1\xi}}{\partial\tau}+A_1\psi_{1\xi}\frac{\partial\psi_{1\xi}}{\partial\xi}+B_1\frac{\partial^3\psi_{1\xi}}{\partial\xi^3}\right)d\eta-\int\left(\frac{\partial\psi_{1\eta}}{\partial\tau}-A_1\psi_{1\eta}\frac{\partial\psi_{1\eta}}{\partial\eta}-B_1\frac{\partial^3\psi_{1\eta}}{\partial\eta^3}\right)d\xi$$
$$+\iint\left(C\frac{\partial P_0}{\partial\eta}-D\psi_{1\eta}\right)\frac{\partial^2\psi_{1\eta}}{\partial\xi^2}d\xi d\eta-\iint\left(C\frac{\partial Q_0}{\partial\xi}-D\psi_{1\xi}\right)\frac{\partial^2\psi_{1\eta}}{\partial\eta^2}d\xi d\eta \quad (14)$$

where

$$A_1=\frac{3a_1^2-a_2}{2a_1^{3/2}},\quad B_1=\frac{1}{2a_1^{3/2}},\quad C=\frac{2}{\sqrt{a_1}},\quad D=\frac{a_1^2-a_2}{2a_1^{3/2}},$$

$$a_1 = \frac{q+1+2A+2p\sigma}{2(1-p)}, \quad a_2 = \frac{(q+1)(3-\mathrm{q})+4\,\mathrm{A}(\mathrm{q}+1)+8\,\mathrm{B}-4\,\mathrm{p}\,\sigma^2}{8(1-p)}.$$

The first term in the right hand side of Eqs (14) will be proportional to η because the integrand is independent of η and the second term in the right hand side of Eqs (14) will be proportional to ξ because the integrand is independent of ξ. These two terms of Eqn (14) are secular terms, which must be eliminated in order to avoid spurious resonances. Hence, we have

$$\frac{\partial \psi_{1\xi}}{\partial \tau} + A_1 \psi_{1\xi} \frac{\partial \psi_{1\xi}}{\partial \xi} + B_1 \frac{\partial^3 \psi_{1\xi}}{\partial \xi^3} = 0, \tag{15}$$

$$\frac{\partial \psi_{1\eta}}{\partial \tau} - A_1 \psi_{1\eta} \frac{\partial \psi_{1\eta}}{\partial \eta} - B_1 \frac{\partial^3 \psi_{1\eta}}{\partial \eta^3} = 0, \tag{16}$$

The third and fourth terms in Eq. (14) are not secular terms at this order, they could be secular for the next order. Hence we have

$$C \frac{\partial P_0}{\partial \eta} = D \psi_{1\eta}, \tag{17}$$

$$C \frac{\partial Q_0}{\partial \xi} = D \psi_{1\xi}, \tag{18}$$

Equation (15) is a KdV equation. This wave is traveling in the ξ direction. Eq. (16) is also a KdV equation. This wave is propagating in the η direction which is opposite to ξ. Using Hirota's method [19] one soliton solution of the KdV Eq. (15) and Eq. (16) are respectively

$$\psi_{1\xi} = \frac{12 B_1}{A_1} \frac{\partial^2}{\partial \xi^2} (\log \ \mathrm{f}) \tag{19}$$

$$\psi_{1\eta} = \frac{12 B_1}{A_1} \frac{\partial^2}{\partial \eta^2} (\log \ \mathrm{f}_1) \tag{20}$$

where $f = 1 + e^{\theta_1}$, $f_1 = 1 + e^{\phi_1}$, $\theta_1 = k_B B_1^{-1/3} \xi - k_1^3 \tau + \alpha$, $\phi_1 = -k_B B_1^{-1/3} \eta - k_1^3 \tau + \alpha$,

