Effect of ion temperature on arbitrary amplitude quantum dust ion acoustic solitary waves

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(Received 20 August 2010)

Using Sagdeev’s pseudopotential technique, the effect of ion temperature and dust density on the arbitrary amplitude dust ion acoustic solitary waves (DIASWs) in quantum electron-ion (e-i) plasma is studied. The effect of ion temperature and dust density on the region of existence, as well as on the shape of the solitary waves are investigated. It is also shown that the quantum parameter $H$ does not play any role in determining the region of existence and on the amplitude of the large amplitude solitary wave. However, $H$ has significant effect on the width of the solitary wave. It is worth noting that our results are in agreement with previous investigations when the effect of ion temperature is neglected.

I. INTRODUCTION

The study of linear and nonlinear wave phenomena in plasma [1,2] is one of the most important areas in present day plasma research. Study on solitary wave, both theoretically and experimentally, is one such wave studies. Small amplitude solitary waves and double layers are mainly studied using the reductive perturbative technique (RPT). On the other hand, Sagdeev’s pseudopotential technique [3] is employed to study large amplitude solitary waves and double layers.

Quantum plasma research has gained much attention because of its applications in different areas like the fabrication of semiconductor devices [4], quantum dots and quantum wires [5], quantum wells, carbon nanotubes and quantum diodes [6], ultra-cold plasmas [7], micro-plasmas [8], biophotonic [9], intense laser-solid plasma experiments [10], in the studies of superdense astrophysical bodies [11] such as white dwarfs, neutron stars, etc. Wave phenomena in quantum plasma have also been investigated by several authors to understand and investigate the new features. Shokri and Rakhadze [12] studied quantum drift waves using kinetic theory approach. Drift solitons and shocks in quantum magnetoplasma were investigated by Haque and Masood [13]. The effect of kinetic viscosity in e-i quantum plasma had been studied by Sahu and Roychoudhury [14]. Later Roy et al. [15] extended their study in an electron- positron-ion (e-p-i) quantum plasma. Ion acoustic and electron acoustic solitary waves and double layers were studied by several authors [16-18]. Hass et al. [19] studied one-dimensional and nonlinear properties of ion-acoustic waves (IAWs) in a collisionless unmagnetized quantum plasma by incorporating the Bohm potential and the Fermi-Dirac pressure distribution. Misra and Bhowmik [20] studied the nonlinear ion-acoustic waves in a quantum plasma in a non-planar geometry in the frame work of Kodomstev-Petriaville (KP) equation. Moslem et al. [21] studied the IAWs in quantum magnetoplasma in the frame work of quantum-Zakharov-Kuznetsov (QZK) equation. Mahmood and Masood [17] had studied the electron acoustic solitary waves in a two electron quantum plasma. Misra and Samanta [18] had also investigated electron acoustic solitary waves and double layers in the small amplitude limit.

Wave phenomena in dusty plasma also get tremendous importance after the investigation of dust acoustic waves, dust ion acoustic waves and dust lattice waves. Dust acoustic solitary waves (DASWs) and dust ion acoustic solitary waves (DIASWs) are also investigated by considering the quantum corrections. Mustaq [22] studied the quantum DASWs in a non-planar geometry in the framework of cylindrical KP equation. Masood et al. [23] studied the quantum dust IASWs in the framework of Korteweg-de Vries (KdV) equation.

But all the studies mentioned above are based on RPT and hence are valid for small amplitude waves only. In order to understand the large amplitude (arbitrary amplitude) solitary wave one has to consider the total nonlinearity of the system and Sagdeev’s pseudo-potential approach is such possible approach which has been applied in different plasma models [24]. Recently in the studies of quantum plasma, Sagdeev’s pseudopotential technique has been used by a several authors. Mahmood and Mustaque [25] had recently studied the ion acoustic solitary waves in unmagnetized quantum e-i plasma. They had shown that the amplitude of the rarefactive solitary waves remained the same but the width became wider with the quantization. Ali et al. [26] had investigated the propagation of IAWs in a collisionless magnetized plasma propagating obliquely against the external magnetic field. They examined the effects of Mach number and the angular dependence on the solitary waves but nevertheless, had neglected the
effect of ion temperature in their calculations. Mahmood and Masood [27] had also investigated the electron acoustic solitary waves in an unmagnetized two-electron quantum plasma using the same technique. Misra et al. [28] studied the large amplitude solitary wave propagation in electron positron plasma. Solitary waves in quantum dusty plasma is also studied by Tribich et al. [29]. But none of the above studies considered the effect of ion temperature except recently Chatterjee et al. [30] had studied the effect of ion temperature on arbitrary amplitude solitary waves in quantum electron ion plasma.

