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# Interaction of two soliton solution (2SS) of Generalised Kadomstev-Petviashvili (KP) equation in magnetised dust-ion-acoustic solitary waves (DIASWs) with Kappa electron velocity distribution

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**Abstract.** Using reduction perturbation technique (RPT) KP equation of magnetised DIASWs are reported in a complex plasma model. Also using Hirota's Bilinear method (HBM) 2SS of KP equation is obtained. It has been observed that various physical parameters i.e. density ratio of ions to electrons ($\alpha$)and spectral index (k) play significant role in the propagation and interaction of 2S. This study may be helpful to interpret the nonlinear characteristics of magnetised dusty plasma waves.

**Keywords:** Magnetised DIASWs, RPT, KP equation, HBM.

## I. Introduction

DIASWs are currently being considered as a major interdisciplinary research field in plasma physics. Many mathematical analysis and practical investgations on this subject can be found in the literatur[1-7]. Plasma wave [8-11] involves a number of eigan modes, such as dust-ion acoustic solitary waves (DIASWs)[12], the dust-acoustic waves(DAWs) [13], dust lattice waves (DLWs) [14], Shukla-Varma mode [15], dust-cyclotron mode [16] and dust-Berstain-Green-Kruskal mode [17] etc. Laboratory based practical observations [18-20] have confirmed the existence of non-linear characteristics of DAs and DIASWs. Theoretically, Shukla and Silin [21] have first noticed that low frequency DIASWs contains dusty plasma with much smaller electron thermal speed. They derived KdV-Burger equation with shock waves which is generated through ion-viscosity dissipation damping. Nakamura et al. [22] derived Burger equation to observe dissipation of DIASWs. Ghosh et al. [23] noticed the shock structure of DIA waves in collisionless plasma and they derived the KdVB equation. Xue [24] showed the influence of bounded non planar geometry on DIASWs in an unmagnetised plasma. They used the RPT and obtained cylindrical/spherical KdVB equation and observed how the sturucture change due to dust density and temperature. Alinejad [25] studied theoretically the 1D dynamics of nonlinear DIASWs in an unmagnetized dusty plasma with the effect of electrostatic nature. The fundamental characteristics of DIASWs are studied by KdV and KdVB eqautions respectively. Xiao et al. [26] investigated the evolution of nonlinear DIASWs in the plasma which is inhomogeneous. They derived the MKdVB equation using higher order correction through RPT. They showed that the various plasma parameter has a significant effect on the propagation and interaction of DIASWs. Using the non extensivity of the elctron Bacha et.al. [27] studied the DIASWs in dusty plasma. Das et al. [28] derived the DZK equation of DIASWs. They showed that the entire plasma system exhibits quasiperiodic behaviour in absence of collision and system became chaotic when collision present. Moslem et al. [29] derived Burger equation for DIASWs in a magnetised plasma. Seadawy et al. [30] derived three dimension ZKB equation for DIASWs. Using RPT Das et al. [31] also derived the DKdV equation and observed the periodic structure at critical values. Paul et al. [32] derived DKdVB equation to study the DIASWs for non extensive plasmas. Gao et al. [33] derived the cylindrical KP equations for cosmic supernova shells and Saturn's F-ring in DIASWs. Gao et al. [34] also derived (3+1)-dimensional KP equation in spherical geometry with both azimuthal and Zenith perturbations with symbolic computation in DIASWs. The generalized Lorentzian velocity distribution [35] or Kappa distribution is $f_0(v) = (1+\frac{v^2}{(k/\theta^2)})^{-(k+1)}$ , where $v = v_x^2+v_y^2+v_z^2$, $\theta$ is effective thermal speed related to usual thermal speed given by $\theta^2 = [\frac{(k-3/2)}{k}]v_{th}^2$ and k is known as a spectral index. The spectral index k is a measure of the slope of energy spectrum of superthermal particles forming the tail of the velocity distribution. The physics of Lorentzian distribution has attracted significant attention due to its applications to describe plasmas which are far from thermal equilibrium, for examples, in the planetary magnetospheres, solar wind, whistler emission of Jupiter, and interstellar medium. The presence of a substantially larger number of superthermal particles, which distinguishes kappa distribution from a Maxwellian, can significantly change the rate of resonant energy transfer between particles and plasma waves [36].

