Positron Impact Excitations Of Hydrogen Atom Under Lorentzian Astrophysical Plasmas

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Abstract. The excitations of hydrogen atom from its ground state due to positron impact under Lorentzian astrophysical plasma environments have been studied by applying a second order distorted wave theory which takes into account the effect of dipole polarization. Excitation cross sections are calculated as functions of the spectral index and plasma parameters for incident positron energies ranging from 20eV to 300eV. It is shown that the nonthermal effect of plasma significantly modifies the excitation cross sections.

Keywords: Lorentzian plasma, positron collision, distorted wave approximation.

I. INTRODUCTION

For the last couple of years considerable interest has been paid to investigate positron collision phenomena in plasma environments [1]-[10] (and further references therein). The interest is motivated mostly due to the existence of positrons in several astrophysical environments [11]-[14]. Furthermore, knowledge of positron collision phenomena imparts important information to various fields of astrophysics [11]-[14] and plasma physics [15]-[18]. In most cases, investigations have been made under weakly coupled plasmas (WCP). The characteristic properties of a plasma can be studied by deriving the velocity dependence of the distribution function of plasma particles. In WCP, the coupling parameter, defined by the ratio of the Coulomb energy to the thermal one, is much less than unity. Hence, the screened interaction between the plasma particles in WCP is described by the standard Debye-Hückel potential, deduced from the thermal Maxwellian distribution [19]. However, non-Maxwellian distributions are also common both in space plasmas and in laboratory plasmas. The interaction of the external field with the plasma in equilibrium is found to stimulate superthermal particles to depart from the conventional Maxwellian distribution function [20, 21]. This is observed in a number of astrophysical environments, such as in the solar atmosphere, in the solar wind plasma, in the magnetotail, near plasma shock waves. In those environments, plasmas are generally observed having a non-Maxwellian high-energy tail owing to the interaction of the external field with the plasma in equilibrium [22]. This type of plasma is called non-Maxwellian plasma or Lorentzian plasma. It is found that the superthermal particles departed from the Maxwellian velocity distribution can be approximated accurately by the power law type of the generalized Lorentzian distribution or the κ distribution function $f_\kappa(v)$ [23]:

$$f_\kappa(v) = \left(\frac{m}{2\pi\kappa E_\kappa}\right)^{3/2} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \left(1+\frac{mv^2}{2\kappa E_\kappa}\right)^{-\kappa-1} \tag{1}$$

where m is the mass of the particle, v being the velocity of the particle, Γ denotes the gamma function, $\kappa(>3/2)$ is called the spectral index of the plasma, $E_\kappa \equiv (1-3/2\kappa)k_BT$ is called the characteristic energy of the plasma, k_B is the Boltzmann constant and T is the plasma temperature. Lorentzian distribution can have substantial high-energy tails arising out of some external acceleration mechanism [24]. The κ distribution function becomes a Maxwellian distribution function in the absence of the superthermal radiation field, that is in the limit of $\kappa \to \infty$, for all energies. Properties of Lorentzian distribution have been studied well in many investigations [25]-[28]. The effective Debye length in non-Maxwellian plasmas differs from the standard Debye length in Maxwellian plasmas owing to the nonthermal nature of the non-Maxwellian plasma. In Lorentzian plasmas, the effective screening depends on spectral index κ and Debye length λ_D, and can be obtained in the form [22]:

$$V(r;\kappa,\lambda_D) = \frac{1}{r}e^{-\sqrt{\frac{(\kappa-1/2)}{(\kappa-3/2)}}\, r/\lambda_D} = \frac{1}{r}e^{-\sqrt{\frac{(\kappa-1/2)}{(\kappa-3/2)}}\, \mu_D\, r} = \frac{1}{r}e^{-\mu_\kappa\, r} \tag{2}$$

where $\mu_D = 1/\lambda_D$ is the standard Debye screening parameter, and

$$\mu_\kappa = \sqrt{\frac{(\kappa-1/2)}{(\kappa-3/2)}}\, \mu_D$$

is the effective screening parameter. Thus, the scattering processes in nonthermal Lorentzian plasmas would certainly differ from those in weakly coupled Maxwellian plasmas. In recent years, a number of investigation has been made to study atomic processes in generalised Lorentzian plasma environments [25]-[34].

