Effect of Various Parameters on Reflection, Transmission, and Absorption of Microwaves in an Inhomogeneous Plasma Slab

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(Received: 2.1.2018 ; Published: 24.12.2018)

Abstract: This work deals with calculation of the absorbed, transmitted and reflected coefficients in an inhomogeneous magnetized cold plasma slab assumed to have a parabolic density function, and examines the impact of various parameters (plasma density, collision frequency, plasma slab thickness and strength of background magnetic field) on these coefficients. Increased absorption and reflection of microwaves by plasmas of higher density and lower collision frequency were observed. Increasing the magnetic field shifted the resonant frequencies to higher value, and maximum absorption and reflection was observed at optimum plasma slab thickness of 0.20 m.

Keywords: Absorption coefficient, Plasma density, Collision frequency, Magnetized plasma

I. INTRODUCTION

This study examines laboratory and space plasmas and factors affecting it. To check the performance of a phenomenon, a series of parameters are introduced in order to test the performance. For example, to check the heating process of a plasma, coefficients of absorption, reflection and transmission are used. Absorption, reflection and transmission of waves hitting the plasma are of special interest in applications such as ionosphere propagation, plasmas as electromagnetic reflectors and absorbers, and microwaves propagation in magnetized plasma [1-6]. Laroussi and Roth investigated the reflection, absorption, and transmission of microwaves by a magnetized nonuniform plasma slab for different collision frequencies, electron densities and angles of wave propagation [6]. They compared between experimental and simulated results assuming a uniform magnetic field of 0.143 T throughout the plasma of density 10^{15}-10^{16} m^{-3}. Tang et al. [7] also studied the absorption, reflection, and transmission of electromagnetic waves by a nonuniform plasma slab of peak densities 1016-1018 m−3 immersed in an ambient uniform magnetic field of higher value of 0.1-0.5 T. In this paper, instead of taking a single uniform background magnetic field throughout the plasma which corresponds to a certain resonance, we dealt with four different uniform background magnetic fields within the range of 0.125-0.157 T which correspond to different resonances. Changing the background magnetic field shifted the frequency for the maximum absorption of energy by the plasma, and hence, caused variation of the absorption, transmission and reflection coefficients. In this work, the coefficients calculation was done for four different thicknesses of plasma slabs (0.06-0.30 m) instead of only one thickness of 0.12 m as in Refs. [6,7]. With slab thickness (or length) variation, the maximum absorption coefficient of electromagnetic waves varied. Interaction of electromagnetic waves and plasma is of particular interests in military applications related to the radar projects. However, the main purpose of this study is to obtain highest efficiency in plasma heating (maximum absorption and minimum in reflection and transmission). The larger the transmission coefficient, the more produced energy passes through the plasma without heating the plasma. Higher reflection coefficient means more wave energy is reflected and it does not lead to plasma heating. Therefore, when absorption coefficient is higher, more produced energy is spent on heating the plasma and energy loss is avoided.

The plasma slab discussed here is a cold inhomogeneous magnetized plasma slab. This paper assumes a fixed background magnetic field that is applied along the z-axis. The second requirement is that ions are motionless and only electrons are under the influence of electromagnetic wave. It is assumed that the magnetic field of this wave does not affect the electrons. Electric force is the only force considered that accelerates the moving electrons causing the electrons to collide with each other and other ions; therefore, transfer their energy to the plasma and thereby the plasma heats up. The electron density distribution is assumed to be a parabolic function that has maximum in center and minimum on the edges of the slab. This article is divided into four parts. In the first part, plasma heating is introduced. The second part is focused on the determination of absorption, reflection and transmission coefficients. In the third part, results of calculations on the coefficients of absorption, reflection and transmission as well as the effects of various factors on these coefficients are presented. The conclusion is given in the fourth part.

II. CALCULATIONS

We shall consider only the electric field component of the wave that interacts with the plasma. A plane wave with an oscillating electric field having $e^{i\omega t}$ time dependence diffusing into plasma can be written as follows

$$\vec{E}(\vec{r},t) = \hat{E}\; e^{-i(\omega t - \vec{k}.\vec{r})} \tag{1}$$

Using Maxwell's equations

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\overrightarrow{B}}{\partial t} = -i\,\omega\overrightarrow{B} = -i\,\omega\mu\overrightarrow{H} = -i\,\omega\mu_0\mu_r\overrightarrow{H} \tag{2}$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \underset{\vec{J}=\sigma\vec{E}}{\overset{\vec{D}=\varepsilon\vec{E}}{\Longrightarrow}} \vec{\nabla}\times\vec{H} = (\sigma\vec{E} + i\,\omega\varepsilon\vec{E}) \overset{\varepsilon_r=\frac{\varepsilon}{\varepsilon_0}}{\Longrightarrow} \vec{\nabla}\times\vec{H} = (\sigma\vec{E} + i\,\omega\varepsilon_0\varepsilon_r\vec{E}) \tag{3}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}\times(-i\,\omega\mu_0\mu_r\vec{H}) = -i\,\omega\mu_0\,\mu_r\vec{\nabla}\times\vec{H} = -i\,\omega\mu_0\mu_r(\sigma + i\,\omega\varepsilon_0\varepsilon_r)\vec{E} \tag{4}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}.\vec{E}) - \nabla^2\vec{E} = -i\,\omega\mu_0\mu_r(\sigma + i\,\omega\varepsilon_0\varepsilon_r)\vec{E} \tag{5}$$

Since plasma is considered almost neutral, so

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon} = 0 \tag{6}$$

$$-\nabla^2 \vec{E} = -i\,\omega \mu_0\, \mu_r (\sigma + i\,\omega \varepsilon_0 \varepsilon_r)\vec{E} \tag{7}$$

Now the imaginary dielectric coefficient will be defined as follows

$$\tilde{\varepsilon}_r = \varepsilon_r - i\,\frac{\sigma}{\varepsilon_0 \omega} \tag{8}$$

Then, equation (7) will be reduced to

$$\nabla^2 E = \frac{\mu_r\, \tilde{\varepsilon}_r}{c^2} \frac{\partial^2 E}{\partial t^2} \tag{9}$$

After solving equation (9), the diffusion constant will be written as follows

$$\tilde{\gamma}^2 = -\mu_r\, \tilde{\varepsilon}_r\, \frac{\omega^2}{c^2} \tag{10}$$

