Analytical Solitary Wave Solution of Electron Acoustic Waves in Quantum Plasma in the Framework of Korteweg-de-Vries Equation in Presence of External Periodic Force

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Abstract. The analytical solution of electron-acoustic solitary waves (EASWs) are reported for an unmagnetised quantum plasma model considering the effect of external periodic force. Using reduction perturbation technique (RPT) forced Korteweg-de-Vries equation is obtained. The effect of different physical parameters such quantum diffraction parameter (H), speed of travelling wave (λ), peak value of frequency (f_0) and the frequency (w) of external periodic force are observed. This paper may be useful to the non linear features of EASWs in quantum plasma in the presence of external periodic force.

Keywords: EASWs, RPT, External force, KdV equation.

I. INTRODUCTION

Electron acoustic solitary waves (EASWs) are currently being considered as a major interdisciplinary research field in plasma physics. EASWs are the low frequency branch of electrostatic plasma waves. EASWs exist in neutralized plasmas, pure electron plasmas and pure ion plasmas. EASWs appear in space and laboratory plasmas [1-3] where two distinct electron populations exist, i.e. cool and hot electron. The linear and nonlinear properties of EASWs in unmagnetised and also magnetised plasma have been investigated by many authors. Javidan and Pakzad [4] noticed small amplitude analysis of weakly nonlinear EASWs in a three component magnetized plasma with kappa distributed hot electrons. Tribeche and Djebarni [5] investigated arbitrary amplitude EASWs in a plasma having cold fluid electrons. Sah et.al. [6] investigated in a three component unmagnetised dense quantum plasma consisting of two distinct groups of electrons and using RPT to obtained KdV equation. Iqbal et al. [7] observed the non linearity of spin EASWs in spin polarized degenerate quantum plasma. Masood et al. [8] studied the EASWs in a quantum magnetoplasma and they derived ZK equation by using the RPT. They observed that the magnetic field exhibits the wave dispersion through weakly transverse and longitudinal direction in the ZK equation. Sayed et al. [9] obtained the time-fractional KdV equation by using the RPT for EASWs in quantum plasma of two different temperature isothermal ions. In some physical phenomena [10-12], the effect of external periodic force is present. Sen et al. [13] obtained the FKdV equation from a plasma model. They considered the forcing term as a source of charge density scenario. Recently Ali et al. [14] observed an analytical solution for EASWs in the presence of periodic force. Till today no work has been reported in the field of quantum plasma area where the external periodic force is applied. This is the main motivation of this work to noticed the effects of external periodic force on the propagation of EASWs. In this paper our main objective is to derive FKdV equation in EASWs in presence of external periodic force and observed the effects of various physical parameters on the analytical solution of EASWs

The rest of this paper is composed as follows: In Section II, we consider the basic equations. We derived Forced KdV equation using RPT in Section III. In Section IV, we obtain numerical results and present discussions. Section V presents conclusions.

II. BASIC EQUATIONS

The normalized basic equations are [15]

$$\frac{\partial n_{ec}}{\partial t}+\frac{\partial}{\partial x}(n_{ec}v_{ec})=0 \tag{1}$$

$$\frac{\partial v_{ec}}{\partial t}+v_{ec}\frac{\partial v_{ec}}{\partial x}-\frac{\partial \phi}{\partial x}-\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_{ec}}/\partial x^2}{\sqrt{n_{ec}}}\right)=0 \tag{2}$$

$$\frac{\partial n_{eh}}{\partial t}+\frac{\partial}{\partial x}(n_{eh}v_{eh})=0 \tag{3}$$

$$\frac{\partial \phi}{\partial x}-n_{eh}\frac{\partial n_{eh}}{\partial x}+\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_{eh}}/\partial x^2}{\sqrt{n_{eh}}}\right)=0 \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_{ec}+\frac{1}{\delta}(n_{eh}-\delta_1) \tag{5}$$

where n_{ec} and n_{eh} are the number densities, v_{ec} is the cold electron velocity, ϕ is the electrostatic potential, $c_{sh}=\sqrt{\frac{2k_BT_{F_{eh}}}{m_e}}$ is the acoustic speed of quantum hot electron and $\omega_{ec}=\sqrt{\frac{n_{ec0}e^2}{\varepsilon_0 m_e}}$ is the plasma frequency, k_B is the Boltzmann constant, $T_{F_{eh}}$ is the Fermi temperature for hot electron, ε_0 is the permitivity of free space, n_{ec0} is the number density of cold electrons and e, m_e are electronic charge and mass respectively. The non-dimensional quantum parameter H is defined as $H=\sqrt{\delta}\frac{\hbar\omega_{eh}}{2k_BT_{F_{eh}}}$, where $\hbar$