Here, the nonthermal effect of generalized Lorentzian plasmas on the following excitation process is studied:

$$e^+ + H(100) \to e^+ + H(nlm) \tag{3}$$

for various possible n,l,m. The main purpose for initiating such an investigation is to present a systematic and coherent discussion on the nonthermal effect of plasma on the excitation cross sections explicitly. We will also endeavour to examine the nonthermal effect on the total cross section for excitation into all possible nlm states in positron-hydrogen collisions. Such an investigation has several applications in astrophysics [11]-[14] and plasma physics [15]-[18]. We have been aware of the existence of atoms in highly excited states or Rydberg atoms for a long time. Presence of such type of atoms has been observed in interstellar space and stellar atmospheres [17, 18]. Collision cross section datum of highly excited states are frequently required for plasma diagnostics and studies in astrophysics. In vacuum, excitations of H atom by positron impact have been studied by a number of authors [35]-[47]. The excitation process (3) has also been investigated under WCP environments [48]-[50]. As such, the presence of Y_{lm} in the bound state wavefunction in the final channel poses complexity in executing a fully quantum mechanical calculation to study the above excitation process. In the present work, we apply the second order distorted wave theory (DWT), proposed by Ghoshal and Mandal [35], to the study the process (3). The fundamental idea of this method is to eliminate the positron-proton interaction from the perturbation whose matrix elements yield the cross section. This is because of the fact that the positron-proton interaction, as such, plays insignificant role in inducing changes in the internal structure of the target atom during collision, particularly for intermediate and high incident energies. This relatively unimportant $e^+ - p$ interaction can be excluded from the effective perturbation by approximating the distortion potential in a given channel as the average (over internal motions) of perturbation of that channel over the bound states. Atomic units (a.u.) will be used in the remaining part of this paper.

II. METHOD AND CALCULATIONS

We write the non-relativistic Hamiltonian (H) of the e^+−hydrogen system embedded in generalised Lorentzian plasma, described by the parameters κ and λ_D in the form:

$$H = -\sum_{i=1}^{2}\left[\frac{\nabla_i^2}{2} + (-1)^i V(r_i;\kappa,\lambda_D)\right] - V(r_{12};\kappa,\lambda_D) \quad (4)$$

$$\text{with} \quad V(r;\kappa,\lambda_D) = \frac{1}{r}e^{-\sqrt{\frac{(\kappa-1/2)}{(\kappa-3/2)}}\, r/\lambda_D} = \frac{1}{r}e^{-\mu_\kappa\, r}$$

where $\vec{r}_1$ and $\vec{r}_2$ are respectively the coordinates of positron and electron with respect to the proton, and $\vec{r}_{12}$ = $\vec{r}_1$ - $\vec{r}_2$. It is convenient to express H as:

$$H = H_i + V_i = H_f + V_f,$$

where H_i and H_f are the unperturbed Hamiltonians in the incident channel and the final channel respectively, and $V_i = V_f = [(e^{-\mu_\kappa r_1}/r_1) - (e^{-\mu_\kappa r_{12}}/r_{12})]$. The eigenvalue equation of H_c $(c = i, f)$ is given by:

$$H_c\Phi_c = E_c\Phi_c = \left[\frac{k_c^2}{2\mu_c} + \varepsilon_c\right]\Phi_c,$$

where Φ_c, E_c, k_c, μ_c and ε_c are respectively the unperturbed state, total energy, momentum of the positron, reduced mass and bound state energy of the concerned channel. In this work, we utilise the following partial wave scattering amplitude as in Ghoshal and Mandal [35]:

$$A_{fi}^{(L)}(k_f,k_i) = g_B^{(L)}(k_f,k_i) + D_{fi}^{(L)}(k_f,k_i) \quad (5)$$

The quantities $g_B^{(L)}(k_f,k_i)$ and $D_{fi}^{(L)}(k_f,k_i)$ in the above equation can be obtained as [35]:

