Reproduction of Spin-orbit Splitting with the Inclusion of Nuclear Tensor using Seniority Force

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Abstract. Prior to embarking on investigations with nuclear tensor, we performed a benchmark study to check the validity of our Skyrme-Hartree-Fock-BCS code [1]. We repeat the fit of SLy5+tensor parametrization but using the seniority force. Our code reproduced the trend obtained in previous study for both SLy5 and SLy5+tensor.

Keywords: nuclear tensor force; Skyrme interaction; seniority force; single-particle energy

I. INTRODUCTION

It is well known that tensor force plays a role in the shell evolution of the nuclei far from the stability line. Its contribution to the spin-orbit splitting was first examined within the Skyrme Hartree-Fock framework by Stancu *et al*. [2] in 1997. Together with the recent work by Colo *et al.* [3], these authors showed that the experimental spin-orbit splitting can only be described by introducing the tensor term. In 2018, Brink and Stancu [4] showed the importance of tensor force on the spin-orbit splitting of Ni isotopes. The role of nuclear tensor in the Skyrme energy density functionals [5-7] and nuclear structure [8-12] has been widely discussed. Apart from spherical and normally deformed nuclei, the role of tensor force in fission and fusion dynamics has also been examined in [13-17]. All these point to the increasing interest within the nuclear physics community on the impact of nuclear tensor.

The development mentioned above motivates us to look into the impact of tensor force on nuclear properties in our studies. The first step in our work is to make a benchmark study to ensure that the nuclear tensor part is correctly implemented in the code. We take as a reference the work by Colo *et al.* [3]. The single-particle splitting of $Z = 50$ isotopes and $N = 82$ isotones in [3] was calculated within the Hartree-Fock-plus-Bardeen-Cooper-Schrieffer (HF+BCS) framework. They considered the nucleon-nucleon interaction of the Skyrme type. Two sets of parametrizations were used, namely the original SLy5 parametrization and the same parametrization but with perturbative addition of nuclear tensor component, referred to as the SLy5+T. For the pairing interaction, they considered the density-dependent delta interaction (DDDI).

In previous studies of one co-author [18,19], the pairing interaction is approximated by a simpler seniority force. This simple pairing interaction has an advantage of a shorter computational time. We are therefore interested to check how well would the seniority force would perform as compared to the results of [3] with the DDDI, on top of benchmarking the code with respect to the nuclear tensor implementation.

II. TECHNICAL DETAILS OF THE CALCULATION

In this work, we performed calculations on the Z = 50 isotopes (ranging from $106 \leq A \leq 132$) and N = 82 isotones (ranging from $132 \leq A \leq 150$) using the Skyrme SLy5 and SLy5+T parameterizations. The two tensor parameters t_e and t_o entering the Skyrme energy density functional was obtained from [3] with all other Skyrme parameters between the two parametrizations remaining unchanged. The seniority force for the pairing part given as

$$g^{(q)} = -\frac{G_q}{11+N_q} \tag{1}$$

assumes a constant amount of pairing between all single-particle states. The pairing strengths G_q (q referring to the nuclear charge state) are usually fitted based on moment of inertia or odd-even mass staggering. In this preliminary work incorporating nuclear tensor, we chose a simpler option of fixing $G_n = G_p$ = 12 MeV. The symbol N_q refers to the number of nucleons for a given charge state q.

The single-particle wave functions are expanded on the axially deformed harmonic-oscillator basis states. The expansion is truncated following the prescription

$$\hbar\omega_\perp(\omega_\perp + 1) + \hbar\omega_z(\omega_z + \tfrac{1}{2}) \leq \hbar\omega_z(N_0 + 2) \tag{2}$$

where ω_0 is the spherical angular frequency given in terms of the angular frequencies ω_z and $\omega_\perp$ in the z- and r- directions, respectively, such that $\omega_0 = \omega_\perp^2\omega_z$. In this work, we choose a basis size of N_0 =14. The harmonic-oscillator parameters $b = \sqrt{m\omega_0/\hbar}$ (where m is the mean nucleon mass) and the deformation parameter, $q \equiv \omega_\perp/\omega_z$ are optimized in order to yield the lowest-energy solution for the ground state the selected nuclei. However, only nuclei at sphericity is considered in this work, so we set q = 1 for all cases. The numerical integrations were performed by using the Gauss–Hermite and Gauss–Laguerre approximations with 50 and 16 mesh points, respectively. The Coulomb exchange term has been evaluated using the Slater approximation.

III. RESULT

Figure 1 (top panel) shows proton single-particle energy difference between the $h_{11/2}$ and $g_{7/2}$ for a fixed proton number Z=50 as a function of the neutron excess, N - Z. Similar to the work by Colo *et al.* [3] the SLy5 parameterization failed to reproduce the experimental trend both qualitatively and quantitatively. The SLy5+T parameterization with inclusion of tensor force can reproduce the experimental trend as expected. Our SLy5+T results with the seniority force are plotted as filled (blue color) squares in Figure 1. These are to be compared with the red color squares from [3] using DDDI. Our results showed almost similar agreement with DDDI except for the Sn-70 (i.e. N – Z = 20) nucleus where the DDDI result is closer to the experiment.