represents Planck's constant and $\omega_{eh} = \sqrt{\frac{n_{eh0}e^2}{\varepsilon_0 m_e}}$ is the hot electron plasma frequency. The neutrality condition is $\delta = \delta_1 - 1$, where $\delta = \frac{n_{ec0}}{n_{eh0}}$, $\delta_1 = \frac{z_i n_{i0}}{n_{eh0}}$, n_{i0} and n_{eh0} are the equilibrium number densities of ions and hot electrons respectively.Considering the boundary conditions $n_{eh} \to 1$, $\frac{\partial n_{eh}}{\partial x} \to 1$, and $\phi \to 0$ at $\pm\infty$ and integrating Eqs.(4) we have

$$\phi = -\frac{1}{2} + \frac{1}{2}n_{eh}^2 - \frac{H^2}{2}\frac{\partial^2\sqrt{n_{eh}}/\partial x^2}{\sqrt{n_{eh}}} \tag{6}$$

III. DERIVATIVE OF FORCED KdV EQUATION

The independent variables are stretched as

$$\xi = \varepsilon^{1/2}(x - \lambda t) \tag{7}$$

$$\tau = \varepsilon^{3/2} t \tag{8}$$

We make the following perturbation expansion of the field variables:

$$\psi = \psi^{(0)} + \sum_{n=1}^{\infty} \varepsilon^{n+1}\psi^{(n)} \tag{9}$$

where $\psi = (n_{ec}, n_{eh}, v_{ec}, v_{eh}, \phi)$ and $\psi^{(0)} = (1,1,0,0,0)$. We get the dispersion relation after equating the lowest order in ε as,

$$\lambda = \sqrt{\delta} \tag{10}$$

We collect the higher order term,

$$\phi^{(2)} = \phi_{\xi}^2(\xi,\tau) \tag{11}$$

$$n_{ec}^{(2)} = -\frac{1}{\delta}(\phi_{\xi}^2(\xi,\tau)) \tag{12}$$

$$n_{eh}^2 = \phi_{\xi}^2(\xi,\tau) \tag{13}$$

$$v_{eh}^{(2)} = \lambda(\phi_{\xi}^2(\xi,\tau)) \tag{14}$$

$$v_{ec}^{(2)} = -\frac{1}{\lambda}(\phi_{\xi}^2(\xi,\tau)) \tag{15}$$

Further we proceed for the next higher order terms and we get the following equations,

$$2\lambda^2 \frac{\partial^2 V_{ec}^{(3)}}{\partial \xi} = \frac{\partial}{\partial \xi}[\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3}] \tag{16}$$

Integrate the above equation w.r.t. ξ and rearranging all the term and we get the well known KdV equation.

$$\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} = 0 \tag{17}$$

where $A = \frac{\lambda}{2}(1 - \frac{3}{\lambda^2})$ and $B = \frac{\lambda^3}{2}[1 - \frac{H^2(1+\lambda^2)}{4\lambda^4}]$.
Now in presence of external periodic force in the system i.e. $f_0 cos(wt)$, then Eq. (17) takes the form

$$\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} = f_0 Cos(wt) \tag{18}$$

heref_0 is the peak value of the external periodic force and w is the frequency of perturbation. The Eq. (18) is known as the FKdV equation. If $f_0 = 0$ then Eq. (18) becomes Eq. (17) and it's solution is given bellow,

$$\phi = \phi_m Sech^2\left(\frac{\xi - M\tau}{W}\right) \tag{19}$$

where $\phi_m = \frac{3M}{A}$ is the amplitude and $W = 2\sqrt{\frac{B}{M}}$ is the width of the wave, M is the normalized constant speed of the wave frame.

Already we know that when $f_0 = 0$, the quantity $I = \int_{-\infty}^{\infty} \phi^2 d\xi$ is a conserved quantity for the FKdV Eq. (18) but in presence of the small external impressed force we get

$$\begin{aligned}\frac{dI}{d\tau} = \int_{-\infty}^{\infty} 2\phi \frac{\partial \phi}{\partial \tau} d\xi = \int_{-\infty}^{\infty} 2\phi (f_0 Cos(wt) - A\phi \frac{\partial \phi}{\partial \xi} - B\frac{\partial^3 \phi}{\partial \xi^3}) d\xi \\ = 2f_0 Cos(wt)\int_{-\infty}^{\infty} \phi d\xi\end{aligned} \tag{20}$$

$$\frac{dI}{d\tau} - 2f_0 Cos(wt)\int_{-\infty}^{\infty} \phi d\xi = 0 \tag{21}$$

$$\frac{dI}{d\tau} = \frac{24 f_0 \sqrt{B}}{A}\sqrt{M(\tau)} cos(w\tau) \tag{22}$$

