Amplitude and Phase Velocity of Solitary Waves in Non – isothermal Plasma

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Highlights: (i) Higher order(3rd order) solitons are discussed in a four component plasma.

(ii) A perturbative new approach known as "tanh-method" is used.

(iii) Third order spiky and explosive solitary wave solutions along with amplitudes in case of non-isothermal electron with warm ion plasma are found out that Freja satellite observation was unable to explain.

Abstract. In presence of positron, ion-acoustic solitary waves for the plasma (He+,O-) are investigated with the variation of different plasma parameters in single and two-temperature non-isothermal electron plasma . The first (ϕ_1), second (ϕ_2) and third (ϕ_3) order compressive solitary wave solutions in non-isothermal plasma are calculated by well known 'tanh – method'. Besides this, one more important point is the "spiky and explosive" solitary waves obtained from higher order solutions which Freja Scientific Satellite observations cannot explain. Again the condition for the existence of a potential well with warm positive ion, warm negative ion including their drifts and warm positron along with the critical values of negative ion concentration$\left(n_{jc}\right)$, negative ion temperature$\left(\sigma_{jc}\right)$, positron density (χ_c) and phase velocity (V_c) are well defined and analysed. Two types of phase velocities are also highlighted and discussed. We are mainly interested with first(Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes and fast ion acoustic phase velocity (V_F) of the solitary waves. All these are represented graphically.

Key Words: Pseudopotential method; drift motion; non-isothermal electron; single and two-temperature electrons; tanh-method; positron; amplitude; phase velocity.

I. INTRODUCTION

In order to discuss the amplitude and phase velocity of solitary waves in non-isothermal plasma, first of all we are trying to state the sufficient initial definition and background of solitary waves and its propagation. Solitary waves are the most general class of non-linear wave structures having self-consistent wave amplitudes and phases. In Plasma Physics "dispersion" and "non-linearity" are the most two important terms concerned with solitary waves. If only "dispersion" is present in plasma there will be a decay of energy and if only the "non-linearity" is present there will be a growth of energy .The ion-acoustic waves propagating with constant phase velocity do not show any dispersion when the concentration of electron (n_e) is equivalent to the concentration of ion (n_i) but when $n_e \neq n_i$, the plasma becomes a dispersive medium for ion acoustic waves. In plasma when dispersion and non-linearity are both present then under certain conditions there is a possibility of balancing between nonlinearity and dispersion giving rise to the formation of solitary waves. Karpman [1], Karpman and Kruskal [2] showed the amplitude of a plane wave propagating in a non-linear dispersive medium changes slowly and this phenomena describes the envelope properties of a non-linear wave. Washimi and Taniuti [3] considered a plasma medium consisting of isothermally warm electrons and cold ions and derived an ion-acoustic solitary wave of small amplitude by the reductive perturbation technique giving non-linear KdV equation after proper scaling of space (x) and time (t). Murthy et al [4] assumed the plasma medium consisting of adiabatically warm ions and isothermally warm electrons for obtaining an expression for the solitary wave velocity .Propagation of ion-acoustic solitary waves for isothermal plasma together with positive ion, negative ion and positron have been studied theoretically and experimentally by many authors [5-8].Jeffrey and Kakutani [9], Ikezi [10] and Ikezi et al [11] showed the characteristics of solitary waves in plasmas experimentally while theoretical observations have been shown by Das and Tagare [12] , Tran and Hirt [13] , Watanabe [14] , Tran [15] and Lonngren [16]. They have reviewed the characteristics of the solitons in different plasma models, with an emphasis on the close relationship between the theoretical and experimental results. Later on, Nakamura et al [17] and Ludwig et al [18] have investigated experimentally the existence and behaviour of the ion-acoustic solitary waves in a multicomponent plasma with negative ions. In case of non-isothermal situation, Schamel [19-20] first studied the ion-acoustic solitons for non-isothermal electrons. With this concept of non-isothermal electrons, Kalita and Bujarbarua [21] extended the works of many previous authors for higher order contribution to ion-acoustic solitary waves. Moreover, solitary waves have interesting characteristics in presence of negative ions together with positive ions in the plasma and the existence of solitary waves depends on the finite ion-temperature effect[22-23]. It is also observed that the plasma with negative ions exhibits compressive and rarefactive solitons in isothermal situation whereas the existence of the solitary waves in non-isothermal plasmas is completely different. It has been shown that in the presence of a very small percentage of non-isothermality, the solitary wave equation exhibits both kinds arising from the non-isothermality on the solitons from which the cases of various plasmas with respect to the non-isothermality could be studied. Tagare and Reddy [24] also studied the ion-acoustic solitary waves of higher order non-linearity in presence of negative ions together with positive ions and non-isothermal electrons. Actually in presence of resonant electrons, the plasma behaves non-isothermally and more interesting results are found in the case of a multicomponent plasma consisting of non-isothermal electrons. Later on, Das et al [25] and others [26-27] assumed Schamel's plasma model and investigated the effects of non-isothermality of two-temperature electrons [28] on the formation of solitons. Majumdar et al [29] studied the third order contributions to ion-acoustic solitons in a plasma with two types of cold positive ions and two-temperature non-isothermal electrons by the Bogoliubov – Mitropolsky method and obtained a modified KdV (MKdV) solitary wave solution of $Sech^4$ type profile from which first, second and third order amplitudes are obtained. Chattopadhyay et al [30] studied the profiles of first and second order compressive solitary wave solutions of non-isothermal single temperature electron plasma for cold positive and negative ions with a special attention of second order W – type soliton shape in nature. Following Chattopadhyay et al [31], the present author took the warm positive and negative ions with drifts finding the critical negative ion concentration. Moreover the first , second and third order[7,32] compressive solitary wave solutions are obtained by a new approach known as the' tanh – method' with first, second and third order amplitudes for non-isothermal single and two-temperature electron plasma [32-33] after using a hyperbolic transformation $Z = \tanh(\eta)$ and $W(Z) = \phi(\eta)$ for obtaining a Fuchsian-like nonlinear ordinary differential equation from which a Frobenius series solution is obtained and finally the solitary wave solutions of first, second and third orders are gained. It is important to note in this case that for third order solution, the spiky and explosive solitary wave solutions [33] are found. It has been observed that within the certain range of plasma parameters the theoretical observations highlight the occurence of spiky solitary waves along with the explosion of the solitary waves[7,32] which are, in turns, to be related to those satellite observations made in interplanetary space plasmas.

The plan of the paper is arranged in the following manner:

In sec.2, the Sagdeev potential function $[\psi(\phi)]$ for non-isothermal single and two temperature electron plasma are calculated from the basic set of equations and critical values of the negative ion concentration(n_{jc}),negative ion temperature(σ_{jc}) and positron density(χ_c) are obtained from the condition for the existence of a solitary wave solutions. Two types of phase velocities$(V_F \; and \; V_S)$ are also discussed here. Sec.3 contains the first (ϕ_1), second (ϕ_2) and third (ϕ_3) order compressive solitary wave solutions along with the first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes for the non-isothermal single and two temperature electron plasma by employing the perturbative new approach tanh – method (hyperbolic tangent method). The loss of energy for first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes are also calculated here. The entire discussion is done very carefully in sec.4 with the variation of different plasma parameters. Concluding remarks are given in sec.5.

II. BASIC EQUATIONS AND ANALYSIS

We consider a plasma consisting of warm non-isothermal electrons, warm positive and warm negative ions with drifts and warm positron. The governing normalised basic equations[5-10] for non-linear behaviour of ion-acoustic solitary waves along x-axis in a collisionless, unmagnetised warm plasmas are

Equation of continuity: $\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}\left(n_\alpha u_\alpha\right) = 0$ (1)

Equation of motion: $\frac{\partial u_\alpha}{\partial t} + u_\alpha \frac{\partial u_\alpha}{\partial x} + \frac{\sigma_\alpha}{Q_\alpha n_\alpha}\frac{\partial p_\alpha}{\partial x} = -\frac{Z_\alpha}{Q_\alpha}\frac{\partial \phi}{\partial x}$ (2)

Pressure equation: $\frac{\partial p_\alpha}{\partial t} + u_\alpha \frac{\partial p_\alpha}{\partial x} + 3p_\alpha \frac{\partial u_\alpha}{\partial x} = 0$ (3)

Poisson's equation: $\frac{\partial^2 \phi}{\partial x^2} = n_e - \sum_\alpha Z_\alpha n_\alpha - n_p$ (4)

where the subscript α = i is for positive ions and α = j is for negative ions & $n_\alpha, u_\alpha, \sigma_\alpha, Q_\alpha, p_\alpha, Z_\alpha, n_e, \phi$, n_p, t and x are respectively the number density of ions, ion fluid velocity, temperatures of ions, ratio of negative to positive ion masses, pressure of ions, charge of ions, concentration of non-thermal electrons, electrostatic potential, concentration of positron , time and distance. The normalised positron density is

$$n_p = \chi \, e^{-\sigma_p \phi} \tag{5}$$

where $\sigma_p = \frac{T_e}{T_p}$ is the temperature ratio of electron and positron, φ is the electrostatic potential. T_e and T_p are the temperatures of electron and positron, χ is the density of positron at φ = 0 and is connected by the charge neutrality condition

$$\sum_\alpha n_{\alpha 0} Z_\alpha + n_{p0} = n_{e0} \tag{6a}$$

$$\text{or, } \chi + n_{i0} = 1 + Zn_{j0} \tag{6b}$$

[α = i is for positive ion and α = j is for negative ion]

In absence of positron (i.e. χ = 0) equation (6) supports Ref. [30]. In this case $\sigma_\alpha = \frac{T_\alpha}{T_e}$ $\left[\sigma_i = \frac{T_i}{T_e}\ for\ \alpha = i\ (positive\ ion) and\ \sigma_j = \frac{T_j}{T_e}\ for\ \alpha = j\ (negative\ ion)\right]$ $Q_\alpha = \frac{m_\alpha}{m_i} = 1$ and $Z_\alpha = 1$ for positive ion (α = i), $Q_\alpha = Q$ (= mass ratio) and $Z_\alpha = -Z$ (charge of ion = Z) for negative ion (α = j).