Figure 1 (bottom panel) shows the neutron single-particle energy differences between the $i_{13/2}$ and $h_{9/2}$ for nuclei along the N = 82 isotones. Here, the qualitative agreement with the experimental single-particle energy differences is reproduced using the SLy5+T with the seniority force. Overall, the SLy5 and SLy5+T results with the seniority force are in good agreement with those with DDDI.

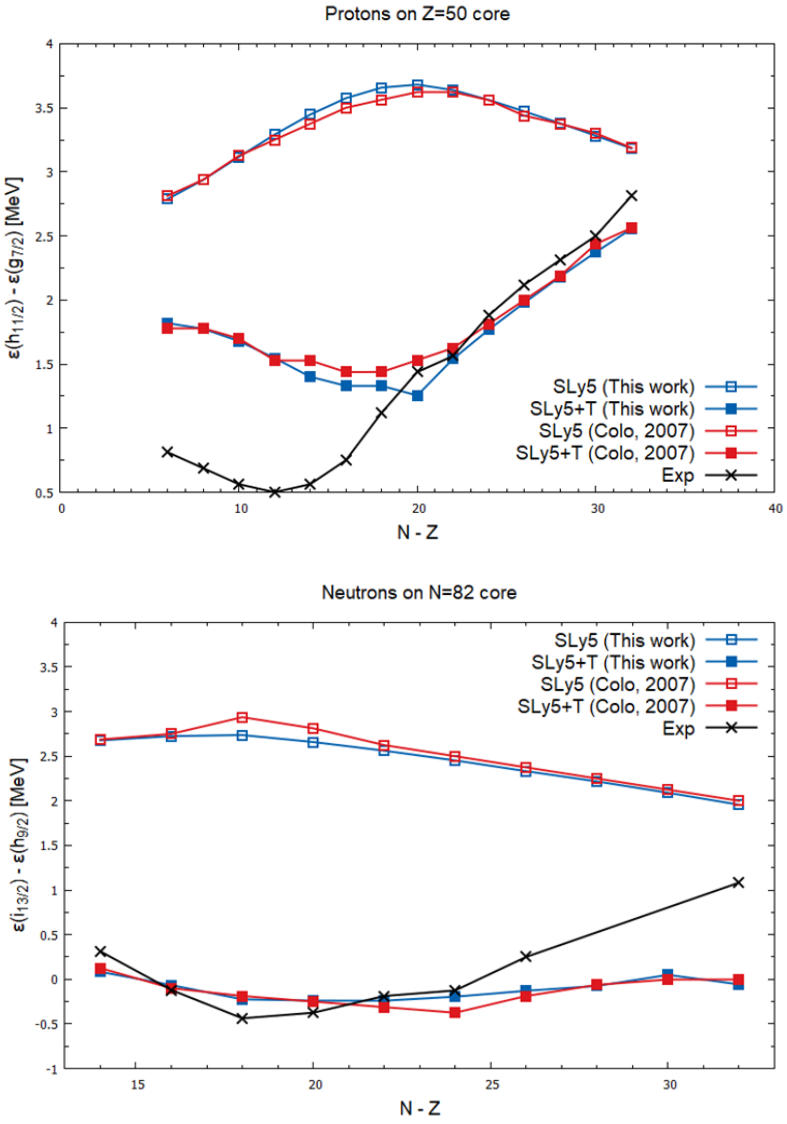

Figure 1. The single-particle energy differences between the h11/2 and g7/2 proton states along the Z = 50 isotopes (top panel) and the $i_{13/2}$ and $h_{9/2}$ neutron states along the N = 82 isotones (bottom panel). Experimental data were taken from [20].

We also compared our calculated proton and neutron single-particle energy levels in ^{132}Sn obtained with the SLy5 and SLy5+T parametrizations with experimental data [21] in Figure 2. For protons single-particle levels, we see that the perturbative addition of tensor force causes an inversion in the $d_{5/2}$ and $g_{7/2}$ states. For neutrons, level crossing does not occur when going from the SLy5 to the SLy5+T parametrization.

Finally, we show the comparison between our calculated binding energies and the experimental values [22] in Table 1 (Z = 50 isotopic series) and Table 2 (N = 82 isotonic series). We found that for the Z = 50 isotopic series, the SLy5+T gave a better agreement with experimental binding energy with a root-mean-square (r.m.s) of about 3 MeV as compared to about 4.1 MeV with the SLy5. However, in the N = 82 isotonic series, the SLy5 parametrization gave results which are closer to experiment with r.m.s of 3.5 MeV.