Now,

$$I=\int_{-\infty}^{\infty}\phi^2 d\xi \tag{23}$$

$$I=\int_{-\infty}^{+\infty}\phi_m^2(\tau)Sech^4\left(\frac{\xi-M(\tau)\tau}{W(\tau)}\right)d\xi \tag{24}$$

$$I=\frac{24\sqrt{B}}{A^2}M^{\frac{3}{2}}(\tau) \tag{25}$$

From Eqs. (22) and (26), the value of $M(\tau)$ is calculated as,

$$M(\tau)=M+\frac{2Af_0}{3w}sin(w\tau) \tag{26}$$

So the analytical solution of EASWs for the FKdV Eq. (18) is,

$$\phi=\phi_m Sech^2\left(\frac{\xi-M(\tau)\tau}{W(\tau)}\right) \tag{27}$$

here $\phi_m(\tau)=\frac{3M(\tau)}{A}$ and $W(\tau)=2\sqrt{\frac{B}{M(\tau)}}$.

IV. NUMERICAL RESULTS AND DISCUSSION

For small amplitude analysis we have plotted a number of graphs. The different physical parameters such quantum diffraction parameter (H), speed of travelling wave (λ), peak value of frequency (f_0) and the frequency (w) of external perturbation on the solitary wave solution of the FKdV (Eq. (18)) have been observed critically in this section. Now using the above parameters we have plotted various solitary wave profile.

We have plotted the solitary wave profile (Fig. 1) i.e. variation in the solitary wave solution of the FKdV (Eq. (18)) for different values of strength of the periodic force (f_0) with fixed values of other parameters $w=0.5$, $H=0.2$, $\lambda=1$, $\tau=1$ and $M=2$. It is seen that when the strength of the force increases, amplitude of the EASWs decreases but width increases.

Variation of the solution of EASWs of the FKdV equation with quantum diffraction parameter H is presented in Fig. 2 and other parameters are same as in Fig. 1. It is critically observed that when H increases, the amplitude of EASWs is near about same but width of the solitary wave decreases. In Fig. 3 we have plotted the variation of the solitary wave solution of the FKdV equation for different values of the frequency (w) of the external periodic force and other parameters are same as in Fig. 1. It is

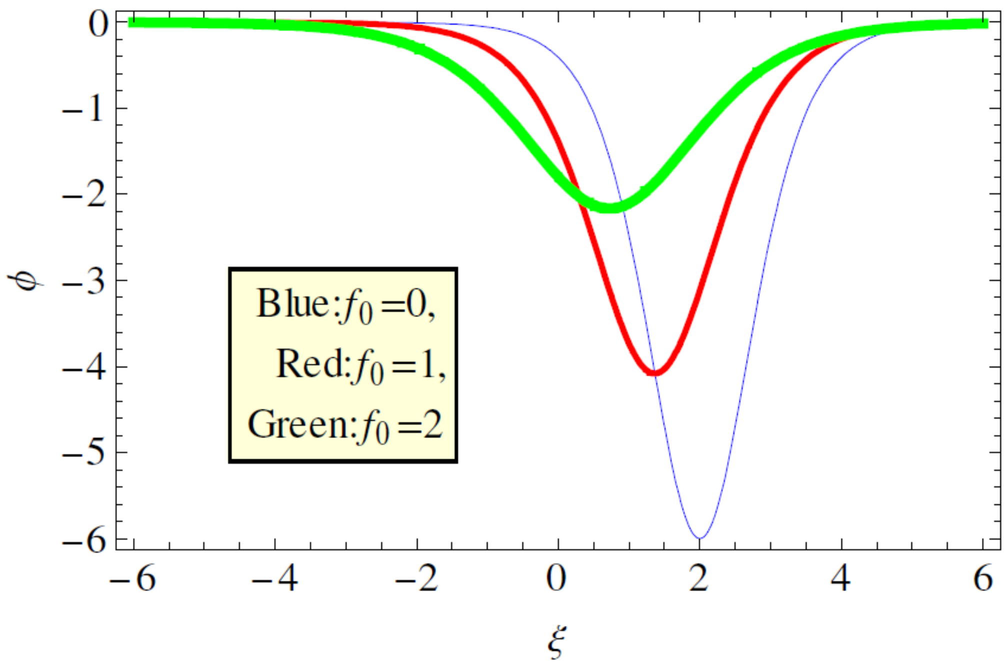

FIGURE 1. Variation in the solitary wave solution of the forced KdV equation (18) for different values of f_0 with λ=1, H=0.2, M=2, w=0.5, and τ=1.