The imaginary dielectric constant, $\tilde{\varepsilon}_r$ for a plane wave with an arbitrary angle hitting the plasma slab can be obtained as follows [8]

$$\tilde{\varepsilon}_r = 1 - \frac{\dfrac{\omega_p^2}{\omega^2}}{\left[1 - i\,\dfrac{\nu}{\omega} - \dfrac{\dfrac{\Omega^2}{\omega^2}\sin^2\theta}{2\left(1 - \dfrac{\omega_p^2}{\omega^2} - i\,\dfrac{\nu}{\omega}\right)}\right] \pm \left[\dfrac{\dfrac{\Omega^4}{\omega^4}\sin^4\theta}{4\left(1 - \dfrac{\omega_p^2}{\omega^2} - i\,\dfrac{\nu}{\omega}\right)} + \dfrac{\Omega^2}{\omega^2}\cos^2\theta\right]^{1/2}} \tag{11}$$

In this relation ω_p, Ω, ν and θ represent the plasma frequency, electron cyclotron frequency, effective collision frequency and the angle between the propagation vector and the uniform background magnetic field inside the plasma respectively. In this relation, the minus sign is used for right hand polarized and the plus sign is used for left hand polarized. This defines that $\tilde{\varepsilon}_r$ has a resonance around the electron cyclotron frequency. The diffusion in the cold plasma has a maximum value of absorption within the range of the electron cyclotron frequency [9].

The slab plasma is assumed to be composed of n thin layers adjacent to each other and each layer is assumed to be homogeneous in density having a certain value that differs from the adjacent layer. The thickness of each plasma layer is L/n where L is the thickness of the slab. The wave passes from one layer to another and there is a reflection in each layer. Since each layer has a specific refractive index, then it has a specific dielectric coefficient. Therefore, the dielectric function relation (11) is calculated for each layer and a constant reflection is obtained from each layer. Powers of absorption, reflection and transmission are then calculated where multiple reflections are ignored. For vertically polarized waves, Fresnel reflection coefficient can be obtained as follows

$$\Gamma = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} \tag{12}$$

where n_1 and n_2 are the refractive indexes of medium 1 and medium 2 respectively. Also, θ_1 and θ_2 are the angle of incidence and the angle of refraction respectively. Based on Snell's law (also known as Snell–Descartes law and the law of refraction) we have

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{13}$$

$$n_1 v_1 = n_2 v_2 \Rightarrow \frac{n_2}{n_1} = \frac{v_2}{v_1} = \sqrt{\frac{\varepsilon_1 \mu_1}{\varepsilon_2 \mu_2}} \tag{14}$$

in which v_1 and v_2 are the traveling velocities of light in medium 1 and medium 2 ($v_1 = c/n_1$ & $v_2 = c/n_2$ and c is the speed of light in vacuum) respectively. Since non-magnetic environment is considered, then

$$\frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \tag{15}$$

$$\cos\theta_2 = \sqrt{1 - \sin^2\theta_2} \tag{16}$$

$$\sin\theta_2 = \frac{n_1}{n_2} \sin\theta_1 = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin\theta_1 \tag{17}$$

Dividing the numerator and denominator of equation (12) by n_2 and substituting the relations (15) and (17), it will be written as

$$\Gamma = \frac{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \cos\theta_1 - \sqrt{1 - \frac{\varepsilon_1}{\varepsilon_2} \sin^2\theta_1}}{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \cos\theta_1 + \sqrt{1 - \frac{\varepsilon_1}{\varepsilon_2} \sin^2\theta_1}} \tag{18}$$

Multiplying the numerator and denominator of equation (18) by $\sqrt{\varepsilon_2/\varepsilon_1}$ and considering that the dielectric constant is imaginary, in its generalized form, it reduces to

$$\Gamma = \frac{\frac{\tilde{\varepsilon}_{i+1}}{\tilde{\varepsilon}_i} \cos\theta_i - \sqrt{\frac{\tilde{\varepsilon}_{i+1}}{\tilde{\varepsilon}_i} - \sin^2\theta_i}}{\frac{\tilde{\varepsilon}_{i+1}}{\tilde{\varepsilon}_i} \cos\theta_i + \sqrt{\frac{\tilde{\varepsilon}_{i+1}}{\tilde{\varepsilon}_i} - \sin^2\theta_i}} \tag{19}$$

In this relation θ_1 is the angle collision of electromagnetic waves, $\tilde{\varepsilon}_i$ and $\tilde{\varepsilon}_{i+1}$ are imaginary dielectric coefficients of i and i+1 layer respectively. The reflection coefficient depends on the location and frequency. The total reflection of the plasma slab is taken as the sum of coefficients of the individual layers.

$$\Gamma_T = \sum_{i=1}^{n} \Gamma_i \times (1 - e^{-\frac{\alpha x}{\sin\theta}})^2 \qquad (20)$$

where α is absorption coefficient (assuming there is no scattering) in location x. Since the wave travels double the distance through the plasma slab as it is reflected back out of the slab, the second term in equation (20) is squared. Finally, the total reflected power is calculated as follows

$$P_r = P_i \left|\Gamma_T\right|^2 \qquad (21)$$

and the total factor of absorption is obtained by multiplying the factor of absorption A_i of each slab that is given by

$$A_i = e^{-\alpha L/\sin\theta_i} \qquad (22)$$

Then,

$$A_T = \prod_i A_i \qquad (23)$$

The power that is transmitted is equal to the power of the incident wave P_i multiplied with the total factor of attenuation. Therefore, by subtracting the power of reflection and transmission from incident power, the total absorption will be obtained as

$$P_a = P_i - P_r - P_t \qquad (24)$$

III. SIMULATED RESULTS AND ANALYSIS

Schematic of the plasma slab and the incident wave is shown in Figure 1. The angle θ_i is the incident angle and θ is the angle between the transferred wave through the plasma and the direction of z coordinate.

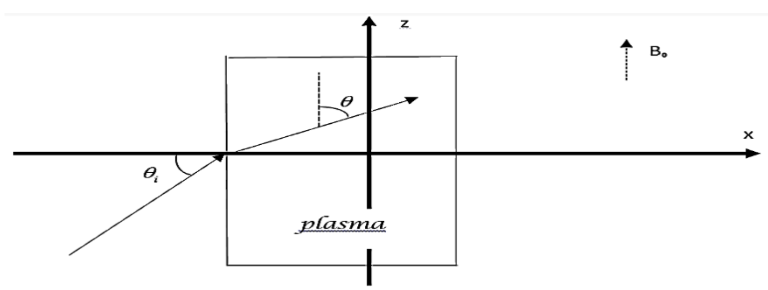

FIGURE 1. Interaction of electromagnetic waves in plasmas

Plasma density is considered as a parabolic function which its maximum density at the center and minimum density on the edges as shown in Figure 2.