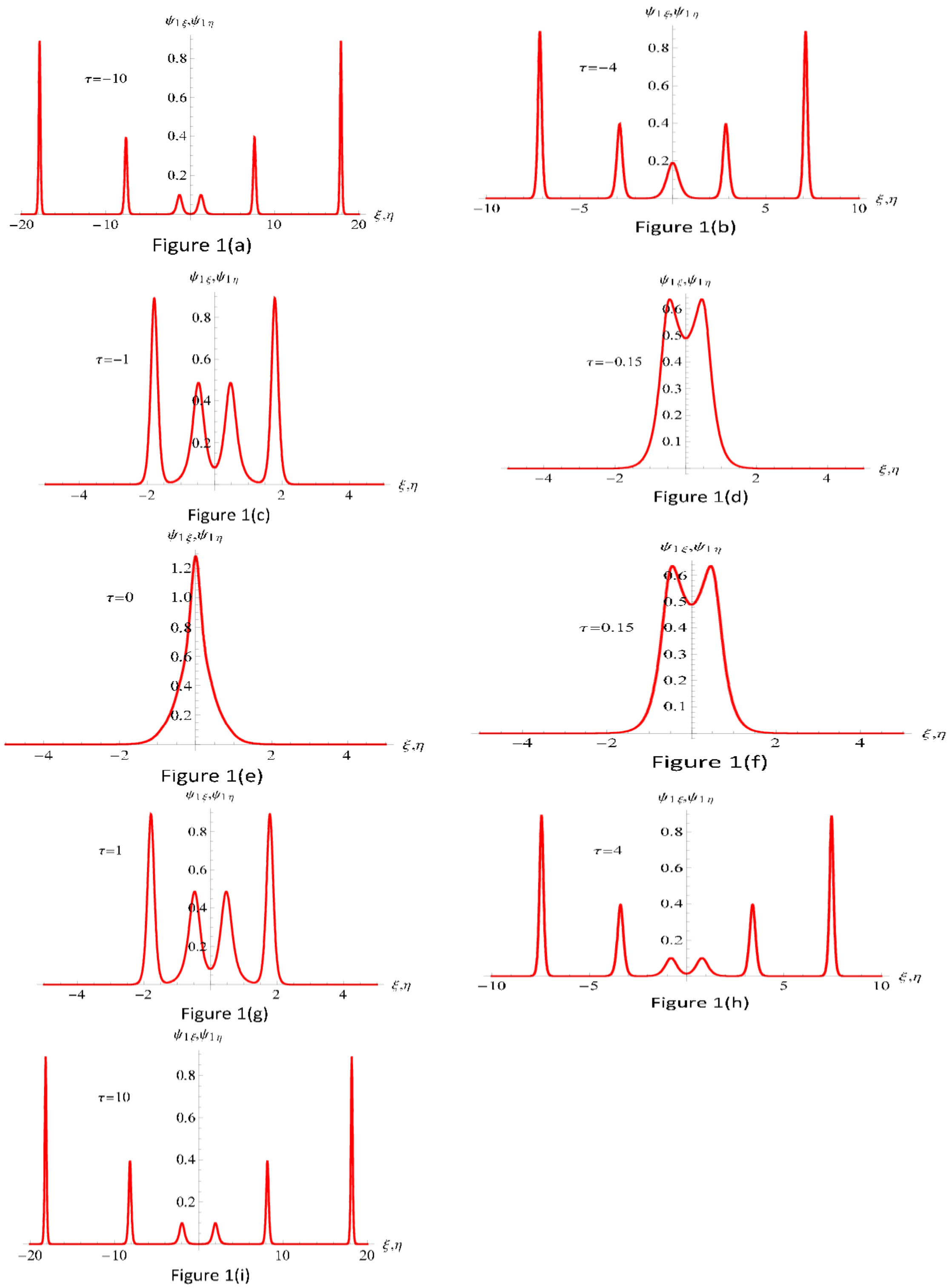

FIGURE 1. Variation of the three soliton profiles $\psi_{1\xi}$ and $\psi_{1\eta}$ for different values of τ with $p = 0.9$, $\alpha = 0.15$, $q = 0.7$ and $\sigma = 0.7$.