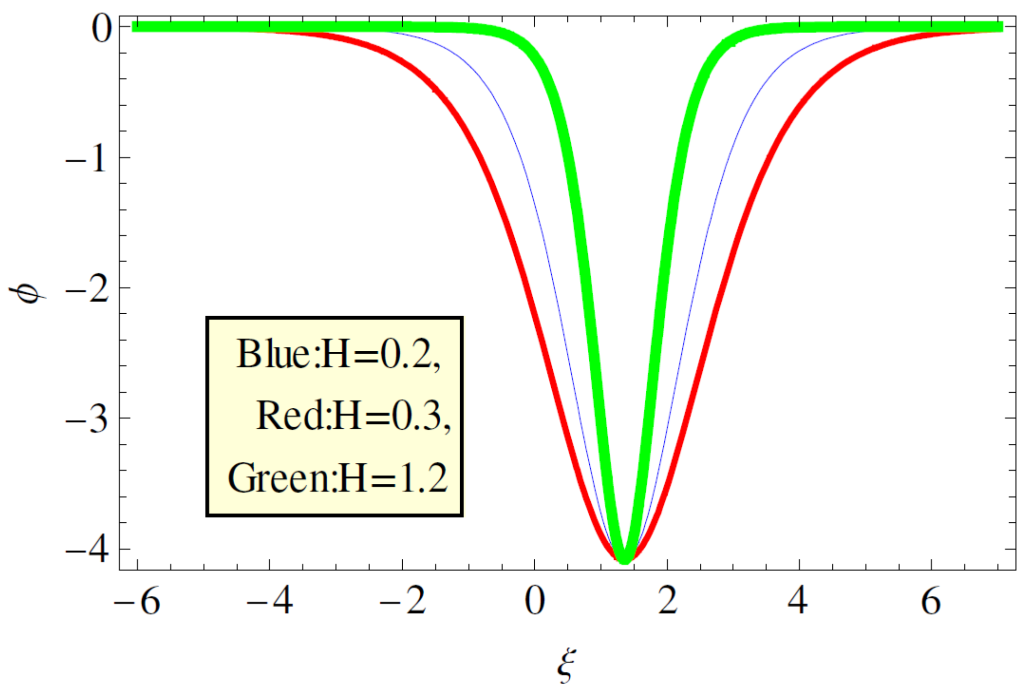

FIGURE 2. Variation in the solitary wave solution of the forced KdV equation (18) for different values of H with λ=1, f_0 =1, M=2, w=0.5, and τ=1.

critically observed that when $w = 2.80$ then the amplitude of solitary wave is large