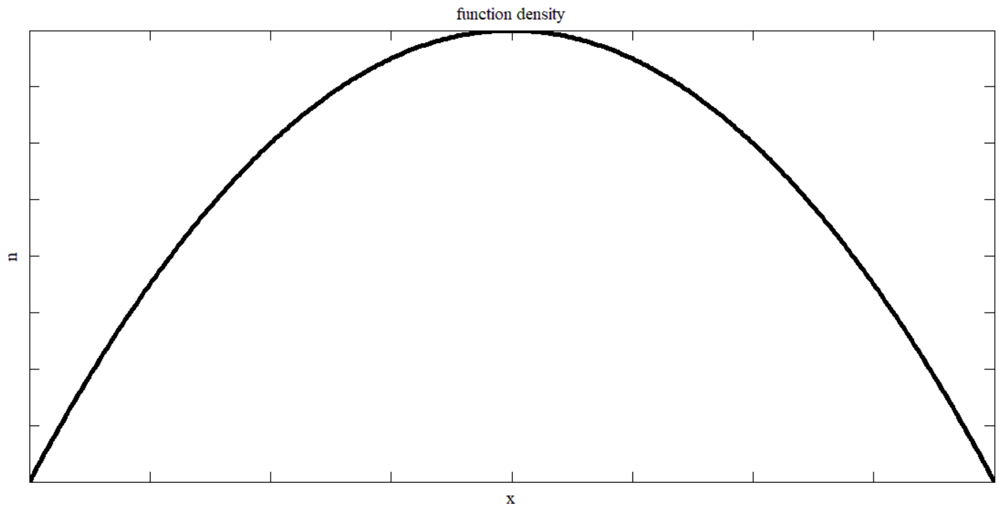

FIGURE 2. The plasma density profile

Plasma is divided into *n* layers and thickness of each layer is *L*/*n*.

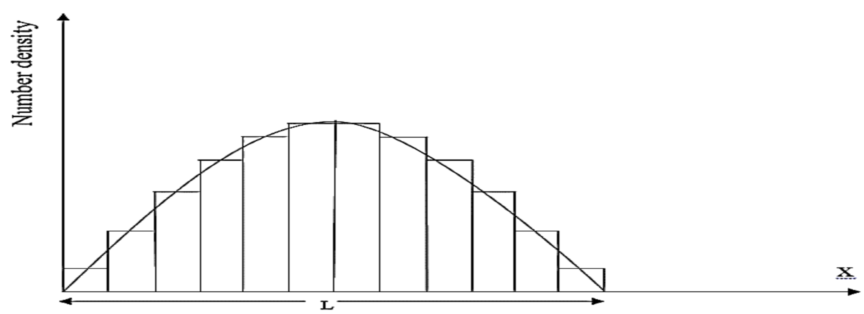

FIGURE 3. Example of division of plasma slab into *n*=12 plasma layers

The cyclotron frequency of an electron moving perpendicular to the direction of an external uniform magnetic field *B* can be calculated from (Ω=2πf). For example when *B* = 0.143 T we have

$$f = \frac{q_e B}{2\pi m_e} = 4\times10^9 Hz = 4GHz \quad (25)$$

Firstly to check the accuracy of the absorbed, reflected and transmitted power calculations, different number of layers was considered, i.e., *n* = 10, 50 and 100.

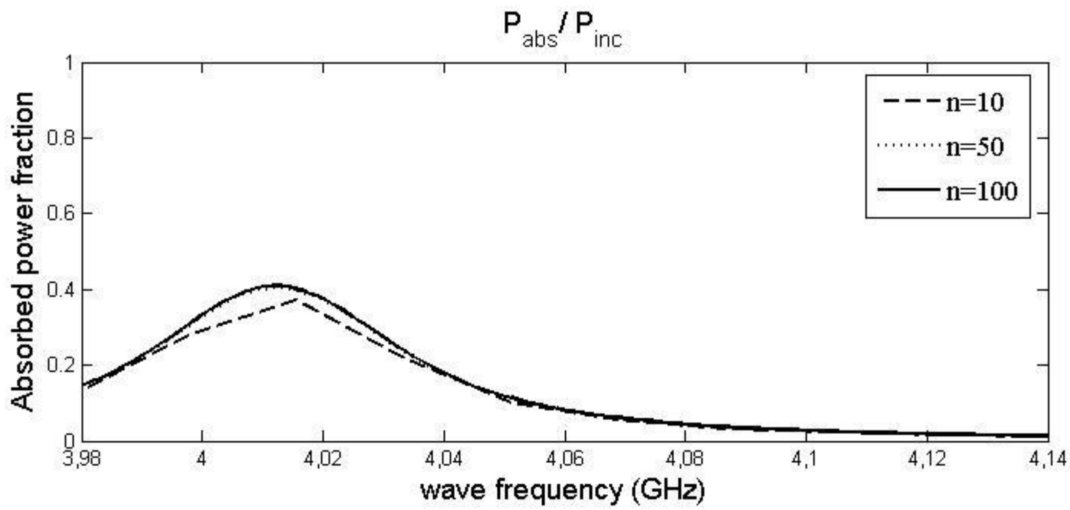

FIGURE 4. The power absorbed for different number of layers. $\nu = 5$ MHz, maximum plasma density $N_0 = 10^{14}$ m^{-3}, $\Omega = 4.0$ GHz, $\theta = 60°$, $L = 0.12$ m.