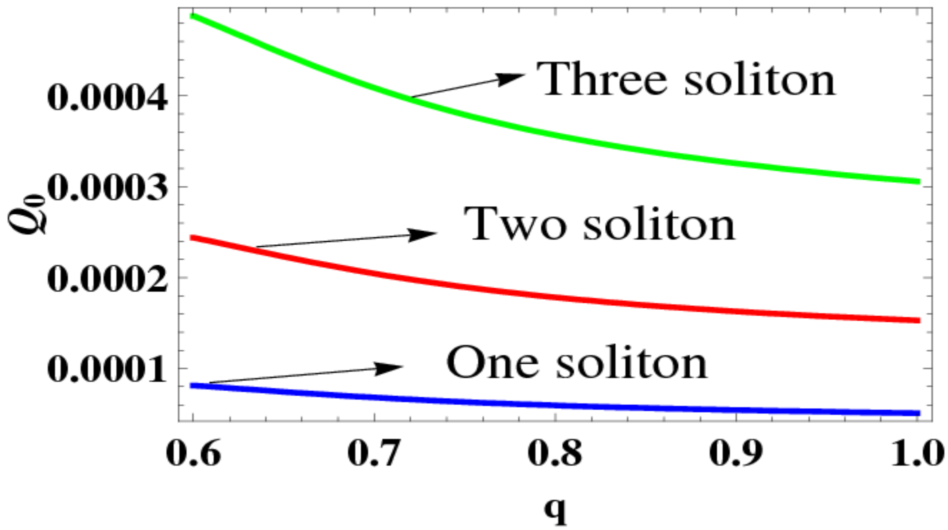

FIGURE 2. Plot of the phase shift of one soliton, two soliton and three soliton against q in the range $0.6 < q \leq 1$. The other parameters are the same as those in Figure 1.

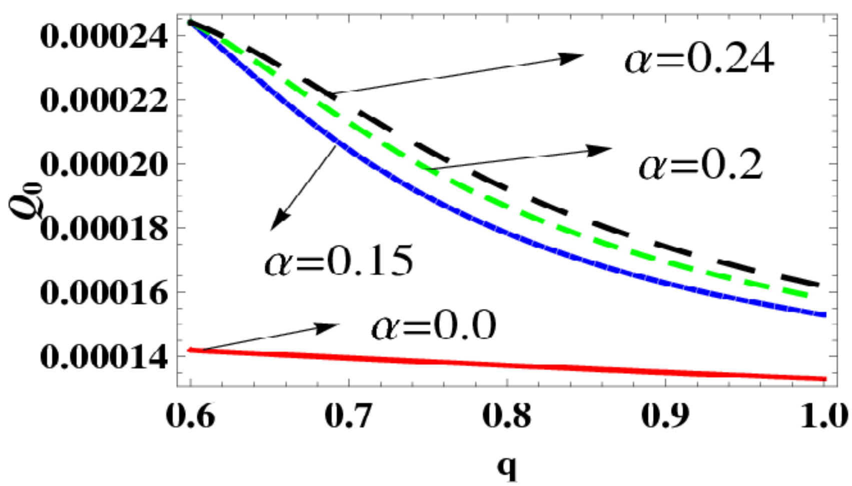

FIGURE 3. Plot of the phase shift of one soliton, two soliton and three soliton against q in the range $0.6 < q \leq 1$ for different values of α. The other parameters are the same as those in Figure 1.

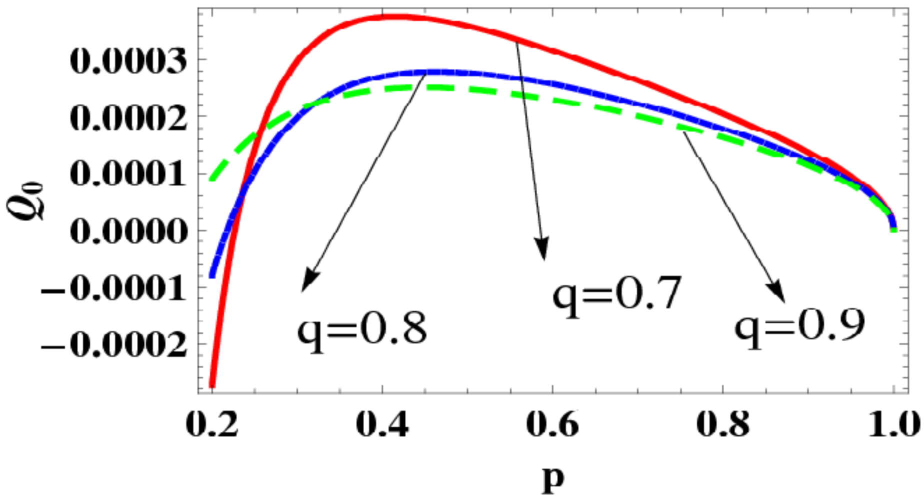

FIGURE 4. Plot of the phase shift against P for different values of q. Other parameters are $\alpha = 0.15$ and $\sigma = 0.7$.

The leading phase changes due to the collision can be calculated from Eqs (17), and (18). To obtain the phase shifts after a head-on collision of the two solitons, we assume that the solitons S_1 and S_2 are, asymptotically, far from each other at the initial time $(t=-\infty)$ i.e. the soliton s_1 is at $\xi=0,\ \eta=-\infty$ and soliton s_2 is at $\eta=0,\ \xi=+\infty$, respectively. After the collision $(t=+\infty)$, the soliton s_1 is far to the soliton s_2, i. e. soliton s_1 is at $\xi=0,\ \eta=+\infty$ and soliton s_2 is at $\eta=0,\ \xi=-\infty$.