$$g_B^{(L)}(k_f,k_i) = \sqrt{k_ik_f}/2 \int_{-1}^{+1} [g_B(\vec{k}_f,\vec{k}_i)]\, P_L(\hat{k}_i\cdot\hat{k}_f)\, d(\hat{k}_i\cdot\hat{k}_f),$$

$$D_{fi}^{(L)}(k_f,k_i) = \sqrt{k_ik_f}/2 \int_{-1}^{+1} [D(\vec{k}_f,\vec{k}_i)]\, P_L(\hat{k}_i\cdot\hat{k}_f)\, d(\hat{k}_i\cdot\hat{k}_f),$$

where P_L denotes the Legendre polynomial of degree L, and

$$\begin{aligned} g_B(\vec{k}_f,\vec{k}_i) &= \left(-\frac{\mu_f}{2\pi}\right) <\Phi_f|V_f|\Phi_i> \\ D_{fi}(\vec{k}_f,\vec{k}_i) &= \frac{1}{(2\pi)^3}\sum_\gamma\left(-\frac{2\pi}{\mu_\gamma}\right)\int\frac{d\vec{k}'}{E-E'_\gamma+i\varepsilon}f_{f\gamma}(\vec{k}_f,\vec{k}')\bar{f}_{\gamma i}(\vec{k}',\vec{k}_i) \\ \text{with}\quad f_{f\gamma}(\vec{k}_f,\vec{k}_i) &= \left(-\frac{\mu_\gamma}{2\pi}\right)<\Phi_f|V_f|\Phi'_\gamma>,\quad \bar{f}_{\gamma i}(\vec{k}_f,\vec{k}_i)=\left(-\frac{\mu_i}{2\pi}\right)<\Phi'_\gamma|U_i|\Phi_i> \end{aligned} \tag{6}$$

In equation (6) summation (over γ) runs over the intermediate states of H atom, and U_c $(c=i,f)$ is the distortion potentials in the concerned channel.

In the present case, we have evaluated the double scattering matrix elements in (6) by considering $\gamma=1s$ only. Omission of the contributions of other values of γ does not make much difference, because, it has been observed from a large number of investigations that such omission has little effects at intermediate and high energy regions [51]. However, effects of other γ values can be offset by including the effect of the dipole polarization potential explicitly. In this paper, we have included that effect by using the following polarization potential [52]:

$$V_{\text{pol}}(r)=-\frac{\alpha_H r^2}{2(r^2+d^2)^3} \tag{7}$$

where d is a variable parameter and α_H stands for the static dipole polarizability of the H atom.

The parameter d is approximately of the order of the size of the atom. With the inclusion of the $V_{\text{pol}}(r)$, only the two-body amplitude $f_B(\vec{k}_f,\vec{k}_i)$ in the elastic channel gets modified to $f_B(\vec{k}_f,\vec{k}_i)+f_{\text{pol}}(\vec{k}_f,\vec{k}_i)$, where $f_{\text{pol}}(\vec{k}_f,\vec{k}_i)$ is the amplitude corresponding to $V_{\text{pol}}(r)$.

As stated earlier, we consider the distortion potentials U_c $(c=i,f)$ as:

$$U_c=<\phi_c|V_c|\phi_c>,$$

where ϕ_c is the bound state wave function in the concerned channel. The wave function ϕ_c of the H atom in Lorentzian plasma environment is constructed variationally by using the following simple trial function:

$$\phi_c(\vec{r}_2)=\phi_{nlm}(r_2,\theta_2,\phi_2)=\left[\left(\frac{2}{n\lambda}\right)^3\frac{(n-l-1)!}{2n[(n+l)!]^3}\right]^{1/2}e^{-\frac{r_2}{n\lambda}}\left(\frac{2}{n\lambda}r_2\right)^l L^{2l+1}_{n+l}\left(\frac{2}{n\lambda}r_2\right)Y_{lm}(\theta_2,\phi_2) \tag{8}$$

where λ is a variational parameter, Y_{lm} is the spherical harmonics and L^{2l+1}_{n+l} is the associated Laguerre polynomial of order $(2l+1)$ and degree $(n+l)$. This type of hydrogenic wave function under a screening environment was considered in a number of investigations [[53]-[56]].