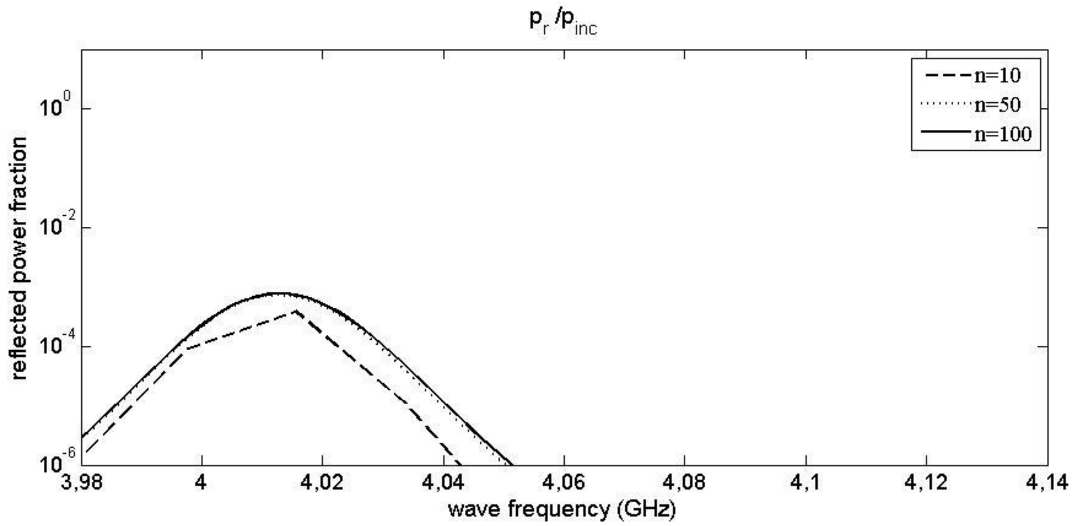

FIGURE 5. The reflected power for different number layers. $\nu = 5$ MHz, maximum plasma density $N_0 = 10^{14}$ m^{-3}, $\Omega = 4.0$ GHz, $\theta = 60°$, $L = 0.12$ m.

As shown in Figures 4, 5 and 6, calculations for 10 plasma layers do not have good accuracy (denoted by largest deviation of resonance from the cyclotron frequency). When the number of layers was increased to 50 and more, there is sufficient accuracy. Hence, $n = 100$ is chosen for all subsequent calculations.

The effect of the collision frequency on the absorption, reflection and transmission coefficient was studied, and the respective simulated power spectra are shown in Figures 7 to 9.

As can be seen with increasing collision frequency, the reflection coefficient reduces, transmission coefficient increases and absorption coefficient decreases. To reduce energy loss and have more plasma heating (more absorbed power), fewer collision frequencies should be used.

Another parameter that was considered is the maximum plasma density in the center of the plasma slab.

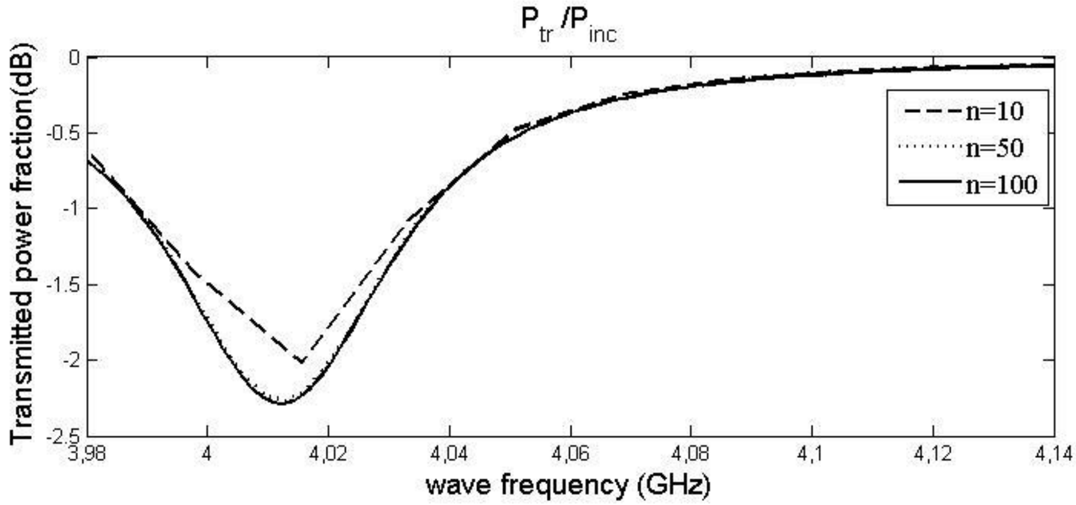

FIGURE 6. Power transmission for the number of different layers. ν = 5 MHz, maximum plasma density $N_0 = 10^{14}$ m^{-3}, Ω = 4.0 GHz, θ = 60°, L = 0.12 m.

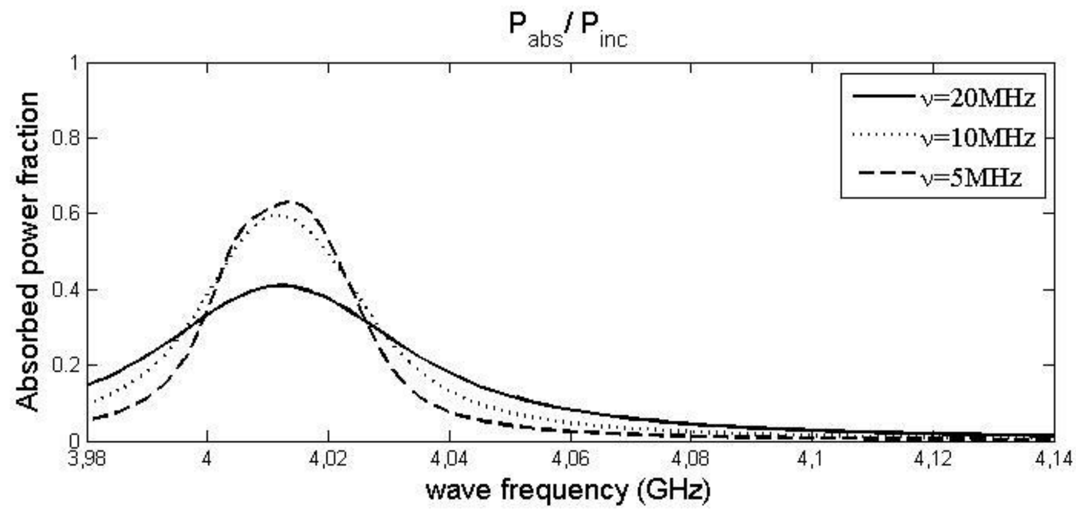

FIGURE 7. Power absorbed for different collision frequencies. $N_0 = 10^{14}$ m^{-3}, Ω = 4.0 GHz, θ = 60°, L = 0.12 m.