Many basic theoretical and practical problems of DIASWs at some critical scenario are still the subject of plasma research field. Till today no work has been reported to the derivation KP equation in this particular mathematical model in a three component magnetised DIASWs contains cold inertial ions, negatively charged stationary dust particles obeying the Kappa (k)distributed electrons. This is the main intention for following up this work. The organisation of the present manuscript is as follows: we introduce model equation in Section 2. Using RPT small amplitude analysis have discussed and derive the KP equation in Section 3. In Section 4 we obtain 2SS of KP equation using HBM. In Section 5, we present the numerical results and discussion and Section 6 is kept for conclusions.

## II. Model Equations

In this paper , we consider a three component magnetised dusty plasma with kappa (k) distribution. The nonlinear dynamics of magnetised DIASWs are governed by following normalised equations:

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}+\frac{\partial (nw)}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t}+(u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z})u = -\frac{\partial \phi}{\partial x}+v, \tag{2}$$

$$\frac{\partial v}{\partial t}+(u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z})v = -v, \tag{3}$$

$$\frac{\partial w}{\partial t}+(u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z})w = -\frac{\partial \phi}{\partial z}, \tag{4}$$

$$(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2})\phi = \alpha[(1-\frac{\phi}{k-3/2})^{-(k-1/2)}-an+b] \tag{5}$$

where $\alpha=\frac{r_g^2}{\lambda_e^2}$, $a=\frac{n_{io}}{n_{e0}}$ and $b=\frac{n_d z_d}{n_{e0}}$. Here $r_g=\frac{C_s}{\omega}$ is the gyroradius of ion, $\lambda_e=\sqrt{\frac{T_e}{(4\pi n_{e0}e^2})}$ is the Debye length of ion, $C_s=\sqrt{\frac{T_e}{m_i}}$ is the acoustic velocity of ion and $\omega=\frac{eB_0}{m_i c}$ ion gyro frequency. Here in the unperturbed state,$n_{e0}$, $n_{i0}$ are the electron and ion densities, respectively.

## III. Small amplitude analysis and derivation of KP Equation

According to RPT, independent variables are stretched as follows:

$$\xi = \varepsilon(z-\lambda t), \tag{6}$$

$$\eta = \varepsilon^2 x, \tag{7}$$

$$\tau = \varepsilon^3 t. \tag{8}$$

The dependent variables are expanded as follows:

$$n = 1+\varepsilon^2 n_1+\varepsilon^4 n_2+\cdots, \quad (9)$$
$$u = \varepsilon^3 u_1+\varepsilon^5 u_2+\cdots, \quad (10)$$
$$v = \varepsilon^3 u_1+\varepsilon^5 u_2+\cdots, \quad (11)$$
$$w = \varepsilon^2 w_1+\varepsilon^4 w_2+\cdots, \quad (12)$$
$$\phi = \varepsilon^2 \phi^(1)+\varepsilon^4 \phi^(2)+\cdots \quad (13)$$

Substituting the above equations into the system of equations (4)-(8) and we get the coefficient of lower power of $\varepsilon$ and also next higher order of $\varepsilon$ as from Eq. (1),

$$n_1 = \frac{1}{\lambda} w_1 \quad (14)$$

$$\frac{\partial n_1}{\partial \tau} - \lambda \frac{\partial n_2}{\partial \xi} + \frac{\partial w_2}{\partial \xi} + \frac{\partial (n_1 w_1)}{\partial \xi} = 0 \quad (15)$$

$$\frac{\partial u_1}{\partial \eta} = 0 \quad (16)$$

Now from equation (2) we get,

$$v_1 = v_2 \quad (17)$$

$$\frac{\partial u_1}{\partial \tau} - \lambda \frac{\partial u_1}{\partial \xi} = 0 \quad (18)$$

$$\frac{\partial u_1}{\partial \tau} - \lambda \frac{\partial u_2}{\partial \xi} + w_1 \frac{\partial u_1}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \eta} = 0 \quad (19)$$