In the above equations, we have normalised the densities by the equilibrium value n_0, the velocities by the characteristic value $\sqrt{\frac{KT_e}{m_\alpha}}$ where m_α is the mass of ion and K is the Boltzmann constant, the pressure by the ion–equilibrium pressure $p_0 = n_0 T_\alpha$, the potential by $\frac{KT_e}{e}$, the time by $\sqrt{\frac{m_\alpha}{4\pi e^2 n_o}}$ and the distance by the Debye length $\sqrt{\frac{KT_e}{4\pi n_o e^2}}$ so that the equations appear totally in dimensionless form.

For solitary wave solution we assume that the dependent variables depend on a single independent variable $\eta = x - Vt$, where V is the velocity of the solitary wave .The boundary conditions are

$$n_\alpha \to n_{\alpha o}\,, u_\alpha \to u_{\alpha o},\ p_\alpha \to 1,\ n_e \to 1, n_p \to \chi \text{ and } \phi \to 0 \text{ at } |x| \to \infty \tag{7}$$

Following Chattopadhyay [34] and using the above boundary conditions we get finally from equations (1) to (4)

$$n_\alpha = \frac{1}{2}\sqrt{\frac{Q_\alpha n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\sqrt{(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}} - \sqrt{(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}}\right] \tag{8}$$

And

$$\frac{d^2\phi}{d\eta^2} = n_e - \frac{1}{2}\Sigma Z_\alpha\sqrt{\frac{Q_\alpha n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\sqrt{(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}} - \sqrt{(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^2 - \frac{2Z_\alpha\phi}{Q_\alpha}}\right] - \chi e^{-\sigma_p\phi} \tag{9}$$

For n_α to be real, the following restrictions on electrostatic potential ϕ [35] is

$$-\frac{Q}{2Z}\left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 < \phi < \frac{1}{2}\left(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 \tag{10}$$

[α = i is for positive ion and α = j is for negative ion]

Following Schamel [19 ,20], the non-isothermal electron density (n_e) is defined as

$$n_e(\phi) = \int_{-\propto}^{\propto} f_e\,(x, v)\mathrm{dv}$$

$$= \exp(\phi)\mathrm{erfc}\left(\sqrt{\phi}\right)\frac{1}{\sqrt{\beta}}\begin{bmatrix} exp(\beta\phi)erf\left[\sqrt{|\beta\phi|}\right] & for\ \beta > 0 \\ \frac{2}{\sqrt{\pi}}\ exp\left[-\left\{\sqrt{(-\beta\phi)}\right\}^2 \int_0^{\sqrt{(-\beta\phi)}} exp(X^2)\, dX\right] & for\ \beta < 0 \end{bmatrix} \tag{11}$$

where f_e(x,v) is the electron distribution function and $\beta = \frac{T_{ef}}{T_{et}}$ is the ratio of free and trapped electron temperatures.

We are now interested to discuss the non-isothermal single and two-temperature electron plasma with positron for first , second and third order solitary wave solutions and amplitudes.

(a) Non-isothermal single temperature electron plasma

After Taylor series expansion from equation (11), the electron density n_e in this case [19,20,32] is given by

$$n_e = 1+ \varphi - \frac{4}{3} b_1 \phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15} b_2 \phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 + \dots\dots\dots\dots \tag{12}$$

where $b_1 = \frac{1-\beta}{\sqrt{\pi}}$, $b_2 = \frac{1-\beta^2}{\sqrt{\pi}}$, $\beta = \frac{T_{ef}}{T_{et}}$, $0<b_1<\frac{1}{\sqrt{\pi}}$ and $0<b_2<\frac{1}{\sqrt{\pi}}$.

T_{ef} is the constant temperature of free electrons and T_{et} is the constant temperature of trapped electrons and β is their ratio and ϕ is the electrostatic potential. When $\beta \to 1$ then $b_1 \to 0$, $b_2 \to 0$ and as a result of this, the expression of electreon density n_e turns into isothermal plasma result.

Now from equation (9) and (12) we can write

$$\frac{d^2\phi}{d\eta^2} = 1+ \varphi - \frac{4}{3} b_1 \phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15} b_2 \phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 + \dots\dots\dots\dots$$
$$-\frac{1}{2}\Sigma Z_\alpha \sqrt{\frac{Q_\alpha n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\sqrt{(V-u_{\alpha 0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}})^2-\frac{2Z_\alpha\phi}{Q_\alpha}}-\sqrt{(V-u_{\alpha 0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}})^2-\frac{2Z_\alpha\phi}{Q_\alpha}}\right]-\chi e^{-\sigma_p\phi} \tag{13}$$

(b) Non-isothermal two- temperature electron plasma

From equation (11) after Taylor series expansion, the electron density n_e in this situation [19,20,32] is given by

$$n_e = 1+\phi-\frac{4}{3}\frac{\left(\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{3}{2}}}\phi^{\frac{3}{2}}+\frac{1}{2}\frac{\left(\mu+\nu\beta_1^2\right)}{\left(\mu+\nu\beta_1\right)^2}\phi^2-\frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{5}{2}}}\phi^{\frac{5}{2}}+\frac{1}{6}\frac{(\mu+\nu\beta_1^3)}{\left(\mu+\nu\beta_1\right)^3}\phi^3-\cdots \tag{14}$$

where $b_l = \frac{1-\beta_l}{\sqrt{\pi}}$, $b_h = \frac{1-\beta_h}{\sqrt{\pi}}$, $b_l^{(1)} = \frac{1-\beta_l^2}{\sqrt{\pi}}$, $b_h^{(1)} = \frac{1-\beta_h^2}{\sqrt{\pi}}$, $\beta_1 = \frac{T_{el}}{T_{eh}}$, $\beta_l = \frac{T_{el}}{T_{el,t}}$, $\beta_h = \frac{T_{eh}}{T_{eh,t}}$, μ , ν , β_l , β_h and β_1 are respectively the unperturbed number density of low temperature and high temperature electrons , temperature ratio of free and trapped electrons in low temperature, temperature ratio of free and trapped electrons in high temperature, temperature ratio of free electrons in low and high temperature.

when $b_l \to b_1$ (i.e. $\beta_l \to \beta$), $\mu \to 1$, $\nu \to 0$, $b_l^{(1)} \to$, b_2 (i.e. $\beta_l \to \beta$) , $b_h \to 0$ (i.e. $\beta_h \to 1$), $b_h^{(1)} \to 0$ (i.e. $\beta_h \to 1$) then the expression for non-isothermal two-temperature electron plasma concentration (n_e) reduces to the result of non-isothermal single temperature electron plasma concentration.

Again from equations (9) and (14) we can write

$$\frac{d^2\phi}{d\eta^2} =$$

$$1+\phi-\frac{4}{3}\frac{\left(\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{3}{2}}}\phi^{\frac{3}{2}}+\frac{1}{2}\frac{\left(\mu+\nu\beta_1^2\right)}{\left(\mu+\nu\beta_1\right)^2}\phi^2-\frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{5}{2}}}\phi^{\frac{5}{2}}+\frac{1}{6}\frac{(\mu+\nu\beta_1^3)}{\left(\mu+\nu\beta_1\right)^3}\phi^3-\cdots$$

$$-\frac{1}{2}\Sigma Z_\alpha\sqrt{\frac{Q_\alpha n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\sqrt{(V-u_{\alpha 0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^2-\frac{2Z_\alpha\phi}{Q_\alpha}}-\sqrt{(V-u_{\alpha 0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^2-\frac{2Z_\alpha\phi}{Q_\alpha}}\right]-\chi e^{-\sigma_p\phi} \tag{15}$$

(c) Condition for solitary wave solution and determination of critical values of negative ion concentration, negative ion temperature, positron density and phase velocity.

Equation (13) or (15) can be written in the following form:

$$\frac{d^2\phi}{d\eta^2} = -\frac{\partial\psi}{\partial\phi} \tag{16}$$

Integrating equation (16) and using the boundary condition (7) we get finally

$$\frac{1}{2}(\frac{d\phi}{d\eta})^2 + \psi(\varphi) = 0 \tag{17}$$

where $\psi(\varphi)$ is the Sagdeev pseudopotential function.