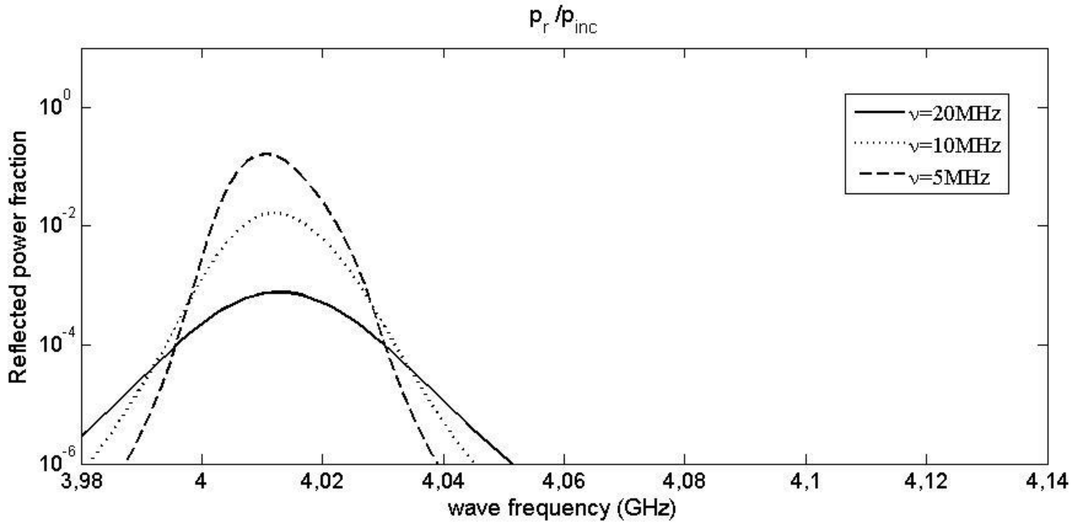

FIGURE 8. The reflected power for different collision frequencies. $N_0 = 10^{14}$ m^{-3}, $\Omega = 4.0$ GHz, $\theta = 60°$, $L = 0.12$ m.

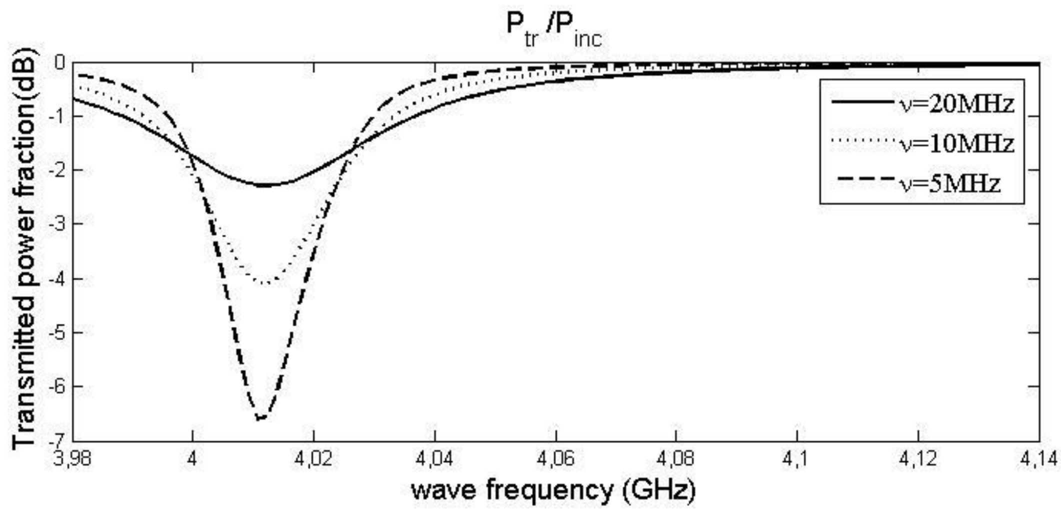

FIGURE 9. Transmission power for different collision frequencies. $N_0 = 10^{14}$ m^{-3}, $\Omega = 4.0$ GHz, $\theta = 60°$, $L = 0.12$ m.

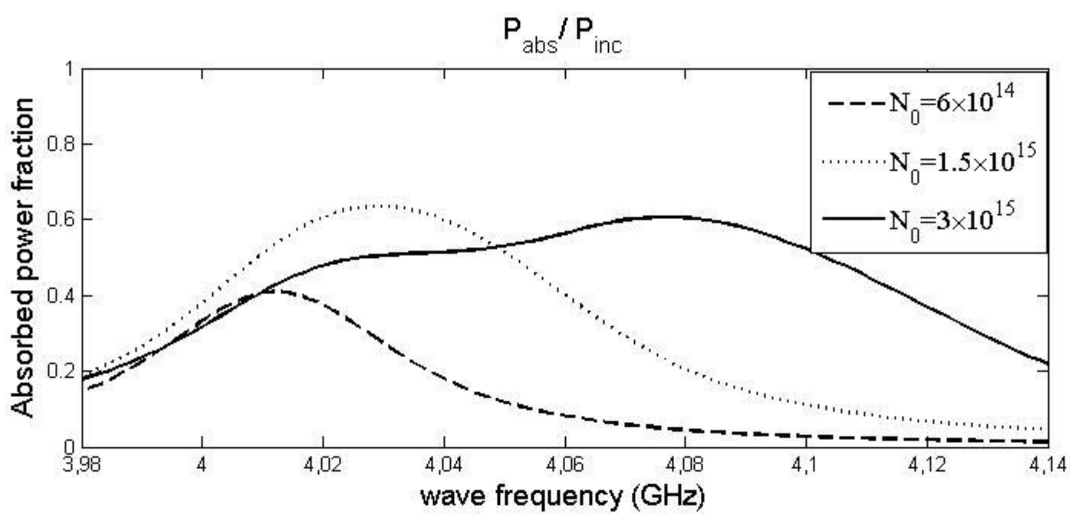

FIGURE 10. The absorbed power for different maximum plasma density (in m^{-3}). $\nu = 5$ MHz, $\Omega = 4.0$ GHz, $\theta = 60°$, $L = 0.12$ m.