From equation (3) we get,

$$u_1 = u_2 = 0 \quad (20)$$

$$\lambda \frac{\partial v_1}{\partial \xi} = 0 \quad (21)$$

$$\frac{\partial v_1}{\partial \tau} - \lambda \frac{\partial v_2}{\partial \xi} + w_1 \frac{\partial v_1}{\partial \xi} = 0 \tag{22}$$

Now from equation (4) we get,

$$\frac{\partial \phi^{(1)}}{\partial \xi} - \lambda \frac{\partial w_1}{\partial \xi} = 0 \tag{23}$$

$$\frac{\partial w_1}{\partial \tau} - \lambda \frac{\partial w_2}{\partial \xi} + w_1 \frac{\partial w_1}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} = 0 \tag{24}$$

From equation (5), we get,

$$\alpha \frac{k-1/2}{k-3/2} \phi^{(1)} - \frac{\alpha n_{i0}}{n_{e0}} n_1 = 0 \tag{25}$$

$$\alpha \frac{k-1/2}{k-3/2} \phi^{(2)} + \frac{\alpha (k-1/2)(k+1/2)}{2(k-3/2)^2} \phi^{(1)} - \frac{\alpha n_{i0}}{n_{e0}} n_2 = \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \tag{26}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(2)}}{\partial \xi^2} = \alpha \frac{(k-1/2)(k+1/2)}{(k-3/2)^2} \phi^{(1)} \phi^{(2)} \tag{27}$$

Now we collecting all 1st order equation to obtain the dispersion relation(Eq. 30),

$$\lambda = \frac{w_1}{n_1} \tag{28}$$

$$\lambda^2 = \frac{\phi^{(1)}}{n_1} \tag{29}$$

$$\lambda^2 = a\left(\frac{k-3/2}{k-1/2}\right) \tag{30}$$

Now using the higher coefficient of $\varepsilon$, we obtain KP equation as:

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)\frac{\partial \phi^{(1)}}{\partial \xi}} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3}\right] + C \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} = 0 \tag{31}$$

where $A = \frac{1}{2\lambda}(3 - \frac{k+1/2}{k-1/2} a)$, $B = \frac{\lambda^3}{2\alpha a}$ and $C = \frac{\lambda}{2}$.

## IV. 2S interaction using HBM

HBM is a powerful technique to obtained multi-soliton solution under integrability condition. Here we describes the of 2SS for direct simulation parameters. Let us recapitulate $\xi$ by $\bar{\xi}$, $\phi^{(1)}$ by $-\frac{6B}{A}\bar{\phi^{(1)}}$, $\eta$ by $\sqrt{\frac{C}{3B}}\bar{\eta}$ and $\tau$ by $\frac{1}{B}\bar{\tau}$, then equation (15) is converted to the following standard KP equation:

$$\frac{\partial}{\partial\xi}[\frac{\partial\phi^{(1)}}{\partial\tau}-6\phi^{(1)\frac{\partial\phi^{(1)}}{\partial\xi}}+\frac{\partial^3\phi^{(1)}}{\partial\xi^3}]+3\frac{\partial^2\phi^{(1)}}{\partial\eta^2}=0 \tag{32}$$

In this section we obtain 2SS of KP equation and to observe the propagation and interaction of DIASWs. For this reason, we shall apply HBM. Using the transformation $\bar{\phi}^{(1)}=-2(logf)_{\bar{\xi}\bar{\xi}}$ in the well known KP equation (16), we bilinearize the Eq. (16) as

$$ff_{\bar{\xi}\bar{\tau}}-f_{\bar{\xi}}f_{\bar{\tau}}+ff_{\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}}-4f_{\bar{\xi}}f_{\bar{\xi}\bar{\xi}\bar{\xi}}+3f^2_{\bar{\xi}\bar{\xi}}+3f^2_{\bar{\xi}\bar{\xi}}-3f^2_{\bar{\eta}}=0 \tag{33}$$