Thus for single temperature non-isothermal electron plasma the exact sagdeev potential $\psi(\phi)$ is

$$\psi(\varphi) = [-\varphi - \frac{1}{2}\phi^2 + \frac{8}{15}b_1\phi^{\frac{5}{2}} - \frac{1}{6}\phi^3 + \frac{16}{105}b_2\phi^{\frac{7}{2}} - \frac{1}{24}\phi^4 + ...]$$

$$+\frac{1}{6}\Sigma\sqrt{\frac{Q_\alpha^3 n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\begin{array}{c}\left\{\left(V-u_{\alpha 0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}}-\left\{\left(V-u_{\alpha 0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}}\\+(V-u_{\alpha 0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^3-(V-u_{\alpha 0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^3\end{array}\right]$$

$$+\frac{\chi}{\sigma_p}(1 - e^{-\sigma_p\phi}) \tag{18}$$

[For positive ion $\alpha = i$ and for negative ion $\alpha = j$]

And for two-temperature non-isothermal electron plasma the exact sagdeev potential $\psi(\phi)$ is

$$\psi(\varphi) = [-\varphi - \frac{1}{2}\phi^2 + \frac{8}{15}\frac{\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}}{(\mu+\nu\beta_1)^{\frac{3}{2}}}\phi^{\frac{5}{2}} - \frac{1}{6}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2}\phi^3 + \frac{16}{105}\frac{\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}}{(\mu+\nu\beta_1)^{\frac{5}{2}}}\phi^{\frac{7}{2}}$$

$$-\frac{1}{24}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3}\phi^4 + ...]$$

$$+\frac{1}{6}\Sigma\sqrt{\frac{Q_\alpha^3\, n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\begin{array}{c}\left\{\left(V-\ u_{\alpha 0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}}-\left\{\left(V-\ u_{\alpha 0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}}\\ +(V-\ u_{\alpha 0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^3-(V-\ u_{\alpha 0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^3\end{array}\right]$$

$$+\frac{\chi}{\sigma_p}(1-e^{-\sigma_p\phi}) \tag{19}$$

[For positive ion α = i and for negative ion α = j]

The term $\frac{1}{2}(\frac{d\phi}{d\eta})^2$ in equation (17) can be regarded as the kinetic energy (K.E.) of a particle of unit mass at position ϕ and time η whereas ψ(φ) is the potential energy (P.E.) of the same particle at that instant. The function ψ(φ) in non-isothermal plasma with positive ion, negative ion and positron may be easily reducible to isothermal plasma if $b_1 = b_2 = 0$ (i.e. for β = 1). For μ = 1, ν = 0 equation (19) is reduced to equation (18).

In absence of positron (χ = 0) equation (18) supports Ref. [30] for cold ion plasma (i.e. for $\sigma_\alpha = 0$). Also from (19) for μ = 1, ν = 0 and χ = 0 with $\sigma_\alpha = 0$, equation (19) supports Ref. [30].

The Sagdeev pseudopotential function ψ(ϕ) must satisfy the following conditions [33] for solitary wave solution:

i) $\psi(\phi) = 0 = \frac{\partial\psi}{\partial\phi}$ for all V at ϕ = 0

ii) $\frac{\partial^2\psi}{\partial\phi^2} < 0$ at ϕ = 0

iii) $\psi(\phi) = 0$ for some $\phi = \phi_m$, ϕ_m is some max. value of ϕ

iv) $\psi(\phi) < 0$ in $0 < |\phi| < |\phi_m|$ (20)

Following chattopadhyay [5] we get the condition for the existence of a solitary wave solution in presence of positron as

$$\sum_\alpha \frac{Z_\alpha^2 n_{\alpha 0}}{Q_\alpha(V-u_{\alpha 0})^2-\frac{3\sigma_\alpha}{n_{\alpha 0}}} < 1+\chi\sigma_p \tag{21}$$

In absence of positron ($\sigma_p = 0$), this inequality (21) supports Ref.[30] for warm ion plasma and also supports Ref.[30] for cold ion plasma (i.e. $\sigma_\alpha = 0$).

From the condition (20) of solitary wave solution ψ(ϕ) = 0 for some $\phi = \phi_m$ we get

$$\frac{1}{6}\Sigma\sqrt{\frac{Q_\alpha^3\, n_{\alpha 0}^3}{3\sigma_\alpha}}\left[\begin{array}{c}\left\{\left(V-\ u_{\alpha 0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}}\right)^2-\frac{2Z_\alpha\phi_m}{Q_\alpha}\right\}^{\frac{3}{2}}-\left\{\left(V-\ u_{\alpha 0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}}\right)^2-\frac{2Z_\alpha\phi_m}{Q_\alpha}\right\}^{\frac{3}{2}}\\ +(V-\ u_{\alpha 0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^3-(V-\ u_{\alpha 0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha 0}}})^3\end{array}\right]$$

$$+\frac{\chi}{\sigma_p}(1-e^{-\sigma_p\phi_m})$$

$$=\phi_m+\frac{1}{2}\phi_m^2-\frac{8}{15}\frac{\mu b_l+\nu\, b_h\beta_1^{\frac{3}{2}}}{(\mu+\nu\beta_1)^{\frac{3}{2}}}\phi_m^{\frac{5}{2}}+\frac{1}{6}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2}\phi_m^3$$

$$-\frac{16}{105}\frac{\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}}{(\mu+\nu\beta_1)^{\frac{5}{2}}}\phi_m^{\frac{7}{2}}$$

$$+\frac{1}{24}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3}\phi_m^4 \tag{22}$$

The above equation gives the value of ϕ_m at which $\psi(\phi_m) = 0$.

Also by $\boldsymbol{\psi}(\phi) < 0$ in $0 < |\phi| < |\phi_m|$ we get from (20)

$$\frac{1}{6}\Sigma\sqrt{\frac{Q_\alpha^3\, n_{\alpha0}^3}{3\sigma_\alpha}}\left[\left\{\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}}-\left\{\left(V-u_{\alpha0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}} + (V-u_{\alpha0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha0}}})^3-(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha0}}})^3\right]$$

$$+\frac{\chi}{\sigma_p}(1-e^{-\sigma_p\phi}) \;<\; \varphi+\frac{1}{2}\phi^2-\frac{8}{15}\frac{\mu b_l+\nu\, b_h\beta_1^{\frac{3}{2}}}{(\mu+\nu\beta_1)^{\frac{3}{2}}}\phi^{\frac{5}{2}}$$

$$+\frac{1}{6}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2}\phi^3$$

$$-\frac{16}{105}\frac{\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}}{(\mu+\nu\beta_1)^{\frac{5}{2}}}\phi^{\frac{7}{2}}+\frac{1}{24}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3}\phi^4 \tag{23}$$

This inequality (23) gives such value of ϕ in $0< |\phi| < |\phi_m|$ where $\psi(\phi) < 0$ is satisfied perfectly with $\phi \neq 0$ and $\phi \neq \phi_m$. Moreover the above value of ϕ from $\psi(\phi) < 0$ depends on μ and ν. Equation (22) and inequation (23) satisfy the condition of solitary wave solution. From inequation (21), effect of μ and ν is not found whether it is single or two temperature electron plasma but inequality (23) gives the effect of μ and ν in addition to give a particular value of ϕ in $0 < |\phi| < |\phi_m|$. So inequation (23) is an important relation in connecting with μ and ν that satisfies the condition of soliton solution.

Now in presence of positron, the critical negative ion concentration (n_{jc}) is obtained from the inequality (21) by the equation

$$a_0 n_{jc}^3 + a_1 n_{jc}^2 + a_2 n_{jc} + a_3 = 0 \tag{24}$$

where $a_0 = Z^2\left[Q(V-u_{j0})^2 + Z(V-u_{i0})^2\right]$

$$a_1 = Z\left[2Q(V-u_{j0})^2 - 3Z\sigma_j + Z(V-u_{i0})^2 - 3Z\sigma_i - Q(V-u_{i0})^2(V-u_{j0})^2 - \chi\sigma_p Q(V-u_{i0})^2(V-u_{j0})^2\right]$$

$$a_2 = Q(V-u_{j0})^2 - 6Z\sigma_j - Q(V-u_{i0})^2(V-u_{j0})^2 + 3Z\sigma_j(V-u_{i0})^2 + 3Q\sigma_i(V-u_{j0})^2$$
$$-\chi\sigma_p Q(V-u_{i0})^2(V-u_{j0})^2 + 3\chi\sigma_p Z\sigma_j(V-u_{i0})^2 + 3\chi\sigma_p Q\sigma_i(V-u_{j0})^2$$

$$a_3 = 3\sigma_j\left[(V-u_{i0})^2 - 1 - 3\sigma_i + \chi\sigma_p(V-u_{i0})^2 - 3\chi\sigma_p\sigma_i\right] \tag{25}$$

After calculating some steps, the value of the critical negative ion concentration n_{jc} is obtained finally from equation (24) as

$$n_{jc} = \left[\frac{1}{2}\left(\frac{a_1 a_2}{3a_0^2} - \frac{2a_1^3}{27a_0^3} - \frac{a_3}{a_0}\right) + \frac{1}{2}\sqrt{\left(\frac{a_1 a_2}{3a_0^2} - \frac{2a_1^3}{27a_0^3} - \frac{a_3}{a_0}\right)^2 - \frac{4}{729}\left(\frac{a_1^2}{a_0^2} - \frac{3a_2}{a_0}\right)^3}\,\right]^{\frac{1}{3}}$$

$$+ \left[\frac{1}{2}\left(\frac{a_1 a_2}{3a_0^2} - \frac{2a_1^3}{27a_0^3} - \frac{a_3}{a_0}\right) - \frac{1}{2}\sqrt{\left(\frac{a_1 a_2}{3a_0^2} - \frac{2a_1^3}{27a_0^3} - \frac{a_3}{a_0}\right)^2 - \frac{4}{729}\left(\frac{a_1^2}{a_0^2} - \frac{3a_2}{a_0}\right)^3}\,\right]^{\frac{1}{3}} - \frac{a_1}{3a_0} \tag{26}$$

In absence of positron (n_p =0 i.e. χ = 0), the co-efficients a_0 , a_1 , a_2 and a_3 in equation (24) will be the same as in Ref.[6] so that the critical negative ion concentration (n_{jc}) supports Ref. [6].

From inequation (21), the critical positron density parameter (χ_c) is obtained as

$$\chi_c = \frac{1}{\sigma_p}\left[\frac{n_{io}}{(V-u_{io})^2 - \frac{3\sigma_i}{n_{io}}} + \frac{Z^2 n_{jo}}{Q(V-u_{jo})^2 - \frac{3\sigma_j}{n_{jo}}} - 1\right]$$

From the above equation, it is observed very easily that as the ratio of electron and positron temperature (σ_p) increases, the positron density (χ) decreases and vice – versa when all other plasma parameters are kept as same. As a consequence of this result, we can say that the ratio of electron and positron temperature (σ_p) will be greater when electron temperature (T_e) is greater than positron temperature (T_p) and that ratio σ_p will be lesser when $T_e < T_p$ i.e. electron temperature (T_e) is less than positron temperature (T_p).