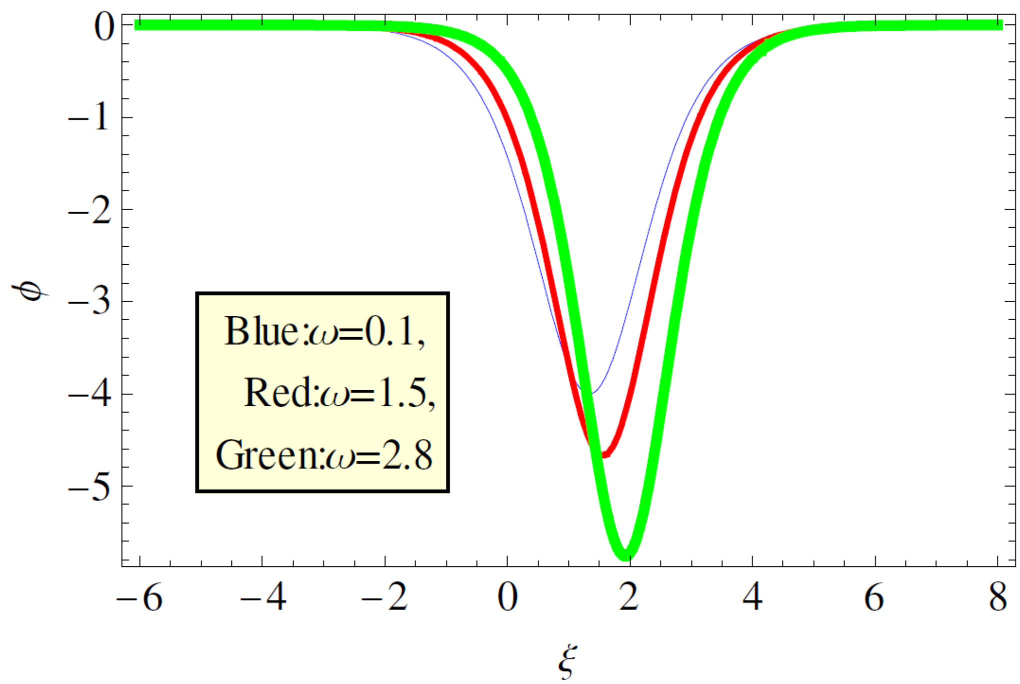

FIGURE 3. Variation in the solitary wave solution of the forced KdV equation (18) for different values of w with λ=1, H=0.2, f_0=1,M=2, and τ=1.

and when $w = 0.10$ then the amplitude of solitary wave is small. So it is found that when w increases then the amplitude and width of the EASWs increases. In Fig. 4, we have plotted the variation of the solitary wave solution of the FKdV (Eq. (18)) for different values of λ with other parameters are same as in Fig. 3. It is observed that when the parameter λ amplitude of EASWs increases, but the width of the solitary wave decreases. The variation of the amplitude of the solitary wave solution w.r.t. the various values of frequency of the external periodic force is depicted in Fig. 5 and other parameters are same as in Fig. 4. It is critically observed that when $w = 0.10$, EASWs has larger amplitude than $w = 0.80$ and $w = 1.50$. So it is found that when w increases then the amplitude of the solitary wave decreases rapidly.

In Fig. 6, the variation of amplitude of the solitary wave solution of the forced KdV (Eq. (18)) with respect tof_0 is presented for different values of λ and the other parameters are same as in Fig. 5. Here we noticed that when $\lambda = 0.40$, the EASWs has larger amplitude but when $\lambda = 1.50$ then it has smallest amplitude comparable to the values of $\lambda = 0.80$ and 0.40. So when λ increases, the amplitude of EASWs decreases rapidly. In Fig.7, we have plotted the variation in the width of the EASWs solution with respect f_0 of the forced KdV (Eq. (18)) is presented for different parameters of w, and other parameters are same as in Fig. 6. It is noticed that as the parameter w increases, the width of the EASWs increases rapidly.

In Fig. 8, the variation in the width of the solitary wave solution with respect f_0 of the forced KdV (Eq. (18)) is presented for different values of quantum diffraction

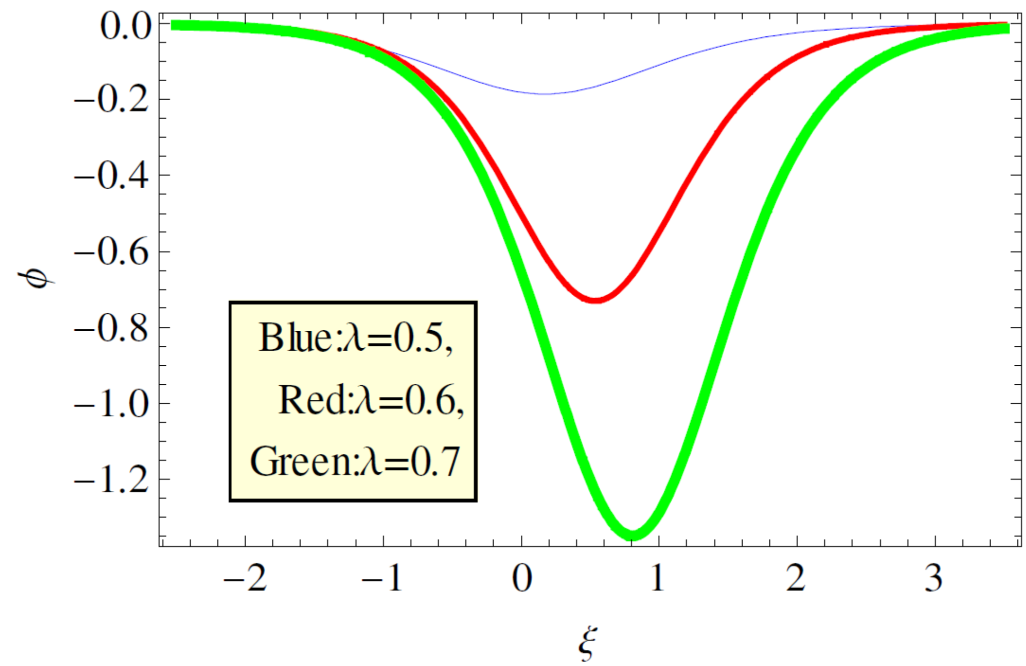

FIGURE 4. Variation in the solitary wave solution of the forced KdV equation (18) for different values of λ with H=0.2, f_0 =1, M=2,w=0.1, and τ=1.