In this paper, we study the effect of ion temperature on the existence of solitary waves in dusty e-i quantum plasma. The organization of the paper is as follows. In Section II the basic equations are given and Sagdeev’s pseudopotential is derived. The results are given in Section III, while Section IV contains the discussions and conclusions.

II. BASIC EQUATIONS

Following the quantum hydodynamic model, we consider the homogeneous and unmagnetized dust-electron-ion quantum plasma consisting of inertia-less electrons. In order to study the effect of ion temperature on the DIAWs in quantum plasma, the ions are taken to be dynamic. The dust grains are considered to be immobile and negatively charged, \( q_d = -Z_d e \), where \( Z_d \) is the number of charges residing on the dust grain.

The basic equations describing the nonlinear dynamics of the DIAWs in the quantum plasma system are

\[
0 = \frac{\partial \phi}{\partial x} - n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2} \frac{\partial}{\partial x} \left( \sqrt{n_e} \frac{\partial \sqrt{n_e}}{\partial x} \right),
\]

\[
\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = - \frac{\partial \phi}{\partial x} - 3\sigma n_i \frac{\partial n_i}{\partial x},
\]

\[
\frac{\partial n_i}{\partial t} + v_i \frac{\partial n_i}{\partial x} = 0,
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = \frac{n_e}{\mu} - n_i + \left( 1 - \frac{1}{\mu} \right).
\]

The densities \( n_e \) and \( n_i \) are normalized by \( n_{e0} \) and \( n_{i0} \), respectively. The quantum ion-acoustic velocity \( C_s = (2k_B T_e/m_e)^{1/2} \), and \( (2k_B T_e/e) \), where \( k_B \) is the Boltzmann constant and \( T_e \) is the electron Fermi temperature and \( e \) is the charge of the electron. \( x \) and \( t \) are normalized by \( C_s/\omega_{pi} \) and \( \omega_{pi}^{-1} \) = \( (m_e 4\pi n_{i0} e^2)^{1/2} \) respectively. \( \mu = \frac{n_{i0}}{n_{e0}} = 1 + \frac{Z_d n_{e0}}{n_{i0}} \) measures the unperturbed ion and electron density imbalance. Meanwhile, \( H = \hbar \omega_{pe}/(2k_B T_e) \) is the quantum parameter determining the ratio between the electron plasmon energy and the electron Fermi energy. \( \phi \) is the electrostatic wave potential and is normalized by \( 2k_B T_e/\epsilon \). \( p_e \) represents the pressure effect and \( \sigma = T_i/T_e \). At equilibrium, we have \( -n_{i0} + n_{e0} + \left( 1 - \frac{1}{\mu} \right) n_{d0} \). We assume that the electrons obey the equation of state pertaining to a one-dimensional zero temperature Fermi gas, thus

\[
p_e = \frac{m_e v_F e^2}{2n_e^2}.
\]

Here \( n_e \) is the equilibrium density and \( v_F \) is the Fermi velocity, which is related to the Fermi temperature \( T_F \) by \( m_e v_F^2/2 = k_B T_F \), where \( k_B \) is the Boltzmann’s constant.

Using quasi-neutrality condition we have

\[
n_e = \left( 1 - \frac{1}{\mu} \right) n_i + n_e = n. \quad (6)
\]

For localized stationary solution, we assume that all dependent variables are function of a single independent variable \( \zeta = x - Mt \), where the Mach number \( M \) is defined as \( M = V/C_s \) and \( V \) is the velocity of the nonlinear structure moving with the frame. By integrating Eq. (1) once, with the boundary condition \( n_e \rightarrow 1 \) and \( \phi \rightarrow 0 \) at \( \zeta \rightarrow \pm \infty \), we obtain

\[
\phi = -\frac{1}{2} + \frac{n_{e0}^2}{2} - \frac{\mu H^2}{2} \frac{\partial^2}{\partial n_e \partial x^2} \sqrt{n_e}.
\]