Now using the D operator of Hirota, we get $D_{\bar{\xi}}D_{\bar{\tau}}[f.f]=2(ff_{\bar{\xi}\bar{\tau}}-f_{\bar{\xi}}f_{\bar{\tau}})$, $D^4_{\bar{\xi}}[f.f]=2(ff_{\bar{\xi}\bar{\xi}\bar{\xi}\bar{\xi}}-4f_{\bar{\xi}}f_{\bar{\xi}\bar{\xi}\bar{\xi}}+3f^2_{\bar{\xi}\bar{\xi}})$, $D^4_{\bar{\eta}}[f.f]=2(3ff_{\bar{\eta}\bar{\eta}}-3f^2_{\bar{\eta}})$, Using the above Eqs., we get the Hirota Bilinear(HB) form $(D_{\bar{\xi}}D_{\bar{\tau}}+D^4_{\bar{\xi}}+D^2_{\bar{\eta}})[f.f]=0$. Now to develop 2SS, we use the Hirota perturbation and we insert $f=1+\varepsilon(e^{\bar{\theta}_1}+e^{\bar{\theta}_2}+\varepsilon^2f_2)$ into the Hirota bilinear form, where $\bar{\theta}_i=k_i\bar{\xi}+\omega_i\bar{\tau}+l_i\bar{\eta}+\alpha_i$, i=1,2. The coefficient of various powers of $\varepsilon$ will give $\omega_i=-(k_i^4+3l_i^2)/k_i$, $f_2=a_{12}e^{\bar{\theta}_1+\bar{\theta}_2}$ with the phase shift $a_{12}=(k_1w_2+k_2w_1+4k_1^3k_2-6k_1^2k_2^2+4k_1k_2^3+6l_1l_2)/(k_1w_2+k_2w_1+4k_1^3k_2+6k_1^2k_2^2+4k_1k_2^3+6l_1l_2)$.

The two solitons solution of KP equation is,

$$\begin{aligned}\bar{\phi}^{(1)} &= -2(\log f)_{\xi\xi}\\ &= -2[\log(1+e^{\bar{\theta}_1}+e^{\bar{\theta}_2}+a_{12}e^{\bar{\theta}_1+\bar{\theta}_2})]_{\xi\xi}\\ &= [\frac{k_1e^{\bar{\theta}_1}+k_2e^{\bar{\theta}_2}+a_{12}(k_1+k_2)e^{\bar{\theta}_1+\bar{\theta}_2}}{1+e^{\bar{\theta}_1}+e^{\bar{\theta}_2}+a_{12}e^{\bar{\theta}_1+\bar{\theta}_2}}]_\xi\\ &= \frac{(e^{\bar{\theta}_1}+e^{\bar{\theta}_2}+a_{12}e^{\bar{\theta}_1+\bar{\theta}_2})({k_1}^2e^{\bar{\theta}_1}+{k_2}^2e^{\bar{\theta}_2}+a_{12}(k_1+k_2)^2e^{\bar{\theta}_1+\bar{\theta}_2})}{(1+e^{\bar{\theta}_1}+e^{\bar{\theta}_2}+a_{12}e^{\bar{\theta}_1+\bar{\theta}_2})^2}\\ &- \frac{(k_1e^{\bar{\theta}_1}+k_2e^{\bar{\theta}_2}+a_{12}(k_1+k_2)e^{\bar{\theta}_1+\bar{\theta}_2})^2}{(1+e^{\bar{\theta}_1}+e^{\bar{\theta}_2}+a_{12}e^{\bar{\theta}_1+\bar{\theta}_2})^2}\end{aligned} \tag{34}$$

$$\bar{\phi}^{(1)}=-\frac{2}{(1+e^{\bar{\theta}_1}+e^{\bar{\theta}_2}+a_{12}e^{\bar{\theta}_1}+\bar{\theta}_2)^2}(k_1^2e^{\bar{\theta}_1}+k_2^2e^{\bar{\theta}_2}+e^{\bar{\theta}_1+\bar{\theta}_2}). \tag{35}$$

with,

$$\phi^{(1)} = 12(B/A)\frac{(k_1^2 e^{\theta_1} + k_2^2 e^{\theta_2} + [(k_1 - k_2)^2 + a_{12}(k_1 + k_2)^2 + k_1^2 e^{\theta_2 + k_2^2 e^{\theta_1}}]e^{\theta_1 + \theta_2})}{(1 + e^{\bar{\theta}_1} + e^{\bar{\theta}_2} + a_{12}e^{\theta_1 + \theta_2})^2} \quad (36)$$

with $\theta_i = (k_i\xi - \frac{k_i^4 + 3l_i^2}{k_i}B\tau + l_i\sqrt{\frac{3B}{C}\eta} + \alpha_i)$.