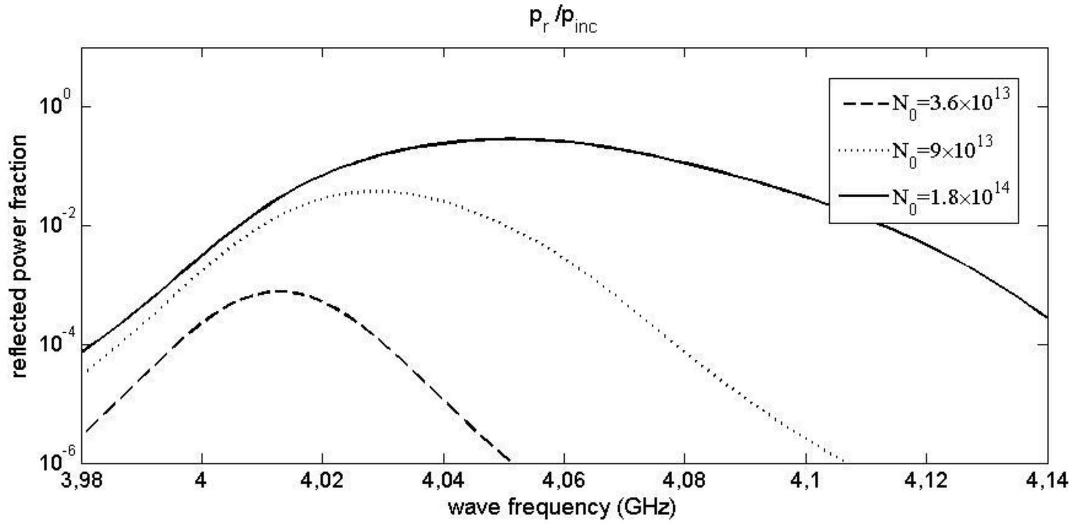

FIGURE 11. The reflected power for different maximum plasma density (in m^{-3}). ν = 5 MHz, Ω = 4.0 GHz, θ = 60°, L = 0.12 m.

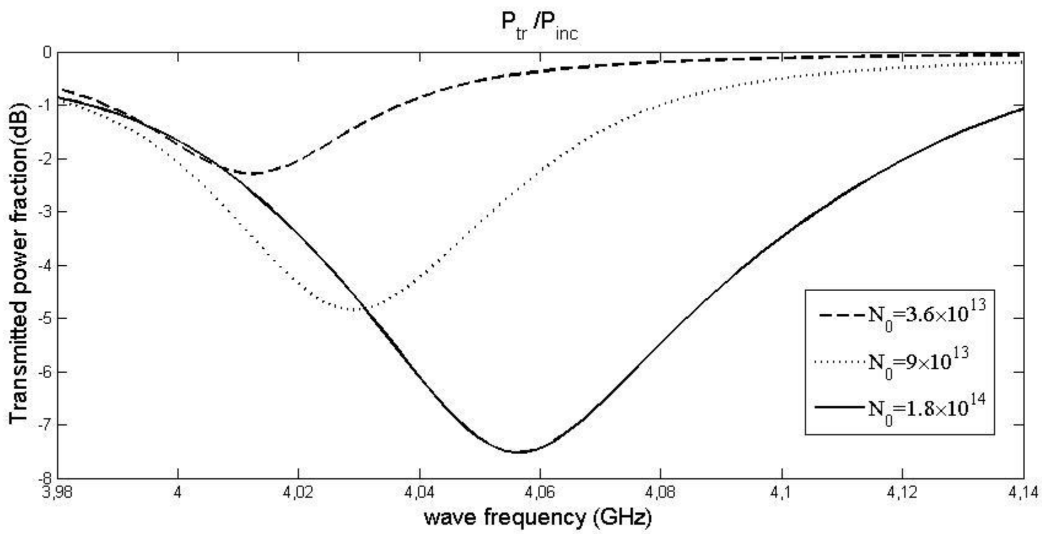

FIGURE 12. Power transmission for different maximum plasma density (in m^{-3}). ν = 5 MHz, Ω = 4.0 GHz, θ = 60°, L = 0.12 m.

It can be predicted that by increasing density of plasma, reflection coefficient increases, but transmission coefficient decreases. As the plasma density increases from 3.6×10^{13} to 1.8×10^{14} m^{-3}, the peak absorbed power increases up to a certain value before decreasing slightly. There is a critical plasma density at which maximum absorption occurs. Increasing the plasma density means more collisions will occur, and the absorbed power would decrease. The absorption spectrum also broadens with increasing plasma density.

Next, we investigate the dependence of absorption, reflection and transmission by the plasma when subjected to different background magnetic fields. Changing the background (uniform) magnetic field, the resonant cyclotron frequency will vary according to equation (25).

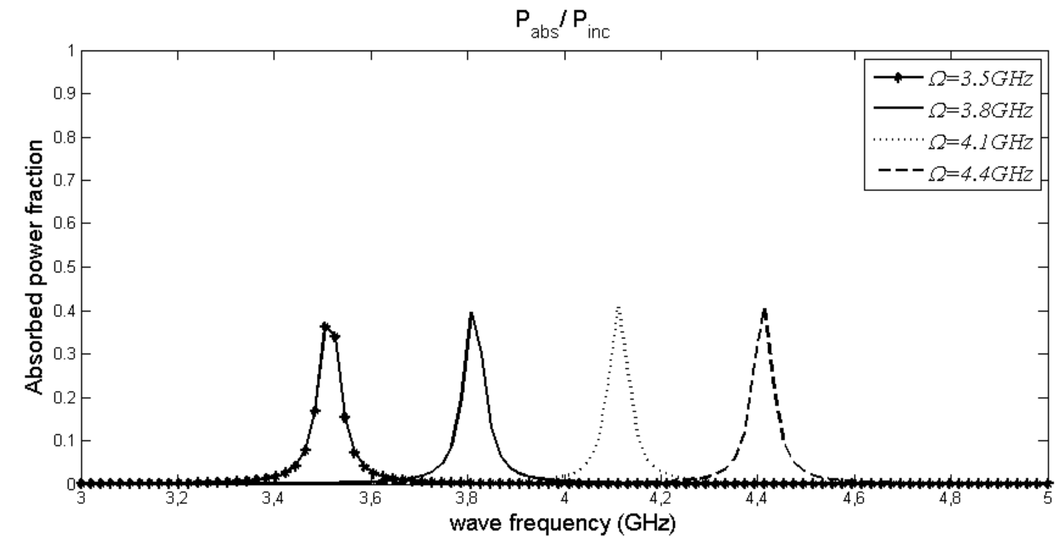

FIGURE 13. Absorbed power for different background magnetic field (in terms of its corresponding cyclotron frequency). $\nu = 5$ MHz, $N_0 = 10^{14}$ m^{-3}, $\theta = 60°$, $L = 0.12$ m.