Thus the Eq. (18) we have

$$\frac{\partial Q_0(\xi,\tau)}{\partial \xi}=\frac{12B_1D}{A_1C}\frac{\partial^2}{\partial \xi^2}(\log\ f)$$

$$\Rightarrow Q_0(\xi,\tau)=\frac{12B_1D}{A_1C}\frac{\partial}{\partial \xi}(\log\ f)\quad =\frac{12B_1^{2/3}D}{A_1C}\frac{k_1e^{\theta_1}}{1+e^{\theta_1}} \tag{21}$$

and the corresponding phase shift is

$$\begin{aligned}\Delta Q_0 &= \varepsilon(x+c_2t)\Big|_{\xi=-\infty,\eta=0}-\varepsilon(x+c_2t)\Big|_{\xi=\infty,\eta=0}\\ &=\varepsilon^2Q_0(\infty,\tau)-\varepsilon^2Q_0(-\infty,\tau)\\ &=-\frac{12\varepsilon^2DB_1^{2/3}}{A_1C}k_1\end{aligned} \tag{22}$$

Similarly the other phase shift

$$\Delta P_0=-\frac{12\varepsilon^2DB_1^{2/3}}{A_1C}k_1 \tag{23}$$

Phase shifts in Eq.(22) and Eq. (23) are similar to that in the investigations [10]-[15] in different plasma models but the approaches are different.

Again each of the KdV equations (15) and (16) has a number of soliton solutions, we consider here two-soliton solutions of each of the KdV equations. The two-soliton solutions for a particular KdV equations move in the same directions and eventually fast moving soliton overtakes the slower one, and in case of two-soliton solutions of (15) and (16) propagate from the opposite directions, although they are far from each other initially, after a certain time they came together and the head-on collision will takes place and then depart from each other. Using Hirota's method[19] two-soliton solutions of the KdV Eq.(15) and Eq.(16) are given by

$$\psi_{1\xi} = \frac{12B_1}{A_1}\frac{\partial^2}{\partial\xi^2}(\log\ g) \tag{24}$$

$$\psi_{1\eta} = \frac{12B_1}{A_1}\frac{\partial^2}{\partial\eta^2}(\log\ g_1) \tag{25}$$

where $g = 1 + e^{\theta_1} + e^{\theta_2} + \alpha_{12}e^{\theta_1+\theta_2}$, $g_1 = 1 + e^{\phi_1} + e^{\phi_2} + \alpha_{12}e^{\phi_1+\phi_2}$,
$\theta_i = k_i B_1^{-1/3}\xi - k_i^3\tau + \alpha_i$,
$\phi_i = k_i B_1^{-1/3}\eta - k_i^3\tau + \alpha_i$, i=1,2 and $a_{12} = (k_1 - k_2)^2/(k_1 + k_2)^2$.

As in case of two soliton solution from Eq. (18) we have

$$\frac{\partial Q_0(\xi,\tau)}{\partial\xi} = \frac{12B_1 D}{A_1 C}\frac{\partial^2}{\partial\xi^2}(\log\ g)$$

$$\Rightarrow Q_0(\xi,\tau) = \frac{12B_1 D}{A_1 C}\frac{\partial}{\partial\xi}(\log\ g)$$

$$\Rightarrow Q_0(\xi,\tau) = \frac{12B_1 D}{A_1 C}\frac{k_1 e^{\theta_1} + k_2 e^{\theta_2} + a_{12}(k_1 + k_2)\,\mathrm{e}^{\theta_1+\theta_2}}{1 + e^{\theta_1} + e^{\theta_2} + a_{12}\,\mathrm{e}^{\theta_1+\theta_2}} \tag{26}$$

and the corresponding phase shift

$$\begin{aligned}\Delta Q_0 &= \varepsilon(x + c_2 t)\big|_{\xi=-\infty,\eta=0} - \varepsilon(x + c_2 t)\big|_{\xi=\infty,\eta=0} \\ &= \varepsilon^2 Q_0(\infty,\tau) - \varepsilon^2 Q_0(-\infty,\tau) \\ &= \frac{12\varepsilon^2 D B_1^{2/3}}{A_1 C}\frac{a_{12}(k_1 + k_2)}{a_{12}} \\ &= \frac{12 D B_1^{2/3}}{A_1 C}(k_1 + k_2)\end{aligned} \tag{27}$$