## V. Numerical Results and Discussions

In this section we now present our numerical simulations. For small amplitude analysis we have plotted a number of graphs. The various physical parameter i.e. density ratio of ions to electrons ($\alpha$)and spectral index (k) play significant role on the solution of DIASWs of the KP equation (16) have been observed critically in this section. Relying on the above typical parameters, we have plotted various solitary wave profile. Two solitons have different amplitudes will collide when the largest one of them is overtaking the smaller. If the **scenario** is different i.e. when they have different amplitudes, they also have different velocities and asymptotically they will be separated in the system. And then **2SS** of KP equation have attained the local structures. In Figure 1 we have plotted the time evolution of solitary wave profile i.e. amplitude of the DIASWs of two soliton $\phi^{(1)}$ vs $\xi$ for the various values of $\tau$ with fixed values of the other physical parameters $k_1$=1, $k_2$=2, $l_1$=1,$l_2$=2, $\eta$=1,k=3, a=0.50 and $\alpha$=1. At $\tau$=-20 the bigger amplitude soliton is behind the smaller amplitude soliton. After that when $\tau$=-6 two solitons start to coincide and formed one soliton at $\tau$=0. When the value of $\tau$=6, the two soliton start to differentiate to each other and finally at $\tau$=20 each appears as a single soliton which preserves their speed and shape i.e. these two dynamical variables remain unchanged. In Fig. 2 we have plotted the joint combination of compressive two soliton of DIASWs for different values of $\tau$ with same physical parameters as in Fig. 1. In Fig. 3 we have plotted the variation of compressive two soliton profiles for different values of $\alpha$ , with the values of $\tau$=-20, k=5 and other physical parameters are same as in Fig. 1. It is observed that the amplitude of the two soliton of DIASWs increases with the increasing value in k. Dynamically, it happens due to reason that the increase of $\alpha$ makes the coefficient of the nonlinear term of the KP equation (16) to increase. And for this circumstances, the soliton amplitude of DIASWs goes down. In Fig. 5 we have plotted the amplitude of DIASWs two soliton i.e. $\phi^{(1)}$ against $\xi$ for various values of $\alpha$. It is found that the amplitude of larger soliton increases with the increasing value of $\alpha$. Practically, this happens due to the fact that the growing value of a makes the nonlinear coefficient of Generalised KP equation to decline and for that phenomena, the two soliton amplitude goes up. It is noticed that the soliton with the bigger amplitude over takes the small soliton amplitude of DIASWs eventually. Finally we observed that the larger soliton of DIASWs moves faster and approaches smaller one and after the overtaking collision both soliton remain keep their initial physical and dynamical properties. Thus, the KP equation of dense magnetised plasma gives a good estimate of the nature of nonlinear structures in DIASWs.

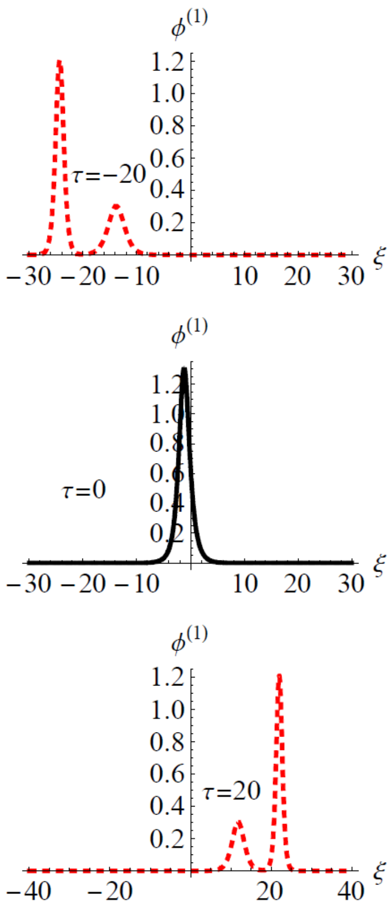


**FIGURE 1.** Variation of compressive two-soliton profiles for different values of $\tau$, with $k_1 = 1$, $k_2 = 2, l_1 = 1, l_2 = 2, \eta = 1, k = 3, a = 0.5, \alpha = 1$.