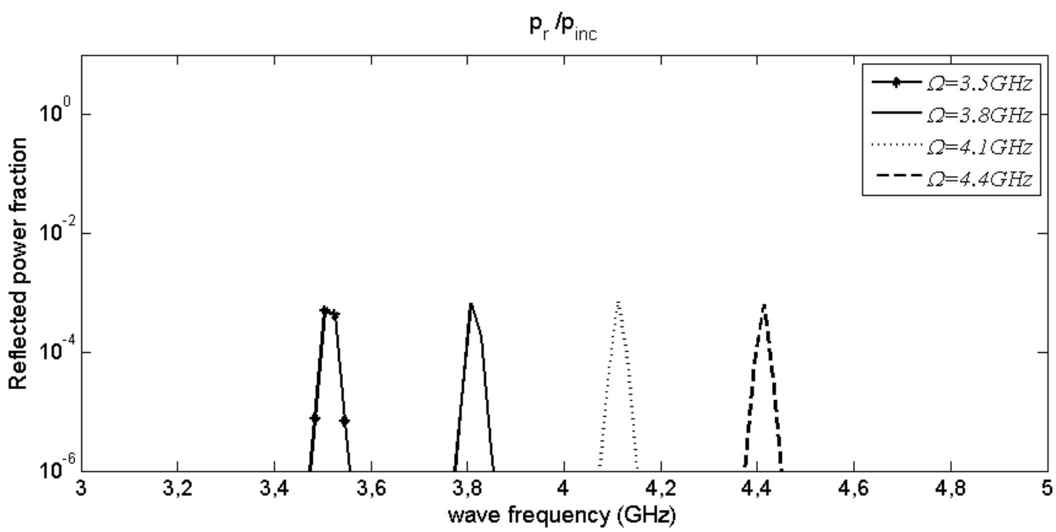

FIGURE 14. Reflected power for different background magnetic field (in terms of its corresponding cyclotron frequency). $\nu = 5$ MHz, $N_0 = 10^{14}$ m^{-3}, $\theta = 60°$, $L = 0.12$ m.

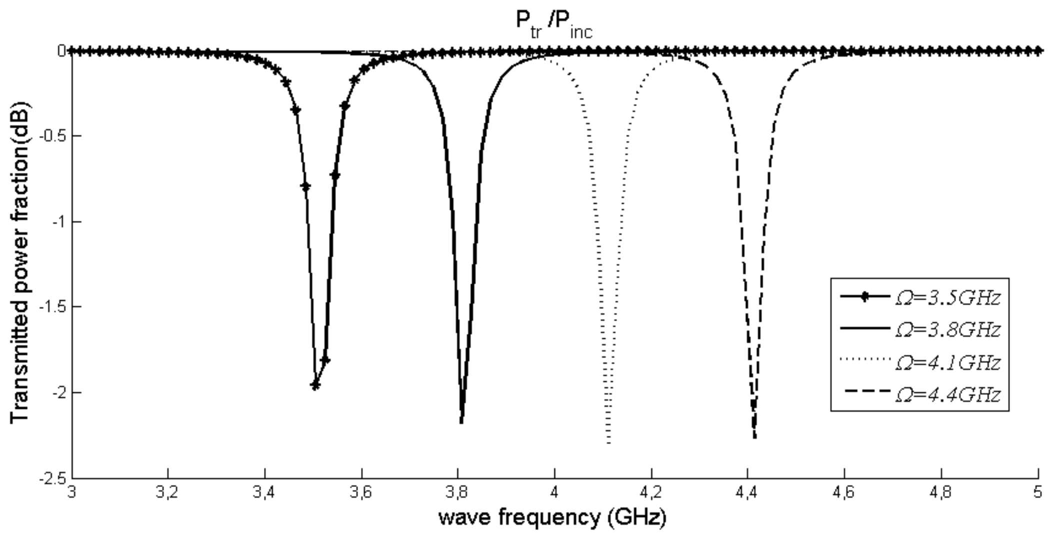

FIGURE 15. Transmission power for different background magnetic field (in terms of its corresponding cyclotron frequency). ν = 5 MHz, $N_0 = 10^{14}$ m^{-3}, θ = 60°, L = 0.12 m.

For resonance cyclotron frequencies of 3.5, 3.8, 4.1 and 4.4 GHz, the corresponding background magnetic fields are 0.125, 0.136, 0.147, and 0.157 T. In Figures 13, 14 and 15, it can be seen that the amount of absorbed, reflected and transmitted powers did not vary much with the background magnetic field. However, maximum absorbed and reflected power peaks and correspondingly minimum transmitted power peak could be observed at 0.147 T. More pronounced is the frequency shift of the respective peaks being associated to the cyclotron frequency.

Finally, the effect of thickness of the plasma slab was investigated. Four different thicknesses of plasma slabs, L = 6, 12, 20 and 30 cm were used in the simulation.

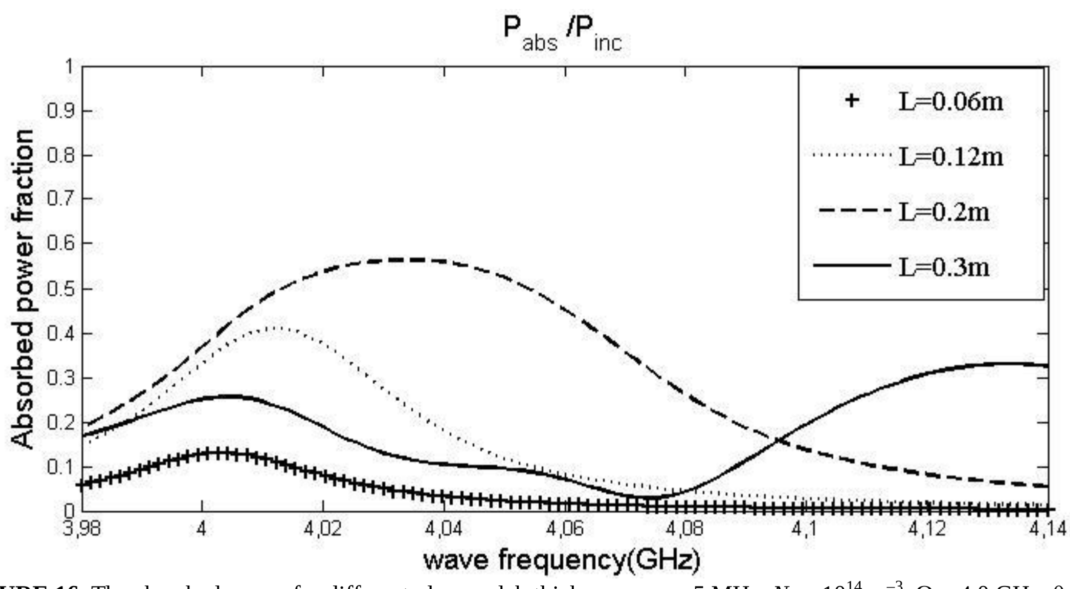

FIGURE 16. The absorbed power for different plasma slab thicknesses. ν = 5 MHz, $N_0 = 10^{14}$ m^{-3}, Ω = 4.0 GHz, θ = 60°.