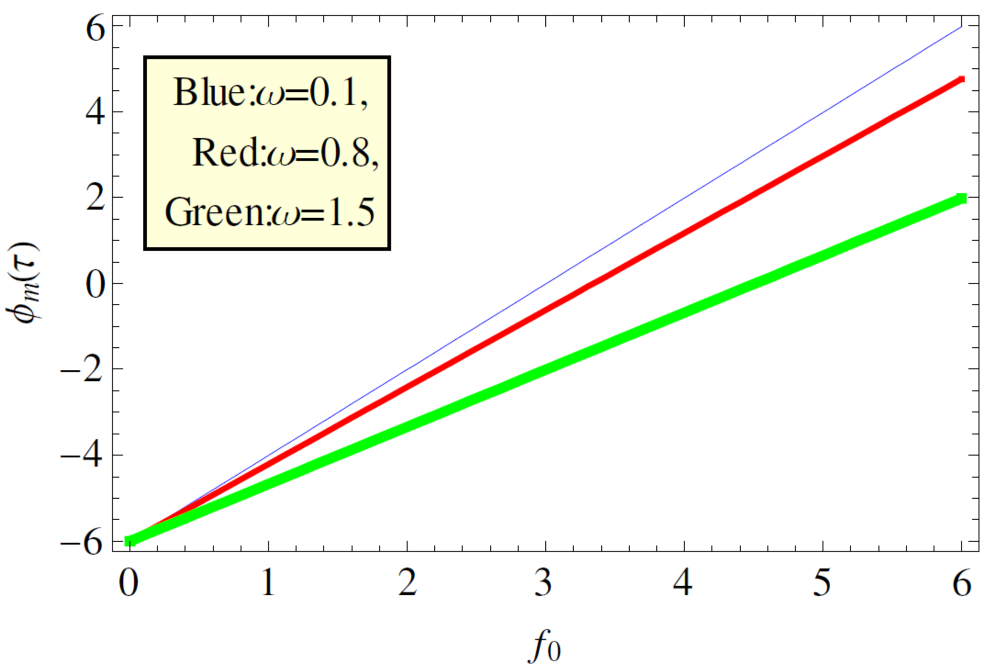

FIGURE 5. Variation in the amplitude of solitary wave solution of the forced KdV equation (18) for different values of w with respect to f_0 for λ=1, H=0.2, M=2, and τ=1.

and the other parameters are same as in Fig. 7. We have noticed that when the quantum parameter H increases the width of the EASWs increases rapidly. In Fig. 9, the variation in the width of the EASWs solution with respect to f_0 of the forced KdV