Employing the wave function (8) of H atom, all two-body amplitudes appearing in the distorted wave amplitude (DWA) (5) can be obtained in closed forms.

III. RESULTS AND DISCUSSION

The partial-wave DWA (5) has been used to calculate the excitation cross sections. All significant contributions from partial waves to the DWA have been included and the higher partial wave contributions to the DWA have been approximated by the corresponding 'First Born Approximation (FBA)' values. It is noticed that the convergence of the series of partial-wave cross section for the transition to the p-states is rather slow than the transitions to the other states. For instance, partial-wave cross section series corresponding to the 2s-state converges (up to six places of decimal) after sixteen terms, whereas convergence for 2p-state requires as many as fifty terms. Table 1 shows the first five partial-wave contributions to the $|100> \rightarrow |200>$ and $|100> \rightarrow |210>$ excitation cross sections in vacuum for three different incident positron energies. For comparison, the corresponding results obtained in some of the previous works ([36]-[42],[44, 45]) are also shown in this table. From Table 1 we notice that our present results are in close agreement with those results.

TABLE 1. Partial wave contributions to the $1s \rightarrow 2s$ and $1s \rightarrow 2p$ excitation cross sections (in units of πa_0^2) of hydrogen atom by positron impact in vacuum. σ denotes the corresponding total excitation cross section obtained by summing up all significant partial wave contributions. The notation x[-y] stands for $x \times 10^{-y}$.

	$1s \rightarrow 2s$			$1s \rightarrow 2p$		
l	E=54.4 eV	E=100 eV	E=200 eV	E=54.4 eV	E=100 eV	E=200 eV
0	0.1614[-1]	0.5236[-2]	0.1463[-2]	0.5811[-1]	0.2228[-1]	0.6826[-2]
1	0.3458[-1]	0.1264[-1]	0.3580[-2]	0.1511	0.5699[-1]	0.1710[-1]
2	0.2629[-1]	0.1370[-1]	0.4813[-2]	0.1795	0.8006[-1]	0.2610[-1]
3	0.1406[-1]	0.1065[-1]	0.4894[-2]	0.1603	0.8702[-1]	0.3211[-1]
4	0.6262[-2]	0.6977[-2]	0.4257[-2]	0.1245	0.8270[-1]	0.3502[-1]
σ	0.1012	0.5768[-1]	0.2956[-1]	0.9807	0.7262	0.4631
(A)	0.1270	0.6100[-1]	0.3000[-1]	0.9480	0.7140	0.4710
(B)	0.1240	0.8000[-1]	0.4000[-1]	0.8750	0.7290	0.4710
(C)	0.1260			0.9530		
(D)	0.1095	0.5353[-1]		0.9210	0.7172	
(E)	0.1078	0.6131[-1]		0.8920	0.7026	
(F)	0.0810	0.6000[-1]	0.2800[-1]			
(G)		0.6100[-1]	0.3000[-1]			
(H)	0.0810	0.5500[-1]	0.2800[-1]	0.8450	0.7690	0.4700
(I)	0.1135					

(A): close-coupled pseudostate approximation by Walters [36], (B): coupled-channel optical model of Bransden *et al* [37], (C): 12-state close coupling approximation of Morgan [38], (D): convergent close-coupling approach of Kadyrov and Bray [39] (data taken from graph within 5% of accuracy), (E): 33-state approximation of Kernoghan *et al* [40] (data taken from graph within 5% of accuracy), (F): distorted wave polarized orbital model of Lugosi *et al* [41], (G): unitarised Eikonal-Born series version 1 of Byron *et al* [44], (H): eigen model of Mukherjee *et al* [45], (I): distorted wave approximation of Nayek and Ghoshal [42].