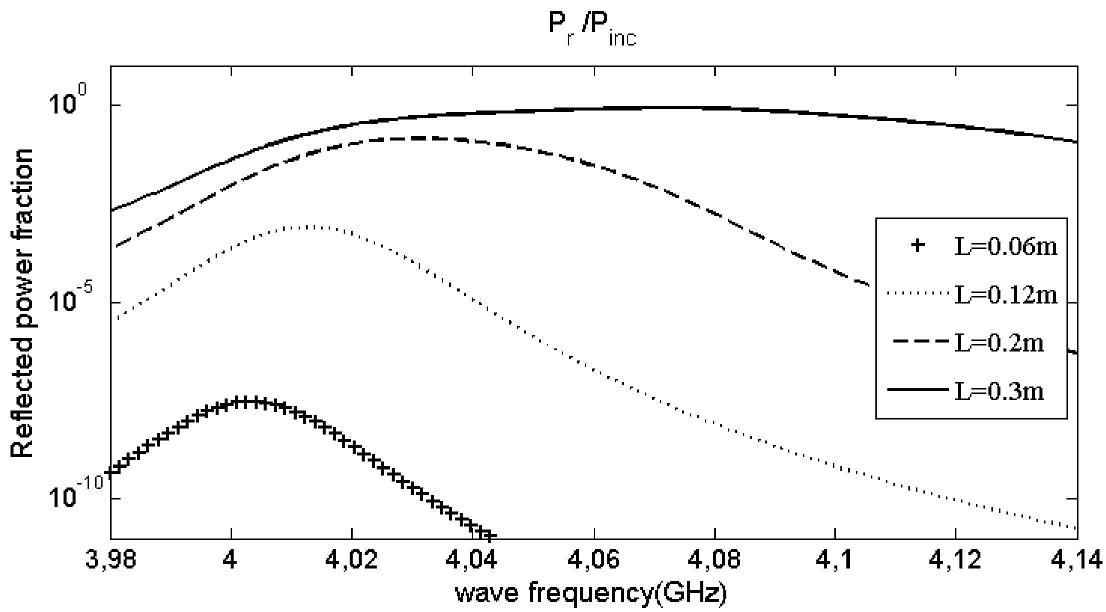

FIGURE 17. The reflected power for different plasma slab thicknesses. $\nu = 5$ MHz, $N_0 = 10^{14}$ m^{-3}, $\Omega = 4.0$ GHz, $\theta = 60°$.

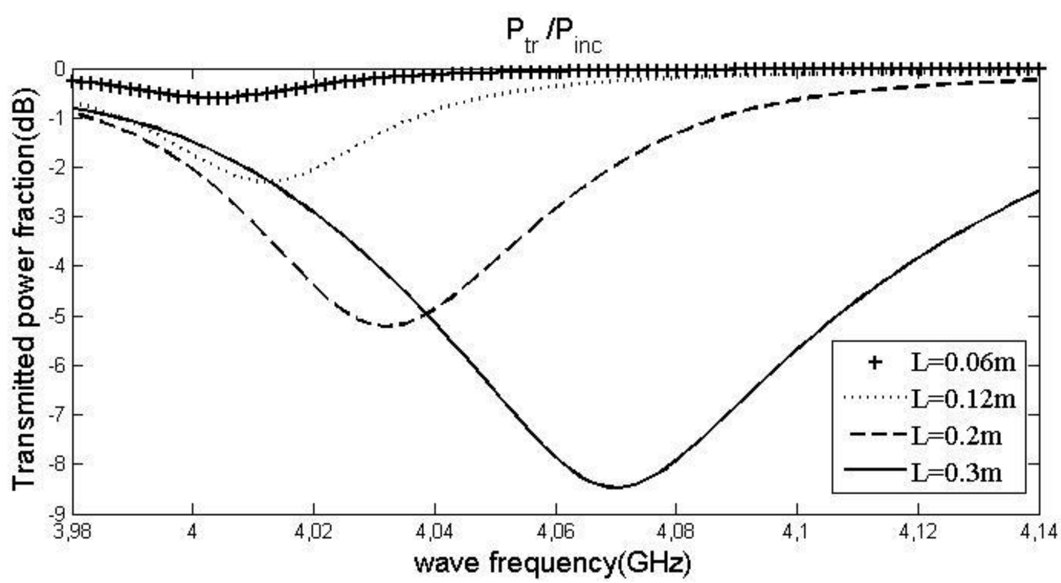

FIGURE 18. The transmitted power for different plasma slab thicknesses. $\nu = 5$ MHz, $N_0 = 10^{14}$ m^{-3}, $\Omega = 4.0$ GHz, $\theta = 60°$.

As expected, Figures 17 and 18 show that by increasing the thickness of the plasma, the reflection coefficient increases and transmission coefficient decreases. However, for the absorption curves in Figure 16, the absorption coefficient increases with slab thickness up to 0.2 m, and thereafter decreases.

IV. CONCLUSION

This study first calculated the reflection, transmission and absorption coefficients and then curves of absorbed, reflected and transmitted power fractions versus frequency of the microwaves (around 4 GHz) were drawn. The effects on these power fractions by a nonuniform magnetized plasma in terms of the plasma density, collision frequency, thickness of plasma slab and background magnetic field were shown. It was observed that less collisional and higher density plasma resulted in higher absorption and reflection (correspond to lower transmission). Highest absorbed power fraction was obtained at an optimum plasma slab thickness of 0.20 m though reflected power increased with thickness. Maximum absorption and reflection (and minimum transmission) of the microwaves were observed at B = 0.147 T. Increasing the magnetic field shifted the resonant frequency to higher value. Increased absorption of the microwaves would be useful for enhanced heating in the magnetized plasma.

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