The condition for the existence of solitary wave solution from the inequation (21) reduces to the form

$$\frac{n_{io}}{(V-u_{io})^2 - \frac{3\sigma_i}{n_{io}}} + \frac{Z^2 n_{jo}}{Q(V-u_{jo})^2 - \frac{3\sigma_j}{n_{jo}}} < 1 + \chi\sigma_p \tag{27}$$

This is more generalised condition than that of the condition without positron for the existence of a solitary wave solution. The critical negative ion temperature (σ_{jc}) from the above inequation is obtained as

$$\sigma_{jc} = \frac{n_{j0}}{3}\left[Q(V - u_{j0})^2 - Z^2 n_{j0}\left\{(1 + \chi\sigma_p) - \frac{n_{io}}{(V-u_{io})^2 - \frac{3\sigma_i}{n_{io}}}\right\}^{-1}\right]$$

In absence of negative ion (i.e. n_{j0}= 0, σ_j = 0) we have from the inequation (21)

$$\frac{n_{io}}{(V-u_{io})^2 - \frac{3\sigma_i}{n_{io}}} < 1 + \chi\sigma_p$$

The critical velocity (V_c) is obtained from the above inequality in presence of positron as

$$V_c = u_{io} \pm \sqrt{\frac{1-\chi}{1+\chi\sigma_p} + \frac{3\sigma_i}{1-\chi}} \qquad \text{where } \chi < 1$$

This is more general result than Ref. [33]. In absence of positron i.e. when χ = 0 then

$V_c = u_{io} \pm\sqrt{1 + 3\sigma_i}$ that supports Ref. [22], The above critical velocity (V_c) is termed as the phase velocity and in this case two types of phase velocities are produced.

(i) Phase velocity of fast ion-acoustic mode (V_F)

$$V_F = u_{io} + \sqrt{\frac{1-\chi}{1+\chi\sigma_p} + \frac{3\sigma_i}{1-\chi}} \quad \text{where } \chi < 1$$

(ii) Phase velocity of slow ion-acoustic mode (V_S)

$$V_S = u_{io} - \sqrt{\frac{1-\chi}{1+\chi\sigma_p} + \frac{3\sigma_i}{1-\chi}} \quad \text{where } \chi < 1$$

It is also found very clearly from the two results (i) and (ii) that $V_F > V_S$ when all other plasma parameters are kept constant.

The condition for slow ion-acoustic mode is

$$\sigma_i < \frac{1}{3}(1-\chi)\left[u_{i0}^2 - \frac{1-\chi}{1+\chi\sigma_p}\right] \quad \text{where } \chi < 1$$

Moreover the critical positive ion temperature (σ_{ic}) is found as

$$\sigma_{ic} = \frac{n_{i0}}{3}\left[(V-u_{i0})^2 - \frac{n_{i0}}{1+\chi\sigma_p}\right]$$

III. SOLITARY WAVE SOLUTIONS WITH RELATED AMPLITUDES

In order to find the nature and behaviour of solitary wave solution, different orders and kinds of solitary wave solutions are essential to be discussed. Now from equation (13) or (15) connecting with equation (16) or (17) after Taylor series expansion of the respective function $\psi(\phi)$ in terms of ϕ and keeping terms upto order ϕ^3we get finally the standard form [7] after simplification

$$\frac{d^2\phi}{d\eta^2} = A_1\,\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 - A_4\phi^{\frac{5}{2}} + A_5\,\phi^3 - \ldots\ldots \quad (28)$$

where A_1 , A_2 , A_3 , A_4 and A_5 are the respective parameters of the equation (28) connected with the following given relations for single temperature non-isothermal electron plasma:

$$A_1 = \left[1 - n_{i0}\left\{(V-u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}\right\}^{-1} - Z^2 n_{j0}\left\{Q(V-u_{j0})^2 - \frac{3\sigma_j}{n_{j0}}\right\}^{-1} + \chi\sigma_p\right]$$

$$A_2 = \frac{4}{3}b_1 = \frac{4}{3}\frac{1-\beta}{\sqrt{\pi}}$$

$$A_3 = \frac{1}{2}\left[\begin{array}{c} 1 - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-3} - (V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-3}\right\} \\ + \frac{Z^3 n_{j0}^{\frac{3}{2}}}{2Q\sqrt{3\sigma_{jQ}}}\{(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-3} - (V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-3}\} - \chi\sigma_p^2 \end{array}\right]$$

$$A_4 = \frac{8b_2}{15} = \frac{8}{15}\frac{1-\beta^2}{\sqrt{\pi}}$$

$$A_5 = \frac{1}{2}\left[\begin{array}{c} \frac{1}{3} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-5} - (V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-5}\right\} \\ + \frac{Z^4 n_{j0}^{\frac{3}{2}}}{2Q^2\sqrt{3\sigma_{jQ}}}\{(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-5} - (V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-5}\} + \frac{\chi\sigma_p^3}{3} \end{array}\right] \quad (29)$$

And for two-temperature non-isothermal electron plasma, the above following parameters are

$$A_1 = \left[1 - n_{i0}\left\{(V-u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}\right\}^{-1} - Z^2 n_{j0}\left\{Q(V-u_{j0})^2 - \frac{3\sigma_j}{n_{j0}}\right\}^{-1} + \chi\sigma_p\right]$$

$$A_2 = \frac{4}{3}\frac{\left(\mu b_l + \nu b_h \beta_1^{\frac{3}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{3}{2}}}$$

$$A_3 = \frac{1}{2}\left[\begin{array}{l} \frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-3} - (V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-3}\right\} \\ + \frac{Z^3 n_{j0}^{\frac{3}{2}}}{2Q\sqrt{3\sigma_{jQ}}}\{(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-3} - (V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-3}\} - \chi\sigma_p^2 \end{array}\right]$$

$$A_4 = \frac{8}{15}\frac{\left(\mu b_l^{(1)} + \nu b_h^{(1)} \beta_1^{\frac{5}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{5}{2}}}$$

$$A_5 = \frac{1}{2}\left[\begin{array}{l} \frac{1}{3}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-5} - (V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-5}\right\} \\ + \frac{Z^4 n_{j0}^{\frac{3}{2}}}{2Q^2\sqrt{3\sigma_{jQ}}}\{(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-5} - (V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-5}\} + \frac{\chi\sigma_p^3}{3} \end{array}\right] \quad (30)$$

From the above expressions of A_1, A_2 , A_3 , A_4 and A_5 for two-temperature electron plasma we can say the following:

(i) The expression of A_1 for both single and two-temperature electron plasma is same.

(ii) For A_2 and A_3 in two-temperature electron plasma, when μ = 1, $b_l = b_1$, ν = 0 [$b_l = b_1$ i.e. β_l = β] , $b_h \rightarrow 0$ i.e. $\beta_h \rightarrow 1$ then the expression of A_2 and A_3 in two-temperature electron plasma reduces to single temperature electron plasma result.

(iii) For A_4 and A_5 in two-temperature electron plasma, when μ = 1 , $b_l^{(1)} = b_2$,

ν = 0 [$b_l^{(1)} = b_2$, i.e. β_l = β] , $b_h^{(1)} \rightarrow 0$, i.e. $\beta_h^2 \rightarrow 1$ then the expression of A_4 and A_5 in two-temperature electron plasma case also reduces to single temperature electron plasma result.

For cold positive $[\sigma_i = 0]$ and negative $[\sigma_j = 0]$ ions with two-temperature electron plasma and positron, the expressions for the parameters A_1 , A_2 , A_3 , A_4 and A_5 in (30) are modified with the following form given below:

$$A_1 = \left[1 - \frac{n_{i0}}{(V-u_{i0})^2} - \frac{Z^2 n_{j0}}{Q(V-u_{j0})^2} + \chi\sigma_p\right]$$

$$A_2 = \frac{4}{3}\frac{\left(\mu b_l + \nu b_h \beta_1^{\frac{3}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{3}{2}}}$$

$$A_3 = \frac{1}{2}\left[\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2} - \frac{3n_{i0}}{(V-u_{i0})^4} - \frac{3Z^3 n_{j0}}{Q^2(V-u_{j0})^4} - \chi\sigma_p^2\right]$$

$$A_4 = \frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{5}{2}}}$$

$$A_5 = \frac{1}{2}\left[\frac{1}{3}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3} - \frac{5n_{i0}}{(V-u_{i0})^6} - \frac{5Z^4 n_{j0}}{Q^3(V-u_{j0})^6} + \frac{\chi\sigma_p^3}{3}\right] \quad (31)$$

In absence of positron (n_p =0 i.e. χ = 0) the above expression A_1 , A_2 , A_3 , A_4 and A_5 in (31) for two-temperature electron plasma follows single temperature electron plasma result [Ref.30] when μ = 1, ν = 0 , $b_l = b_1$, $b_l^{(1)} = b_2$ and β_l = β .

The equation (28) for non-isothermal situation reduces to isothermal case when $A_2 = 0 = A_4$ i.e. β = 1 [b_1= 0=b_2]. Under this condition we can easily obtain the first and second order isothermal solitary wave solution and amplitude [5].

We are now briefly discussing the solution procedure of the differential equation (28) upto certain powers of ϕ by a simplified wave solution technique known as tanh- method (hyperbolic tangent method). Actually we now solve sagdeev potential equation [formed by (16) and (28)] step by step with different ordering in ϕ and take the number of nonlinear terms for showing only the soliton features. While doing this, we have neglected other small effects like viscosity, collision, Landau damping etc. Now for finding the soliton solution of equation (28), at first we use a hyperbolic transformation Z = tanh(η) and W(Z) = ϕ (η). After that, the said equation is transformed to a Fuchsian – like nonlinear ordinary differential equation from which a Frobenius series solution is obtained and finally the soliton solutions of first, second and third orders compressive structures of solitary wave profiles are described successively taking terms upto certain powers of ϕ in equation (28).