Similarly the other phase shift is

$$\Delta P_0 = \frac{12 D B_1^{2/3}}{A_1 C}(k_1 + k_2) \tag{28}$$

Finally the three soliton solution of (15) and (16) have the form of Hirota's method [19]

$$\psi_{1\xi} = \frac{12B_1}{A_1}\frac{\partial^2}{\partial\xi^2}(\log\ h) \tag{29}$$

$$\psi_{1\eta} = \frac{12B_1}{A_1}\frac{\partial^2}{\partial\eta^2}(\log\ h_1) \tag{30}$$

$$h = 1 + e^{\theta_1} + e^{\theta_2} + e^{\theta_3} + \alpha^2{}_{12}e^{\theta_1+\theta_2} + \alpha^2{}_{13}e^{\theta_3+\theta_1} + \alpha^2{}_{23}e^{\theta_2+\theta_3} + \alpha^2 e^{\theta_1+\theta_2+\theta_3}$$
$$h_1 = 1 + e^{\phi_1} + e^{\phi_2} + e^{\phi_3} + \alpha^2{}_{12}e^{\phi_1+\phi_2} + \alpha^2{}_{13}e^{\phi_3+\phi_1} + \alpha^2{}_{23}e^{\phi_2+\phi_3} + \alpha^2 e^{\phi_1+\phi_2+\phi_3}$$
$$\theta_i = k_i B_1^{-1/3}\xi - k_i^{\ 3}\tau + \alpha_i,$$
$$\phi_i = -k_i B_1^{-1/3}\eta - k_i^{\ 3}\tau + \alpha_i,\ \text{i=1,2,3;}$$
$$a_{lm}^2 = \left(\frac{k_l - k_m}{k_l + k_m}\right)^2; \quad l, m = 1, 2, 3\ \ l < m.$$
$$a^2 = \prod_{l,m=1,l<m}^{3} a_{lm}^2$$

α_i is the initial phase of the i^{th} soliton in a three-soliton, and the corresponding phase shifts are given by

$$\Delta P_0 = -\frac{12\varepsilon^2 D B_1^{2/3}}{A_1 C}(k_1 + k_2 + k_3) \tag{31}$$

$$\Delta Q_0 = \frac{12\varepsilon^2 D B_1^{2/3}}{A_1 C}(k_1 + k_2 + k_3) \tag{32}$$

III. DISCUSSION AND CONCLUSION:

Overtaking collision is realized by applying two or more consecutive voltage pulses between the two plasmas with amplitudes such that the first pulse generates a small amplitude soliton and the second pulse generates large amplitude than the previous one and so on. Since the larger amplitude soliton propagates faster, it will overtake the smaller one. This has been studied by inverse scattering method. Now we want to study if same number of independent discharges are put at the opposite end of the plasma with same amplitude respectively in which the waves are detected, then a soliton is excited at each end of the main plasma at the same time. The solitons propagate toward each other and interact near the center of the main plasma. The collisional phenomena are shown in figures Fig. 1(a) to Fig. 1(i).

In Figure 1(a) it is seen that the existence three solitons when $\tau = -10$ on the left hand side and are moving to the right side and also there are three solitons on the right hand side and are moving towards the left side as τ approaches to zero. The fast solitons on each side over take their slower partners. Fig. 1(c) and Fig. 1(d) shows the overtaking of solitons. In Fig. 1(e) it shows the merge of six solitons. Fig. 1(f) to Fig. 1(i) are just the mirror images of Fig. 1(d) to Fig. 1(a) respectively. In one dimension, when two or more solitary waves approach near to each other, they interact, exchange their energies and positions with one another, and then separate off, regaining their original wave forms. Throughout the process of the collision, the solitary waves are remarkably stable entities, preserving their identities through interaction. The unique effect due to collision is their phase shift. So eventually each soliton gains two phase shifts, one due to head-on collision and the other one because of overtaking of one soliton by another. We have derived analytically the expression of phase shifts.

Figure 2 shows that the phase shift of one, two and three soliton varies the nonextensive parameter q in the range $0.6 < q \leq 1$ [25]. Figure 3 shows how the phase shift Q_0 of two soliton varies with q in the range $0.6 < q \leq 1$, for several values of α and we can see that the phase shift increases with increasing α. Similarly, Figure 4 shows how the phase shift Q_0 of two soliton varies with p for several values of q in the range $0.6 < q \leq 1$. If the value of q increases then Q_0 rises to a certain limit before it decreases.

It can be concluded that the non-extensive non-thermal parameter and the other parameters play an important role on the phase shift of the soliton.

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