In Fig.1 cross sections for excitation of H atom into the different angular momentum states under Lorentzian plasma are presented. From this figure, we observe that cross sections attain their maxima in the low energy region. Beyond the maxima cross section falls down rapidly with increasing incident energy. Variation of cross sections with incident energy for excitation into the different *nlm* states shows almost same nature for different spectral index κ. One

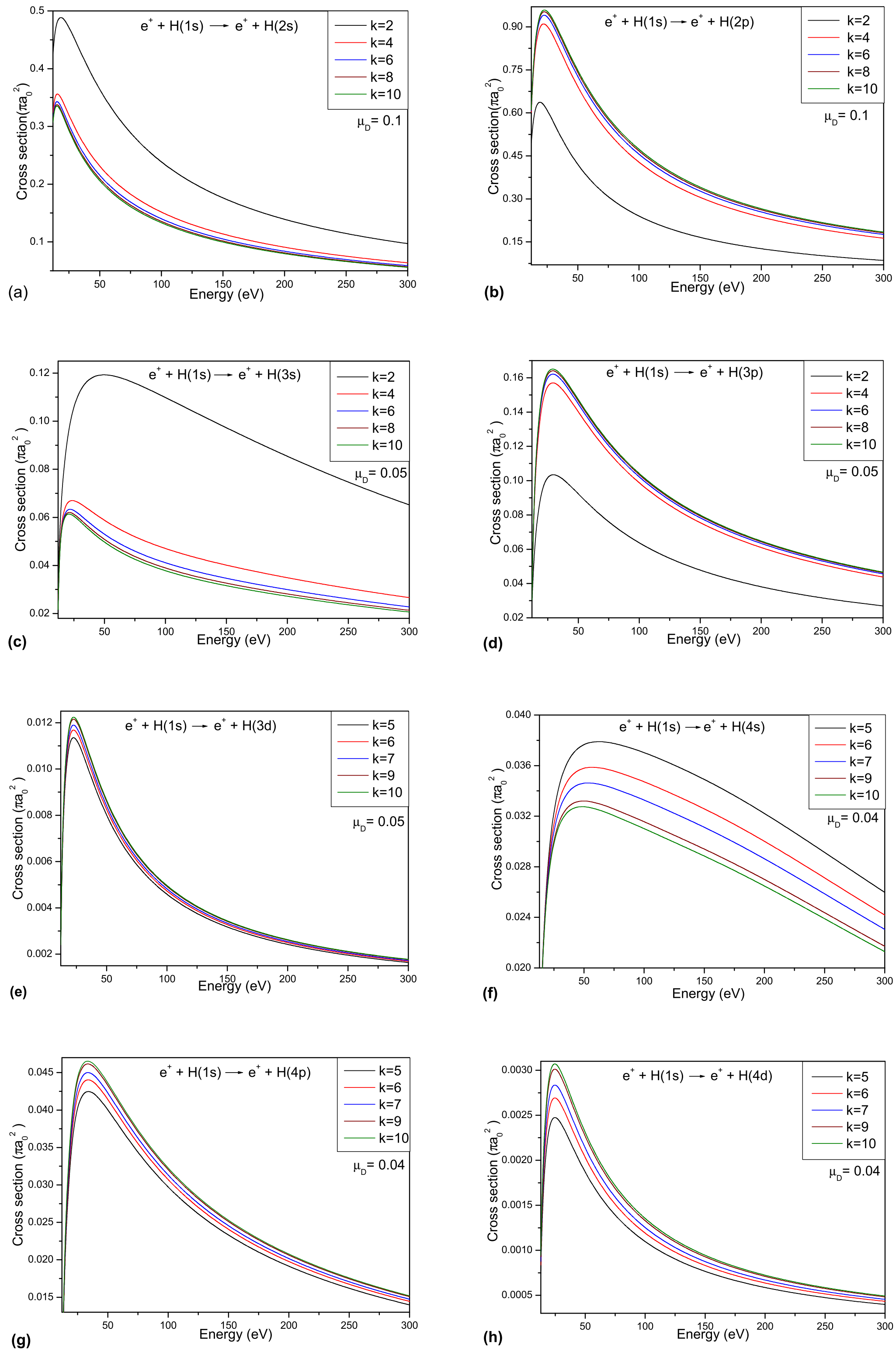

FIGURE 1. Excitation cross section (in πa_0^2) in $e^+ + H(1s)$ collisions under Lorentzian plasma as a function of incident positron energy (in eV) for different values of spectral index for excitation of hydrogen atom into the (a) 2s-state, (b) 2p-state, (c) 3s-state, (d) 3p-state, (e) 3d-state (f) 4s-state, (g) 4p-state and (h) 4d-state respectively.