Now for first order solitary wave solution, taking terms upto $\phi^{\frac{3}{2}}$ [7,32] (lowest order nonlinearity in ϕ) from equation (28) we get

$$\frac{d^2\phi}{d\eta^2} = A_1\,\phi - A_2\phi^{\frac{3}{2}} \quad (32)$$

The equation (32) after integration gives $\left(\frac{d\phi}{d\eta}\right)^2 = A_1\,\phi^2 - \frac{4}{5}A_2\,\phi^{\frac{5}{2}}$ (33)

where the constant of integration is zero by boundary condition.

The usual first order solitary wave solution (ϕ_1) of equation (33) in non-isothermal plasma [7,32,36] is

$$\phi_1 = \left(\frac{5A_1}{4A_2}\right)^2 Sech^4\left(\sqrt{\frac{A_1}{16}}\,\eta\right) \quad (34)$$

For non-isothermal plasma the first order amplitude (Φ_{01}) [7,32,36] of the solitary wave solution (34) is

$$\Phi_{01} = \left(\frac{5A_1}{4A_2}\right)^2 \quad (35)$$

Again for better approximation on the solitary wave solution, similarly taking terms upto ϕ^2[7,32] (next higher order term in ϕ) from equation (28) we get

$$\frac{d^2\phi}{d\eta^2} = A_1\,\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 \quad (36)$$

Integrating equation (36) and using the boundary condition we get

$$\left(\frac{d\phi}{d\eta}\right)^2 = A_1\,\phi^2 - \frac{4}{5}A_2\,\phi^{\frac{5}{2}} + \frac{2}{3}A_3\,\phi^3 \tag{37}$$

The second order solitary wave solution (ϕ_2) of equation (37) as usual way in non-isothermal plasma [7,32,37] is

$$\phi_2 = \left[\frac{3A_2}{5A_3} \pm \sqrt{\left(\frac{9A_2^2}{25A_3^2} - \frac{3A_1}{2A_3}\right)}\,Sech\left(\frac{1}{2}\sqrt{A_1 - \frac{9A_2^2}{25A_3}}\,\eta\right)\right]^2 \tag{38}$$

The second order amplitude (Φ_{02}) [7,32] of the solitary wave solution (38) is

$$\Phi_{02} = \left[\frac{3A_2}{5A_3} \pm \sqrt{\left\{\left(\frac{3A_2}{5A_3}\right)^2 - \frac{3}{2}\left(\frac{A_1}{A_3}\right)\right\}}\right]^2 \tag{39}$$

In order to obtain a still better approximation of the solitary wave solution, we get again from equation (28) taking terms upto $\phi^{\frac{5}{2}}$ [7,32] (next more higher order terms in ϕ) as

$$\frac{d^2\phi}{d\eta^2} = A_1\,\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 - A_4\phi^{\frac{5}{2}} \tag{40}$$

Integrating equation (40) we get

$$\left(\frac{d\phi}{d\eta}\right)^2 = A_1\ \phi^2\ -\ \frac{4}{5}\ A_2\ \phi^{\frac{5}{2}}\ +\ \frac{2}{3}\ A_3\ \phi^3\ -\ \frac{4}{7}\ A_4\ \phi^{\frac{7}{2}} \tag{41}$$

where the constant of integration is zero by boundary condition.

The well known third order solitary wave solution (ϕ_3) [32] of the above equation (41) in non-isothermal plasma [7,32] is

$$\phi_3 = \left(\frac{7A_3}{18A_4}\right)^2 Sech^4\left[\pm\frac{1}{2}\left(\frac{2A_3}{9}\right)^{\frac{1}{2}}\left(\frac{7A_3}{18A_4} - \sqrt{\phi_3}\right)\eta\right] \tag{42}$$

The above solution ϕ_3 is an implicit function of η and is known as the profile of a spiky solitary wave solution [32] defined in the region $\sqrt{\phi_3} > 0$.

In a similar way for the region where $\sqrt{\phi_3} < 0$ we get the profile of an explosive (ϕ_3) solitary wave solution [32] in the implicit form

$$\phi_3 = \left(\frac{7A_3}{18A_4}\right)^2 Cosech^4\left[\pm\frac{1}{2}\left(\frac{2A_3}{9}\right)^{\frac{1}{2}}\left(\frac{7A_3}{18A_4} - \sqrt{\phi_3}\right)\eta\right] \tag{43}$$

The third order amplitude(Φ_{03})[32] of the spiky and explosive solitary wave solution (42) or (43) is

$$\Phi_{03} = \left(\frac{7A_3}{18A_4}\right)^2 \tag{44}$$

The first (ϕ_1), second (ϕ_2) and third (ϕ_3) order solitary wave solutions of the equation (28) are obtained by a perturbative new approach known as tanh-method (hyperbolic tangent method). The Freja scientific satellite confirmed observations [38] of dip and hump solitons in space supported by our theoretical results obtained from first (ϕ_1) and second (ϕ_2) order solitary wave solutions of the sagdeev potential equation (17). The third (ϕ_3) order (higher order) spiky and explosive solitary wave solutions along with its amplitude (Φ_{03}) of the same equation (17) is the new additional observations due to the trapped electrons in the potential well and these two kinds of solitary waves viz. Spiky and explosive solitary waves were not defined by Freja satellite observations [38] so far in space plasmas and that's why it is new findings.

Lemma 1. From equations (16) and (28) we can write

$$\frac{d^2\phi}{d\eta^2} = - \frac{\partial\psi}{\partial\phi} = A_1\,\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 - A_4\phi^{\frac{5}{2}} + A_5\,\phi^3 - ....$$

Integrating and using the boundary condition we get

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 = -\,\psi(\varphi) = \frac{1}{2}A_1\phi^2 - \frac{2}{5}A_2\phi^{\frac{5}{2}} + \frac{1}{3}A_3\phi^3 - \frac{2}{7}A_4\phi^{\frac{7}{2}} + \frac{1}{4}A_5\phi^4$$

Substituting $\phi = \theta^2$ in the above equation we get finally

$$\int \frac{d\theta}{\theta\sqrt{A_1 - \frac{4}{5}A_2\theta + \frac{1}{3}A_3\theta^2 - \frac{4}{7}A_4\theta^3 + \frac{1}{2}A_5\theta^4}} = c \pm \frac{1}{2}\eta \qquad (45)$$

where c = integration constant.

Equation (45) is an elliptic integral of more than third order. After simplifying the elliptic integral we can say that the more higher order solution of θ in terms of η (finally potential ϕ in terms of η) represents the different progressive modes which might be augmented to solitary waves as well as to double layers under different basic conditions.

Now from equation (45) after taking terms upto $A_2\theta$ we get the solution (34) when θ is expressed in terms of ϕ. Again when the terms upto $A_3\theta^2$ are considered in equation (45) we get the solution (38) when θ is expressed in terms of ϕ. Moreover when the terms upto $A_4\theta^3$are considered in equation (45) we get the solutions (42) and (43) when θ is expressed in terms of ϕ. After that when we consider terms upto $A_5\theta^4$ we get the higher order (more than third order) solution. Equation (45) is thus an important integral equation for obtaining more and more higher order equation which will give the result more closer to the experimental results.

Lemma 2. The condition for compressive solitary wave solution is obtained from $\frac{\partial\psi}{\partial\phi} > 0$ for $\phi = \phi_m$ where $\phi_m > 0$.

From equation (28) we get finally after simplification

$$A_5^2\phi_m^4 + (2A_3A_5 - A_4^2)\phi_m^3 + (A_3^2 + 2A_1A_5 - 2A_2A_4)\phi_m^2 + (2A_1A_3 - A_2^2)\phi_m + A_1^2 > 0 \qquad (46)$$

This is a fourth degree inequation in ϕ_m where A_1, A_2 , A_3 , A_4 and A_5 are given in equations (29) & (30). The roots of the inequation in ϕ_m (roots may be real or complex) can be calculated numerically by which the condition for compressive solitary wave solution is established for real positive value of ϕ_m.

IV. ENERGY CALCULATION

It is also observed from non-isothermal solutions (34), (38) and (42) or (43) that no isothermal solution of the mentioned order is obtained easily from non-isothermal solution until the non-isothermal order is removed. On the other hand, soliton amplitude is another important factor regarding this solution. Actually in solitary waves, the amplitude of vibration does not remain constant but it becomes progressively smaller. The decrease in amplitude is due to loss of energy. The loss of energy for first and second order amplitudes is $E_1 = \Phi_{02} - \Phi_{01}$ and the loss of energy for second and third order amplitudes is $E_2 = \Phi_{03} - \Phi_{02}$.

Actually in first case, the loss of energy can be expressed as

$$E_1 = \Phi_{02} - \Phi_{01} = \left[\frac{3A_2}{5A_3} \pm \sqrt{\left\{\left(\frac{3A_2}{5A_3}\right)^2 - \frac{3}{2}\left(\frac{A_1}{A_3}\right)\right\}}\right]^2 - \left(\frac{5A_1}{4A_2}\right)^2 \quad (47)$$

And in the second case, the loss of energy can be similarly expressed as

$$E_2 = \Phi_{03} - \Phi_{02} = \left(\frac{7A_3}{18A_4}\right)^2 - \left[\frac{3A_2}{5A_3} \pm \sqrt{\left\{\left(\frac{3A_2}{5A_3}\right)^2 - \frac{3}{2}\left(\frac{A_1}{A_3}\right)\right\}}\right]^2 \quad (48)$$

V. DISCUSSION

We are now investigating the first (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of the ion-acoustic solitary waves in presence of non-isothermal electrons together with drifting positive ion, negative ion and positron under the variation of different plasma parameters. These are represented graphically by the figs. 1 – 6.