From the ion continuity Eq. (3) and the ion momentum Eq. (2) with the boundary condition \( \phi \rightarrow 0 \), \( v_i \rightarrow 0 \) and \( n_i \rightarrow 1 \) at \( \zeta \rightarrow \pm \infty \), we get

\[
\phi = \frac{M^2}{2} \left( 1 - \frac{1}{n_i^2} \right) + \frac{3\sigma}{2} \left( 1 - n_i^2 \right).
\]

Now, by using quasi-neutrality condition (6) and also substituting \( Z = \sqrt{n} \) in Eq. (7), we get

\[
\frac{H^2}{2} \left( \frac{\partial^2 Z}{\partial \zeta^2} \right) = \frac{Z}{2} - \frac{M^4 Z}{2} \left( 1 - \frac{1}{\mu} + \frac{Z^2}{\mu} \right) + \frac{3\sigma Z}{2} \left( 1 - \left( \frac{1 - 1}{\mu} + \frac{Z^2}{\mu} \right)^2 \right).
\]

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Multiplying both sides of Eq. (9) by $\frac{d^2n}{d\xi^2}$ and integrating once with the boundary conditions

$$(\frac{d^2n}{d\xi^2}) \rightarrow 0, \quad (\frac{dn}{d\xi}) \rightarrow 0 \text{ and } n \rightarrow 0 \text{ as } \xi \rightarrow |\pm\infty|,$$

we obtain the nonlinear differential equation in terms of density as follows:

$$\frac{1}{2} \frac{d}{d\xi} \left( \frac{dn}{d\xi} \right)^2 + U(n) = 0 \tag{10}$$

where the Sagdeev potential $U(n)$ is defined as

$$U(n) = -\frac{8n}{\mu H_0^2} \left[ n^2 - \frac{1 + M^2 + 3\sigma}{4} - \frac{n}{4} + \frac{M^2\mu^2}{4(n + \mu - 1)} \right] + \frac{\sigma}{4\mu^2}((n + \mu - 1)^3 - \mu^3). \tag{11}$$

It is seen that the above equation reproduces Eq. (17) of Ref. [29] provided one considers $\sigma = 0$. Eq. (11) is the well-known Sagdeev’s pseudopotential equation which can be treated as an energy integral of a particle of unit mass moving with velocity $(\frac{dn}{d\xi})$ at position $n$ in a pseudopotential well $U(n)$. The nature of $U(n)$ will then determine the conditions for the existence of solitary wave solution. If between any two roots ($n = 1$ and $n = n_0$) of $U(n)$, $U(n)$ is negative, and if one root is a single root and another is a double root, then a solitary wave is found (see Ref. [2]). Here the initial conditions are so chosen that the double root appears at $n = 1$. Hence, the conditions for the existence of soliton solution are

(i) $U(n) = 0$ at $n = 1$ and $n = n_0$,

(ii) $\frac{du}{dn}|_{n=1} = 0$, but $\left. \frac{du}{dn} \right|_{n=n_0} \neq 0$,

(iii) $\left. \frac{d^2u}{dn^2} \right|_{n=1} < 0$.

The condition (iii) will be $(M^2 - \mu - 3\sigma) < 0$ or $M^2 < (\mu + 3\sigma)$. So if $\mu > (1 - 3\sigma)$ then both subsonic and supersonic solitary waves exist and if $\mu < (1 - 3\sigma)$ then only subsonic solitary wave exists. This indicates the modification of the result obtained by Ref. [29] provided one include the ion temperature in consideration. And also the maximum possible value of $\sigma$ is $(\mu - M^2)\frac{1}{2}$ beyond which $M$ will be complex, i.e. no soliton solution exist, provided the other two conditions hold. Moreover, if $n_m > 1$ then the wave is called compressive solitary wave and if $n_m < 1$, the solitary wave is called the rarefactive solitary wave.