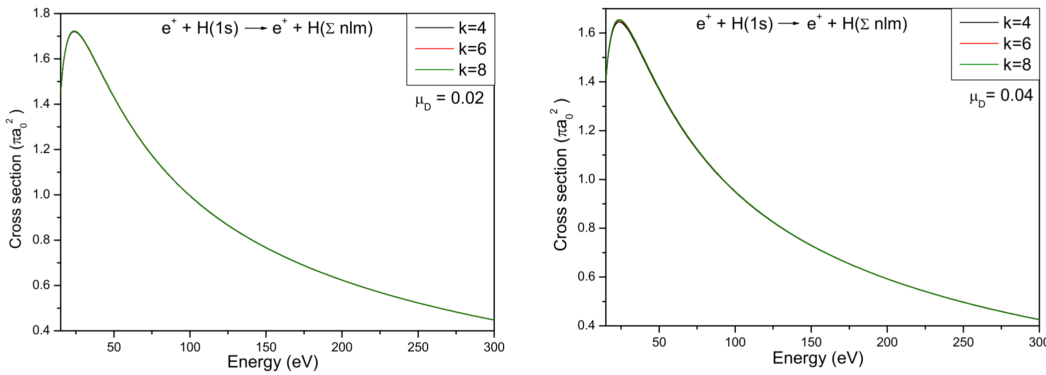

FIGURE 2. Total cross section (in πa_0^2) for excitation in all nlm states in $e^+ + H(1s)$ collisions under Lorentzian plasma as a function of incident positron energy (in eV) for different values of spectral index at $\mu_D = 0.02$ and $\mu_D = 0.04$ respectively.

of the interesting facts to be noted is the effect of the spectral index on the cross sections of various transitions. We note that increasing strength of spectral index reduces the cross section for the transitions into s-states, whereas it enhances cross sections for the excitations into p-states and d-states. The enhancement and the reduction in cross section is apparent for larger values of the Debye screening strength μ_D. Moreover, cross sections for the transition into the p-states dominate over the cross sections for the transition into all other angular momentum states. This enhancement in cross section is due to the "optically allowed" (dipole) transitions (change of orbital angular momentum quantum number of the target H-atom satisfying the selection rule ($|\triangle l| = 1$). The "optically allowed" transitions dominate for discrete excitation and ionization [47] at high energies. It should be emphasised here that cross sections for highly excited transitions in vacuum can be calculated conveniently by using scaling law [47].

It is also important to examine the nonthermal effect on the total cross section for excitation, $\sigma(\sum nlm)$, into all possible nlm states in $e^+ + H(1s)$ collisions. The variation of $\sigma(\sum nlm)$ with incident energy and spectral index is shown in Fig.2 for two different Debye lengths. We have included all significant excitation cross sections to obtain $\sigma(\sum nlm)$. We notice that change in $\sigma(\sum nlm)$ with varying spectral index is not substantial, particularly at high incident energies. At low and intermediate energies $\sigma(\sum nlm)$ suffers a little change due to the nonthermal effect of the plasma.

IV. CONCLUSIONS

In this paper, we make an attempt to study the process: $e^+ + H(100) \rightarrow e^+ + H(nlm)$ under Lorentzian plasma environments. Use of a second order DWT that includes the effect of dipole polarization gives us an opportunity to calculate the excitation cross section accurately in the intermediate and high incident energies. It is found that background plasma environment significantly modifies the excitation cross sections. At a given screening strength, increase in the spectral index can enhance or reduce the excitation cross section depending on the order or type of the excitation. We expect that our present investigation will provide fruitful information to the future researches in plasma physics, astrophysics and positron physics.

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