In Fig. 1, first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) are shown very clearly with the variation of two-temperature electron concentration (μ, ν). These are shown respectively by the figures l_1 , l_2 , l_3 and s_1, s_2, s_3. It is also found that for any particular values of negative ion concentration (n_{j0}), the amplitudes are increasing in nature for both two-temperature [$l_3>l_2>l_1$] and single temperature [$s_3>s_2>s_1$] electron plasma in presence of positron. Moreover the values of these three types of amplitudes in presence of the same positron are larger for single temperature non-isothermal electron than those of two-temperature non-isothermal electron plasma [$s_1>l_1$, $s_2>l_2$, $s_3>l_3$].

Fig. 2 shows the first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) with the variation of the temperature of positive (σ_i) and negative (σ_j) ions. Here Φ_{01}, Φ_{02} and Φ_{03} are shown respectively by l_1 , l_2 and l_3 when $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$ and also by d_1 , d_2 and d_3 when $\sigma_i = \frac{1}{20}$, $\sigma_j = \frac{1}{10}$ in presence of positron. As temperatures of ions are increasing the first (Φ_{01}) and second (Φ_{02}) order amplitudes are decreasing [$d_1<l_1$, $d_2< l_2$ except at n_{j0} = 0.04] with increasing negative ion concentration upto n_{j0}= 0.03 and increases after n_{j0} = 0.03 but the third (Φ_{03}) order amplitudes are increasing [$d_3>l_3$] for increasing temperature of both ions along with increasing negative ion concentration in presence of positron.

The first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) with the variation of the ratio of electron and positron temperature (σ_p) are shown in Fig. 3. In this case l_1 , l_2 and l_3 are represented by Φ_{01}, Φ_{02} and Φ_{03} when σ_p= 0.41 and c_1 , c_2 and c_3 are represented by the same Φ_{01}, Φ_{02} and Φ_{03} when σ_p= 0.90. It is evident from the figure that the first, second and third order amplitudes are increasing [$c_1>l_1$, $c_2>l_2$, $c_3 > l_3$] when positron temperature increases for increasing negative ion concentration (n_{j0}).

In Fig. 4, the first (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitay wave solutions verses negative ion concentration (n_{j0}) with the variation of the ratios of negative to positive ion masses (Q) in presence of positron are shown that are represented by the respective figures l_1 , l_2 and l_3 when Q = 4 and also by g_1 , g_2 and g_3 when Q = 16. When the mass ratio(Q) is increasing then the first (Φ_{01}) , second (Φ_{02}) and third (Φ_{03})

order amplitudes are increasing [$g_1 > l_1$, $g_2 > l_2$ except at n_{j0} = 0.01 and $g_3 > l_3$ except at n_{j0} = 0.04] for increasing negative ion concentration in presence of positron.

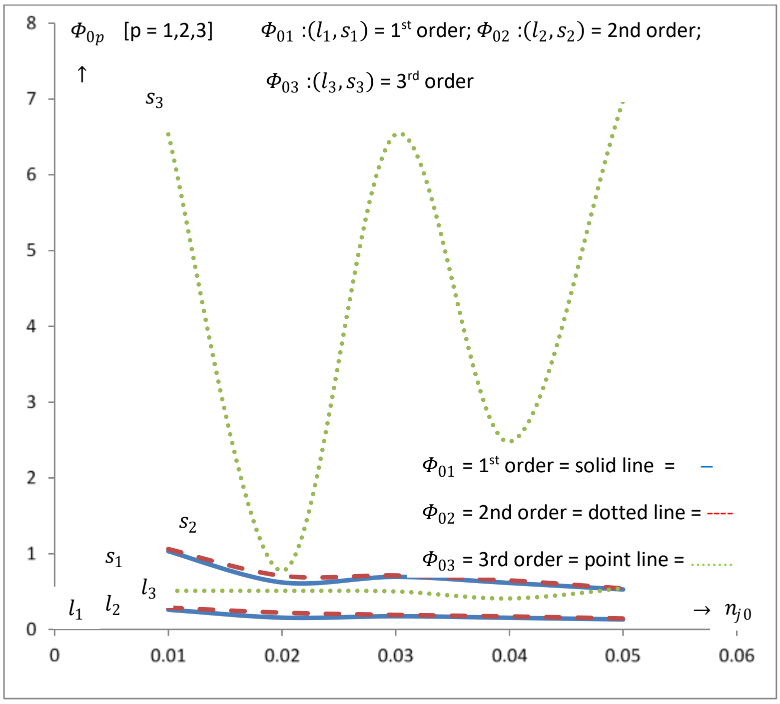

FIGURE 1. First (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) with the variation of two-temperature electron concentration (μ, ν) for V = 1.5, Q = 4, u_{i0} = 0.4, u_{j0} = 0.2 , $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17 , σ_p = 0.41 , b_l = 0.15, b_h = 0.4, β_1 = 0.25 , $b_l^{(1)}$= 0.25, $b_h^{(1)}$ = 0.51, μ = 0.15 , ν = 0.85 , μ = 1 , ν = 0 .

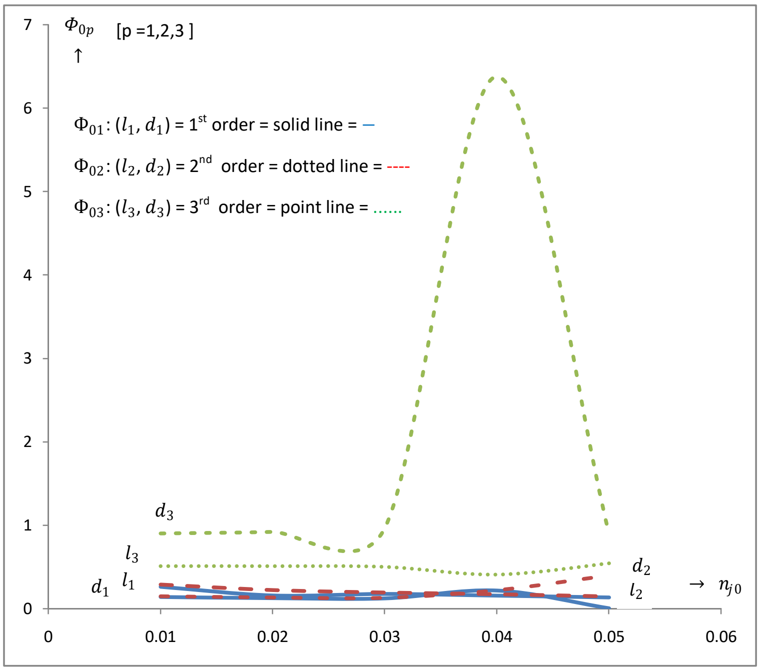

FIGURE 2. First (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) with the variation of temperature of positive (σ_i) and negative (σ_j) ions for V = 1.5, Q = 4, u_{i0} = 0.4, u_{j0} = 0.2 , $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, $\sigma_i = \frac{1}{20}$, $\sigma_j = \frac{1}{10}$, χ = 0.17 , σ_p = 0.41 , μ = 0.15 , ν = 0.85 , b_l = 0.15, b_h = 0.4, β_1 = 0.25 , $b_l^{(1)}$= 0.25, $b_h^{(1)}$ = 0.51 .

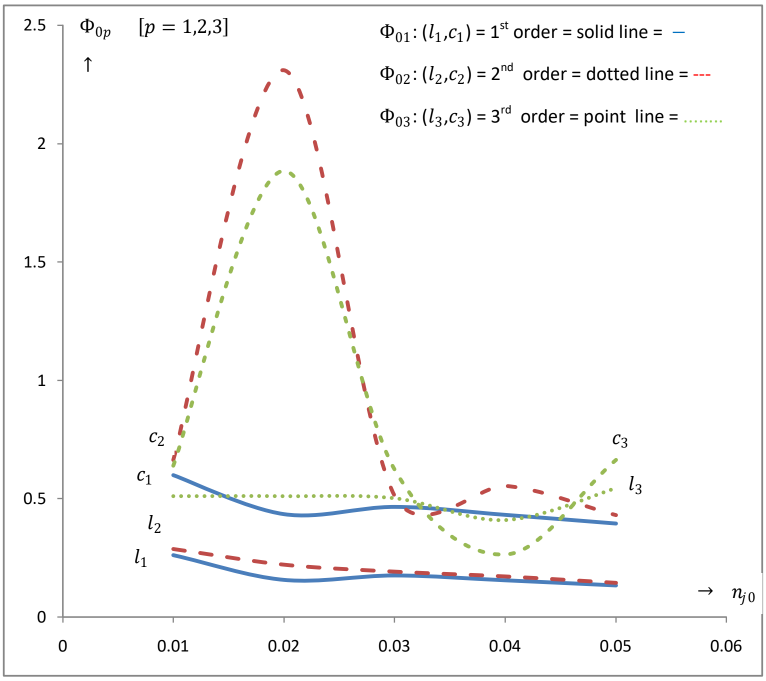

FIGURE 3. First (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) with the variation of the ratio of electron and positron temperature (σ_p) for V = 1.5, Q = 4, u_{i0} = 0.4, u_{j0} = 0.2 , $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17 , σ_p = 0.41 , σ_p = 0.9 , μ = 0.15 , ν = 0.85 , b_l = 0.15, b_h= 0.4, β_1 = 0.25 , $b_l^{(1)}$= 0.25, $b_h^{(1)}$ = 0.51 .