Also from Eq. (10), it is seen that the shape of the solitary wave can be determined from the following relation

$$\xi = \pm \int_{\xi_0}^{\xi} \frac{dn}{\sqrt{-2U(n)}} \tag{12}$$

III. RESULTS AND DISCUSSIONS

The effect of ion temperature on the existence of solitary waves can be seen from Fig. 1. The line $(M^2 - 3\sigma - \mu) = 0$ is plotted in $M-\sigma$ space for different values of $\mu = 2, 1, 0.5$. The region in between $M$-axis, $\sigma$-axis and the said line is the region of existence of solitary waves for this model. It is seen from the Figure that $\mu$ reduces the region significantly. In Fig. 2 $U(n)$ is plotted against $n$ for different values of $\sigma$ viz., $\sigma = 0$ (dashed), $\sigma = 0.1$ (solid), and $\sigma = 0.3$ (dotted). Other parameters are $M = 0.6$, $H = 2.0$ and $\mu = 1.25$. From the Figure, it is shown that for $\sigma_1 = 0$, $U(n)$ satisfies all three conditions for the existence of solitary waves. Hence, solitary waves can exist for $\sigma = 0$. Sagdeev potential $U(n)$ crosses the $n$ axis at $n = n_m = 0.2334$, which is less than 1.

FIG. 1. The line $(M^2 - 3\sigma - \mu) = 0$ is plotted in $M-\sigma$ space for different values of $\mu$ viz. $\mu = 2$ (solid), 1 (dashed), 0.5 (dotted).
FIG. 2. $U(n)$ is plotted against $n$ for different values of $\sigma$ viz. $\sigma = 0$ (dashed), $\sigma = 0.1$ (solid), and $\sigma = 0.3$ (dotted). Other parameters are $M = 0.6$, $H = 2.0$ and $\mu = 1.25$.

Hence we have a rarefractional soliton for $\sigma = 0$. Furthermore, it can be seen from the Figure that with the increase of ion temperature $\sigma$, the amplitude of the solitary wave increases. Finally, the maximum value of $\sigma$ will be 0.298. Therefore, for $\sigma > 0.298$, $U(n)$ does not satisfy the condition (iii) for existence of soliton. That means no solitary waves exists for $\sigma = 0.3$. The effect of finite ion temperature on the shape of rarefraction soliton waves is deduced from Fig. 3 which is drawn using Eq. (12). In this Figure, $n$ is plotted against $\xi$ for different values of $\sigma$ viz. $\sigma = 0$ and 0.1. It can be noticed that with the increase of $\sigma$, the amplitude of the solitary wave decreases but the width increases. Hence, the ion temperature has a significant effect on the shape of the rarefraction soliton waves. Fig. 4 shows the Sagdeev potential $U(n)$ plotted against $n$ for different values of $H$ viz. $H = 2$ (dotted), 3 (dashed) and 5 (solid) with $\sigma = 0.1$ and $\mu = 1.25$. Although $U(n)$ crosses the $n$-axis at a fixed point for different values of $H$, the shape of the Sagdeev potential is different. Furthermore, it is noticed that despite the increase of $H$, the amplitude of the solitary wave remains the same but with the increase of $H$, the width of the solitary wave is found to increase significantly. Overall, the ratio of initial ion density to initial electron density $\mu$, ion temperature $\sigma$, quantum parameter $H$ and Mach number $M$ all have significant effects on the shape of the rarefraction soliton waves (see Ref. [25]).

IV. CONCLUSIONS

The effect of ion temperature on the existence of large amplitude solitary wave in quantum d-e-i plasma has been studied using Sagdeev’s pseudopotential approach. It has been shown that both supersonic and subsonic solitary waves can exist for this system. For rarefraction soliton waves, our results are found to be in agreement with the investigation of Tribeche et al. [29] for zero ion temperature. Mach number, ion temperature and quantum parameter are found to play significant roles in determining the shape of solitary waves in quantum plasmas.

ACKNOWLEDGEMENTS

One of the authors (PC) is grateful to the University of Malaya for the award of Visiting Senior Research Fellow to enable him to participate in this work.
FIG. 3. \( n \) is plotted against \( \xi \) for different values of \( \sigma \) viz. \( \sigma = 0 \) and 0.1. Other parameters are \( M = 0.6 \), \( H = 2.0 \) and \( \mu = 1.25 \).

FIG. 4. \( U(n) \) plotted against \( n \) for different values of \( H \) viz. \( H = 2 \) (dotted), 3 (dashed) and 5 (solid) with \( \sigma = 0.1 \) and \( \mu = 1.25 \).

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