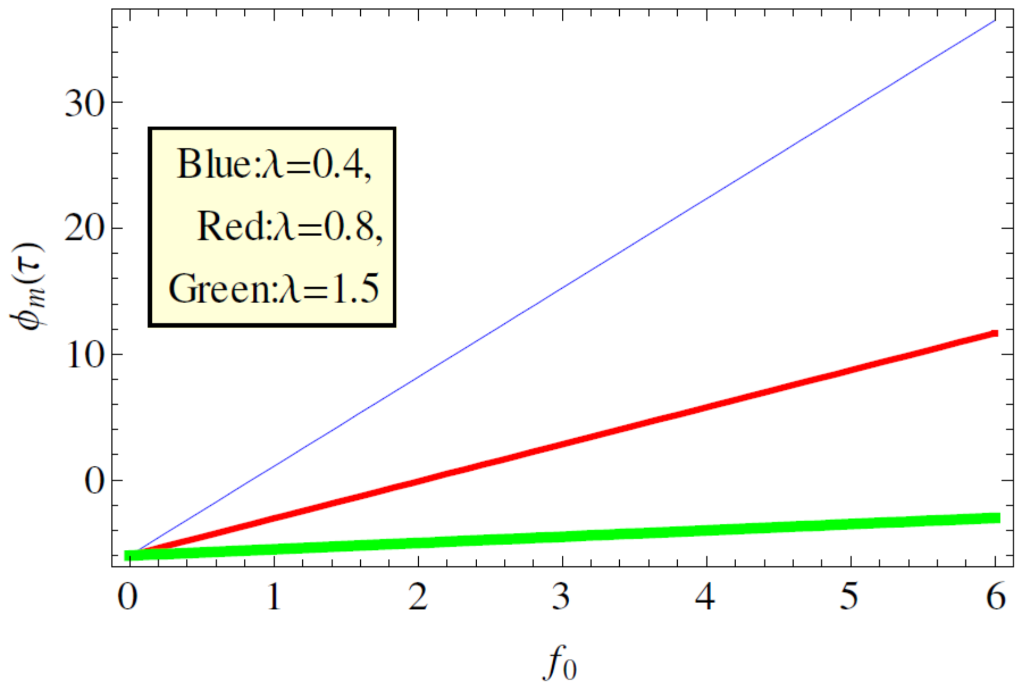

FIGURE 6. Variation in the amplitude of solitary wave solution of the forced KdV equation (18) for different values of λ with respect to f_0 for H=0.2, M=2, w=0.1, and τ=1.

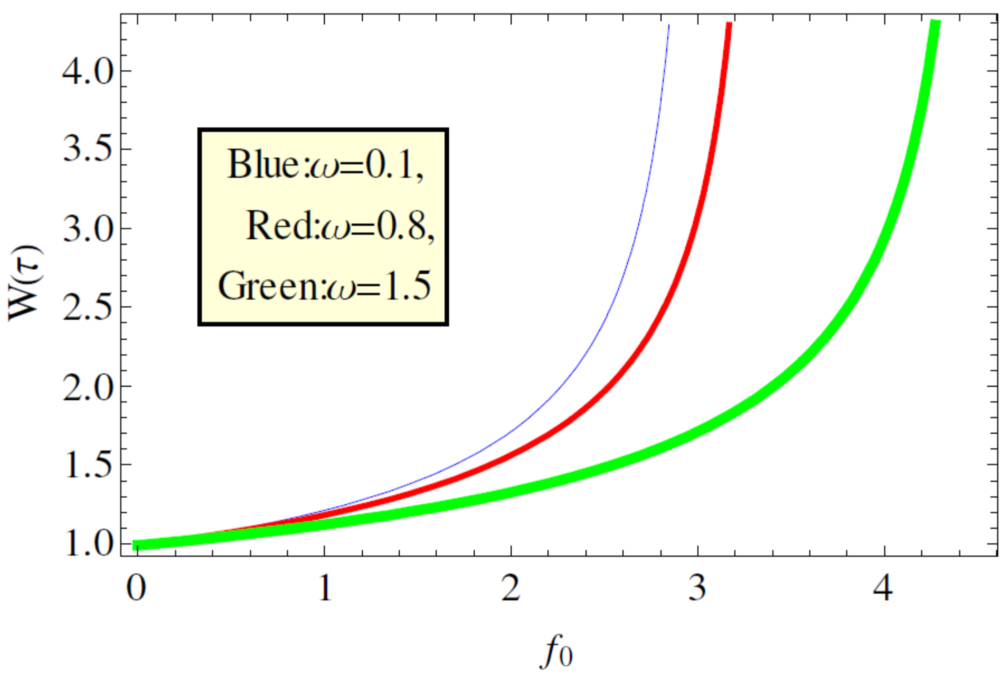

FIGURE 7. Variation in the width of solitary wave solution of the forced KdV equation (18) for different values of w with respect to f_0 for λ=1, H=0.2, M=2, and τ=1.

(Eq. (18)) is presented for different values of λ and the other parameters are fixed as in Fig. 8. It is observed that after crossing the value $f_0 = 0.75$, then the width of the