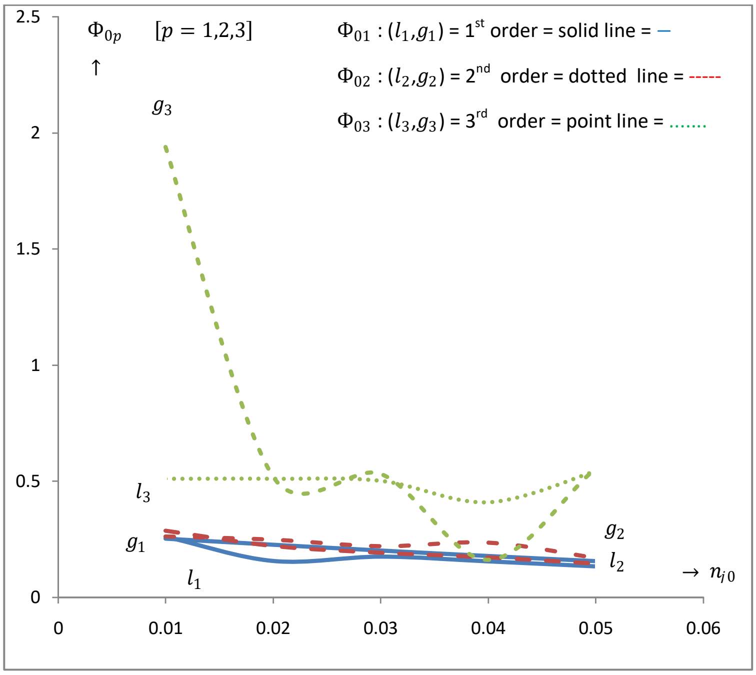

FIGURE 4. First (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) with the variation of the ratios of negative to positive ion masses (Q) for V = 1.5, Q = 4, Q = 16, u_{i0} = 0.4, u_{j0} = 0.2 , $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17 , σ_p = 0.41 , μ = 0.15 , ν = 0.85 , b_l = 0.15, b_h = 0.4, β_1 = 0.25 , $b_l^{(1)}$= 0.25, $b_h^{(1)}$ = 0.51 .

Fig. 5 shows the first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) with the variation of positron density (χ) .The first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes are represented by the curves l_1 , l_2 and l_3 when χ = 0.17 and those by h_1 , h_2 and h_3 when χ = 0 . As positron density (χ) increases, the first (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes are increasing [l_1>h_1, l_2>h_2 and l_3>h_3] for increasing negative ion concentration.

In Fig. 6, the first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary waves verses negative ion concentration (n_{j0}) with the variation of non-isothermal electron parameter are shown in presence of positron. In this case first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes are represented by the curves l_1 , l_2 and l_3 when the non-isothermal parameters are b_l = b_1 = 0.15 , b_h= 0.40, $b_l^{(1)}$= 0.25, $b_h^{(1)}$= 0.51 and again by the curves k_1 , k_2 and k_3 when those of the same non-isothermal parameters are b_l = b_1 = 0.20, b_h= 0.32, $b_l^{(1)}$= 0.15 and $b_h^{(1)}$= 0.24. As non-isothermal parameter changes, the amplitudes are also changing [k_1>l_1, k_2>l_2, k_3 > l_3] . The first (Φ_{01}), second(Φ_{02}) and third (Φ_{03}) order amplitudes are decreasing for increasing negative ion concentration (n_{j0}) in presence of the same positron.

Besides these three types of amplitudes we are now discussing the phase velocities for fast ion-acoustic mode (V_F) in presence of positron which are represented by the figs.7 – 10.

Fig. 7 represents the phase velocity (V_F) for fast ion-acoustic mode against positron density (χ) with the variation of the ratio of electron and positron temperature (σ_p). As σ_p increases the fast ion-acoustic phase velocity (V_F) decreases i.e. V_{F_1}<V_{F_2}<$V_{F_3}$where V_{F_1}, V_{F_2} and V_{F_3} are respectively the phase velocities at σ_p = 0.41, 0.7 and 0.9. Again for increasing χ with a fixed value of σ_p , the said phase velocity (V_F) decreases gradually.

In Fig. 8, the phase velocity (V_F) for fast ion-acoustic mode of solitary waves verses positron density (χ) with the variation of positive ion temperature (σ_i) is shown. When temperature of positive ion (σ_i) increases phase velocity (V_F) always increases [$V_{F_1} > V_{F_2} > V_{F_3}$where $V_{F_1}$is the phase velocity at σ_i= $\frac{1}{20}$, $V_{F_2}$is the phase velocity at σ_i= $\frac{1}{30}$ and $V_{F_3}$is the phase velocity at σ_i= $\frac{1}{100}$]. For a particular positive ion temperature (σ_i), phase velocity (V_F)is decreasing for increasing values of χ .

Again the phase velocity (V_F) for fast ion-acoustic mode of the solitary waves verses positron density (χ) with the variation of the drift velocity of positive ion (u_{i0}) is shown in Fig. 9. When drift velocity of positive ion (u_{i0}) is increasing, the phase velocity (V_F) is increasing for increasing values of χ . Here $V_{F_1} > V_{F_2} > V_{F_3}$where $V_{F_1}$is the phase velocity at u_{i0}= 0.9, $V_{F_2}$is the phase velocity at u_{i0}= 0.7 and $V_{F_3}$is the phase velocity at u_{i0}= 0.4. For a particular drift velocity (u_{i0}), phase velocity (V_F)is decreasing for increasing values of χ .

In Fig. 10, the phase velocity (V_F) for fast ion-acoustic mode of the solitary waves verses positive ion drift velocity (u_{i0}) with the variation of positron density (χ) is shown. The phase velocity (V_F) increases for increasing drift velocity of positive ion (u_{i0}) at a particular χ. Again when χ increases, the phase velocity (V_F) decreases for increasing drift velocity of positive ion (u_{i0}) i.e. V_{F_1}<V_{F_2}<V_{F_3} where $V_{F_1}$is the phase velocity at χ = 0, $V_{F_2}$is the phase velocity at χ = 0.17 and $V_{F_3}$is the phase velocity at χ = 0.22 .

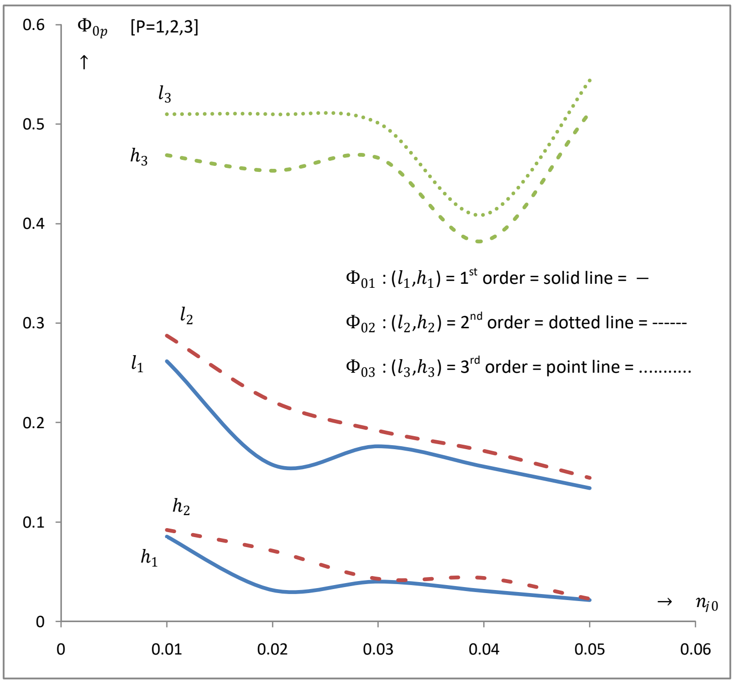

FIGURE 5. First (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) with the variation of positron density (χ) for V = 1.5, Q = 4, u_{i0} = 0.4, u_{j0} = 0.2 , $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0, χ = 0.17, σ_p = 0.41 , μ = 0.15 , ν = 0.85 , b_l = 0.15, b_h = 0.4, β_1 = 0.25 , $b_l^{(1)}$= 0.25, $b_h^{(1)}$ = 0.51.

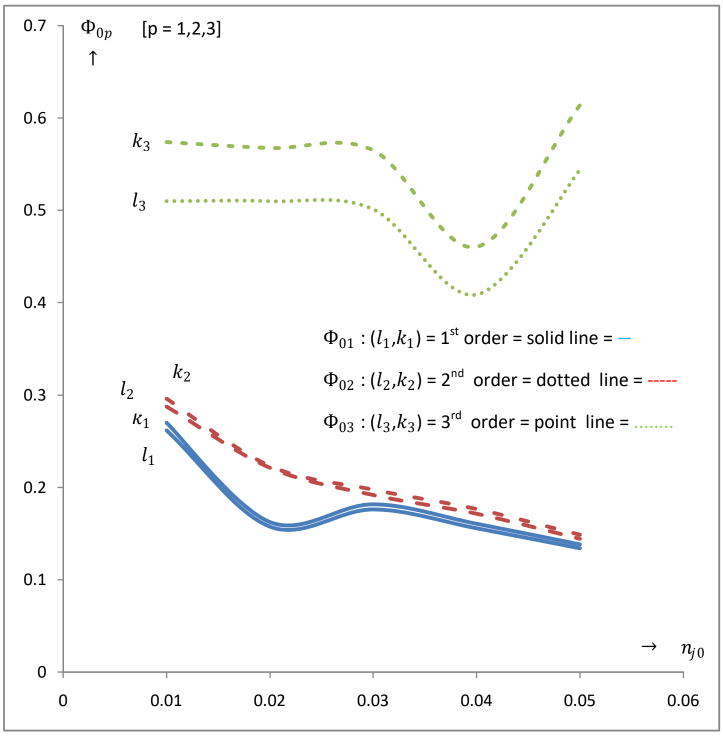

FIGURE 6. First (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of the solitary wave solutions verses negative ion concentration (n_{j0}) with the variation of non-isothermal parameter for V = 1.5, Q = 4, u_{i0} = 0.4, u_{j0} = 0.2 , $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17 , σ_p = 0.41 , μ = 0.15 , ν = 0.85, b_l = 0.15, b_h = 0.4, β_1 = 0.25 , $b_l^{(1)}$= 0.25, $b_h^{(1)}$ = 0.51, b_l = 0.2, b_h = 0.32, $b_l^{(1)}$= 0.15, $b_h^{(1)}$ = 0.24.