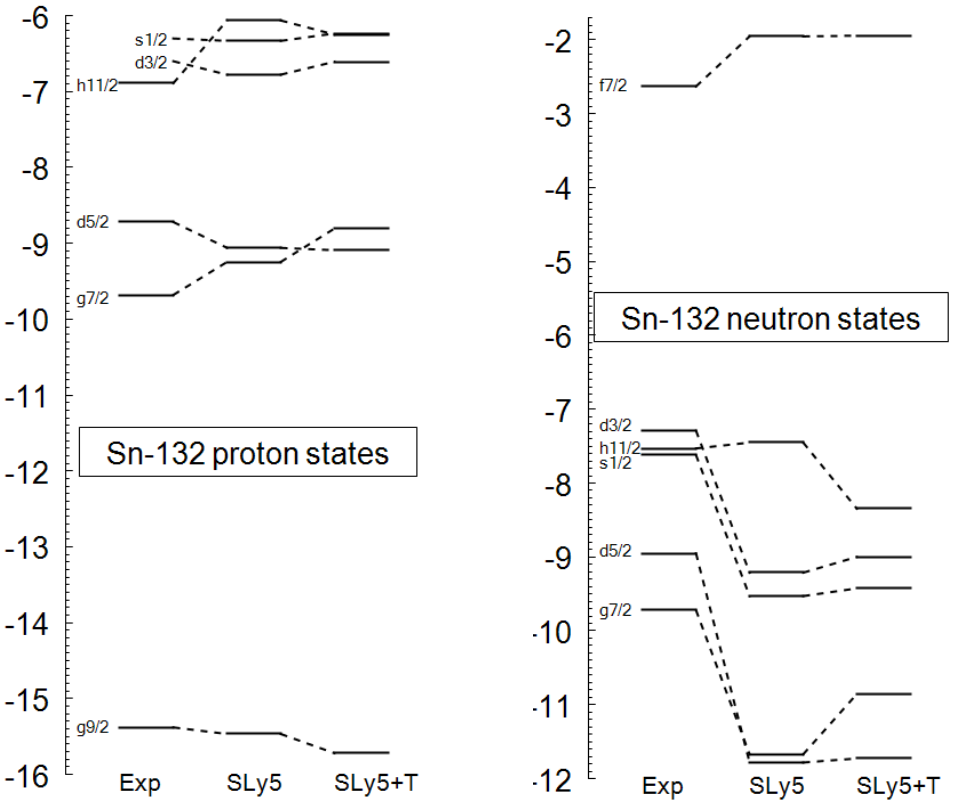

Figure 2. Comparison of proton and neutron single-particle states of ^{132}Sn between calculated results of SLy5 and SLy5+T with the seniority force and experimental data [21].

Table 1. Binding energy (in MeV) for Z = 50 isotopic series between calculated values obtained with SLy5 and SLy5+T as compared to experimental data obtained from [22].

Nucleus	N	Binding energy [MeV]		
		SLy5	SLy5+T	Exp [22]
Sn-106	56	-890.98	-898.69	-893.80
Sn-108	58	-910.60	-916.88	-914.66
Sn-110	60	-929.47	-934.69	-934.46
Sn-112	62	-947.79	-952.47	-953.53
Sn-114	64	-965.62	-970.30	-971.57
Sn-116	66	-982.95	-987.88	-988.68
Sn-118	68	-999.73	-1004.56	-1004.95
Sn-120	70	-1015.91	-1020.75	-1020.54
Sn-122	72	-1031.50	-1035.71	-1035.52
Sn-124	74	-1046.51	-1050.46	-1049.96
Sn-126	76	-1061.05	-1065.18	-1063.88
Sn-128	78	-1075.61	-1080.38	-1077.37
Sn-130	80	-1089.47	-1095.28	-1090.29
Sn-132	82	-1103.08	-1110.32	-1102.84
	RMS	4.1315	2.9802	

Table 2. Similar to Table 1 but for the N = 82 isotonic series.

Nucleus	Z	Binding energy [MeV]		
		SLy5	SLy5+T	Exp [22]
Er-150	68	-1211.19	-1218.04	-1215.33
Dy-148	66	-1205.86	-1213.39	-1210.78
Gd-146	64	-1199.85	-1208.25	-1204.43
Sm-144	62	-1191.33	-1199.13	-1195.73
Nd-142	60	-1180.94	-1188.24	-1185.28
Ce-140	58	-1169.06	-1175.95	-1172.68
Ba-138	56	-1155.57	-1162.24	-1158.29
Xe-136	54	-1139.90	-1146.58	-1141.88
Te-134	52	-1122.53	-1129.40	-1123.41
Sn-132	50	-1103.08	-1110.32	-1102.84
	RMS	3.5488	4.3501	

IV. CONCLUSION

In summary, we have performed a benchmark study of HF+BCS code [1] with and without nuclear tensor entering the Skyrme EDF. We take as our reference the seminal work of Colo *et al.* [3] and perform similar calculations except that the pairing interaction is approximated by the seniority force. In addition to the main purpose of this research note i.e. to ensure that the code with nuclear tensor is properly implemented, we also found that our results with the seniority force reproduced the results obtained by Colo *et al.* with DDDI. This gives us the confidence that making a fit on Skyrme parameters to determine nuclear tensor components entering the Skyrme EDF similar to what was done by [3] can be safely performed using the present code.

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