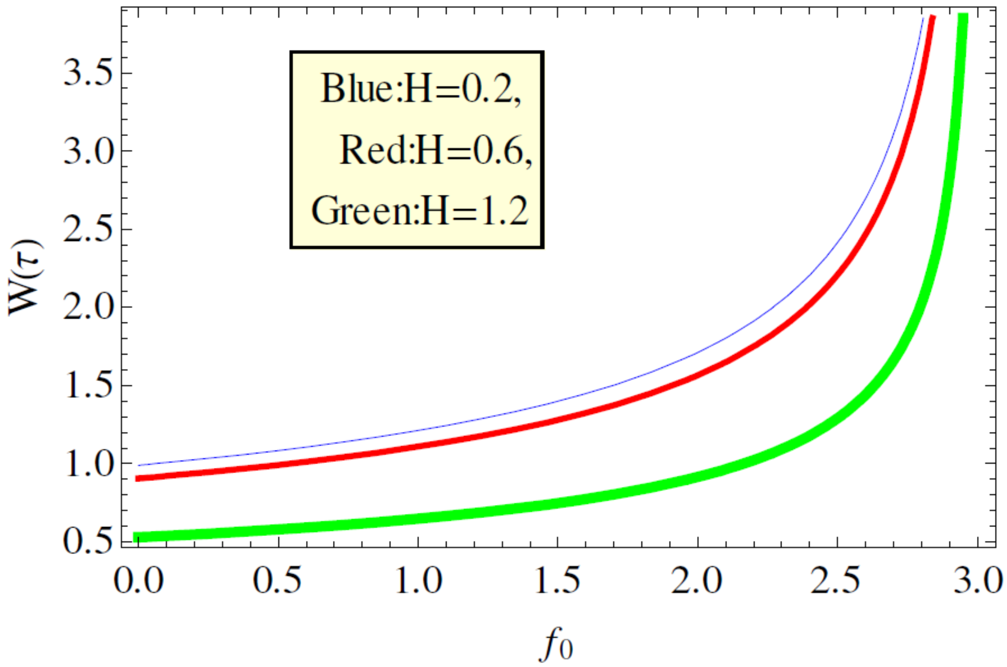

FIGURE 8. Variation in the width of solitary wave solution of the forced KdV equation (18) for different values of H with respect to f_0 for λ=1, M=2, w=0.1, and τ=1.

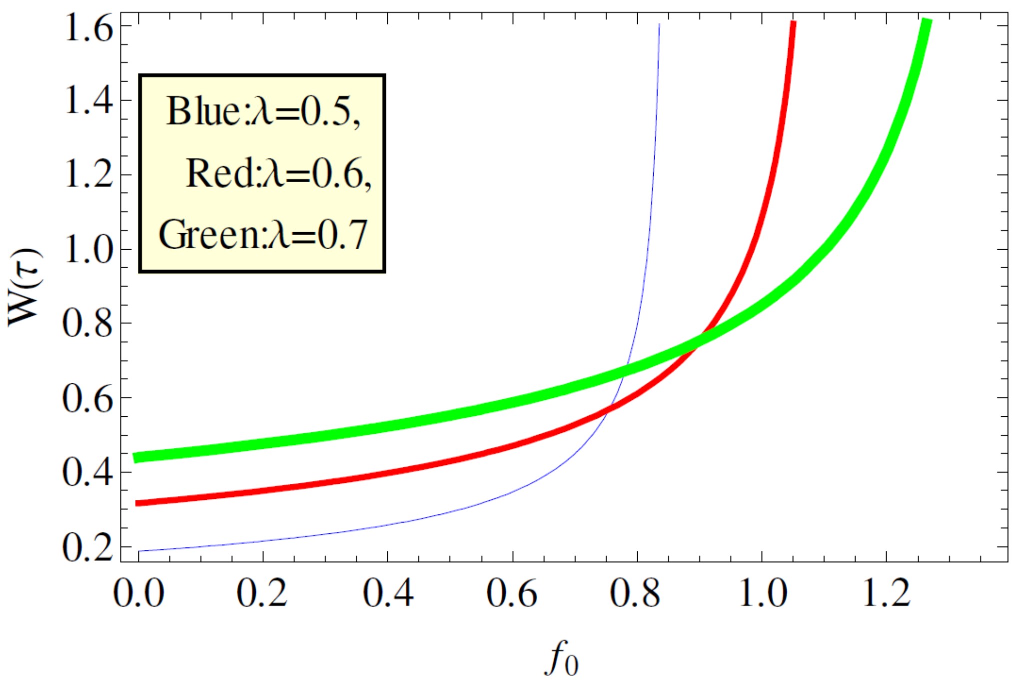

FIGURE 9. Variation in the width of solitary wave solution of the forced KdV equation (18) for different values of λ with respect to f_0 for H=0.2, M=2, w =0.1, and τ=1.

EASWs ($\lambda = 0.50$) profile decrease rapidly than $\lambda = 0.60$ and $\lambda = 0.70$ and at the point $f_0 = 0.90$ the solitary wave decreases rather than the value of $\lambda = 0.70$. So we have seen that when λ increases, the width of the EASWs increases rapidly. Thus, the FKdV equation gives a good estimate of the non linear structures in EASWs in quantum plasma, in the presence of external periodic force.

V. CONCLUSION

In this PAPER we have theoretically investigated the analytical solution of EASWs in quantum plasma in presence of external periodic force. Using RPT we have derived FKdV equation. The effect of different physical parameters such as the speed of traveling wave (λ),strength (f_0) and frequency (w)of the external perturbation have been investigated on the analytical solution of the EASWs. The graphs measures the significant effect of the relevant physical parameters. At last the theoretical result obtained from this paper may be useful to investigate the non linear phenomena of EASWs in quantum plasma in the presence of external periodic force.

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