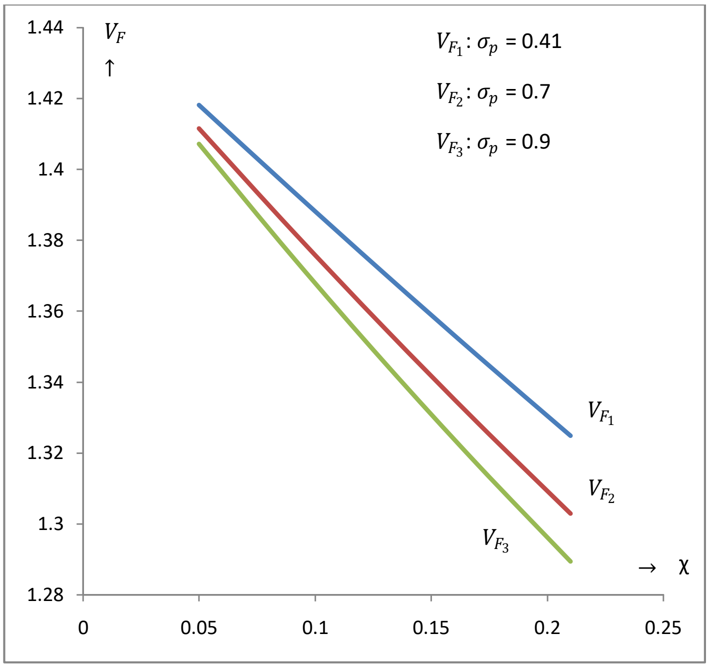

FIGURE 7. Fast ion-acoustic phase velocity (V_F) verses positron density (χ) with the variation of the ratio of electron and positron temperature (σ_p) for u_{i0} = 0.4, $\sigma_i = \frac{1}{30}$, σ_p = 0.41 , 0.7 , 0.9.

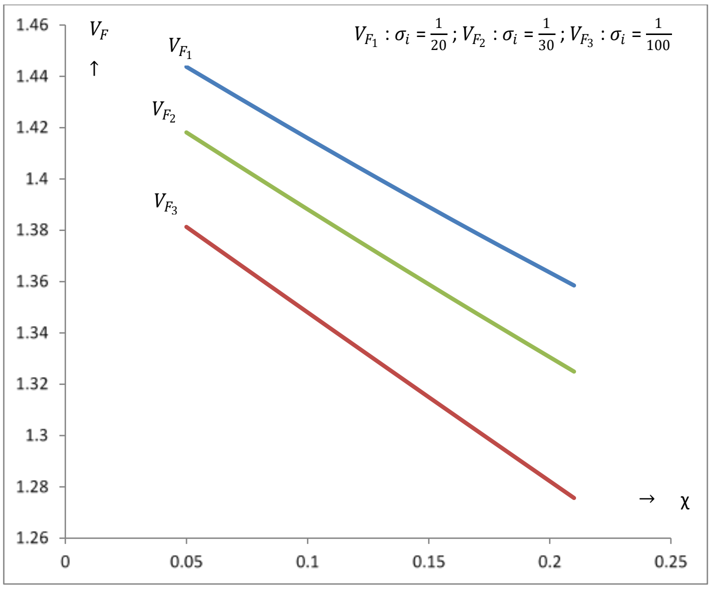

FIGURE 8. Fast ion-acoustic phase velocity (V_F) verses positron density (χ) with the variation of positive ion temperature (σ_i) for u_{i0} = 0.4 , $\sigma_p = 0.41$, $\sigma_i = \frac{1}{30}$, $\frac{1}{20}$, $\frac{1}{100}$.

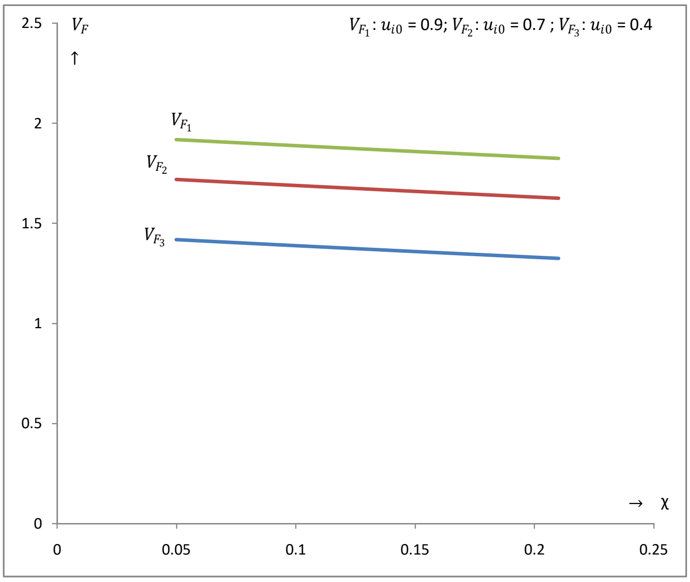

FIGURE 9. Fast ion-acoustic phase velocity (V_F) verses positron density (χ) with the variation of drift velocity of positive ion (u_{i0}) for σ_p = 0.41, $\sigma_i = \frac{1}{30}$, u_{i0} = 0.4, 0.7 , 0.9.

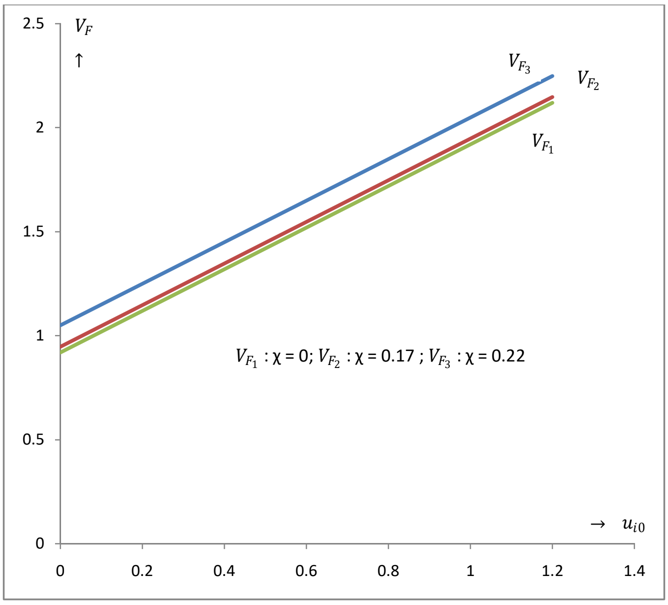

FIGURE 10. Fast ion-acoustic phase velocity (V_F) verses positive ion drift velocity(u_{i0}) with the variation of positron density (χ) for σ_p = 0.41, $\sigma_i = \frac{1}{30}$, χ = 0, 0.17 , 0.22.

VI. CONCLUSION

The non-isothermal electron plasma together with drifting positive ion, negative ion and positron play an important role on the formation of ion acoustic solitary wave solutions. The first(ϕ_1), second (ϕ_2) and third (ϕ_3) order solitary wave solutions are obtained from the sagdeev potential equation (17) by a perturbative new approach known as the tanh-method after using a hyperbolic transformation Z = tanh(η) and W (Z) = ϕ (η). The profiles of first(ϕ_1) and second (ϕ_2) order solitary wave solution do not give any good approximation but as the order becomes higher [say third order (ϕ_3)], non-isothermality expects to have a better approximation on the solution. We therefore retain the terms upto the third order quantities in the expansion of equation (17). Moreover the higher order (third order) theoretical values give more closer to the experimental results and from this third order soliton solution we may also say that the potential, width and Mach number can be expressed as a function of soliton amplitude for which the different order of amplitudes are considered in this problem. It is also remarkable to note theoretically that first, second and third order amplitudes of the solitary wave solutions, two types of phase velocities and loss of energy are observed in this situation.

Moreover it is interesting to note that previous authors [29,30] did not consider the contribution of third order amplitudes in the investigation of ion-acoustic solitary waves in two-temperature and single temperature non-isothermal electron plasma with positron. By our above investigation the present author thus tried to show very easily the effects of positron and loss of energy generated in this problem are most probably a new achievement.

Our motivation is to compare the first, second and third order amplitudes ($\Phi_{01}, \Phi_{02}, \Phi_{03}$) with the variation of two-temperature electron concentration (μ, ν), temperature of both ions (σ_i, σ_j), ratio of electron and positron temperature (σ_p), ratio of negative to positive ion masses (Q), positron density (χ) and finally with the variation of non-isothermal parameter. Fast ion-acoustic phase velocity (V_F) is also considered and analysed properly with the variation of different plasma parameters and they are depicted in Figs. 7 – 10. Energy calculation in this problem is a new idea through the observation of amplitudes measurement.

Generally the plasma acoustic modes in space are caused by the non-isothermality of space plasma. Due to trapped electrons Freja satellite observations could not be able to find spiky and explosive solitary waves [7, 32] along with their amplitudes and widths. This observations could be motivated through the satellite for the new findings in space plasma.

Our future plan is to solve the two-temperature non-isothermal electron plasma with positron along with relativistic warm positive and negative ion.

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