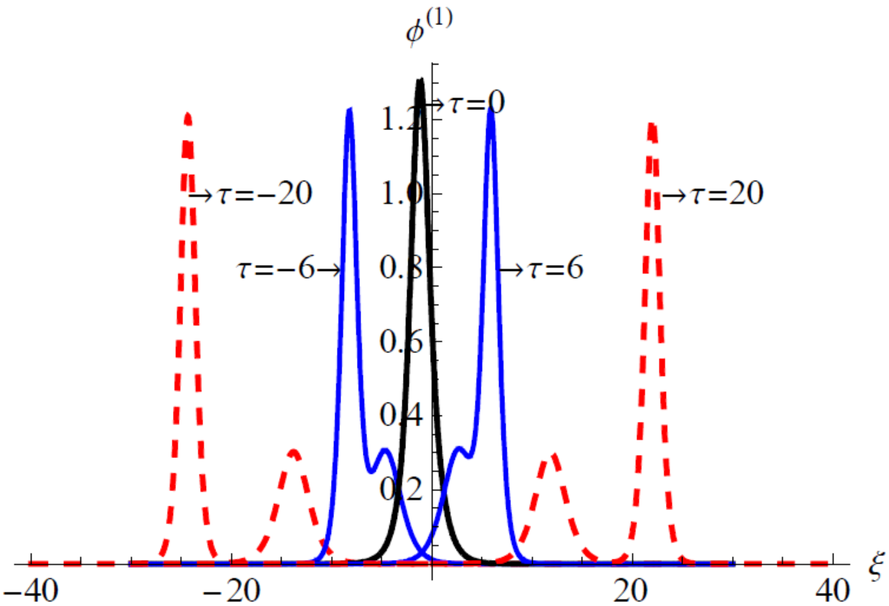


**FIGURE 2.** Variation of the combined compressive two-soliton profiles for different values of $\tau$, with same values of other parameters as Fig 1.

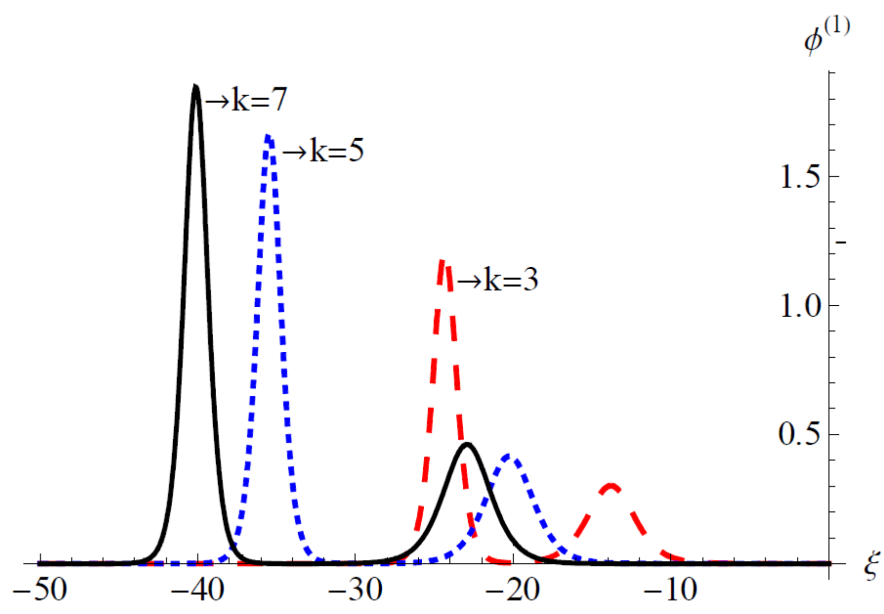


**FIGURE 3.** Variation of the combined compressive two-soliton profiles for different values of $k$, with $\tau = -20$ and other parameters are same as Fig 1.

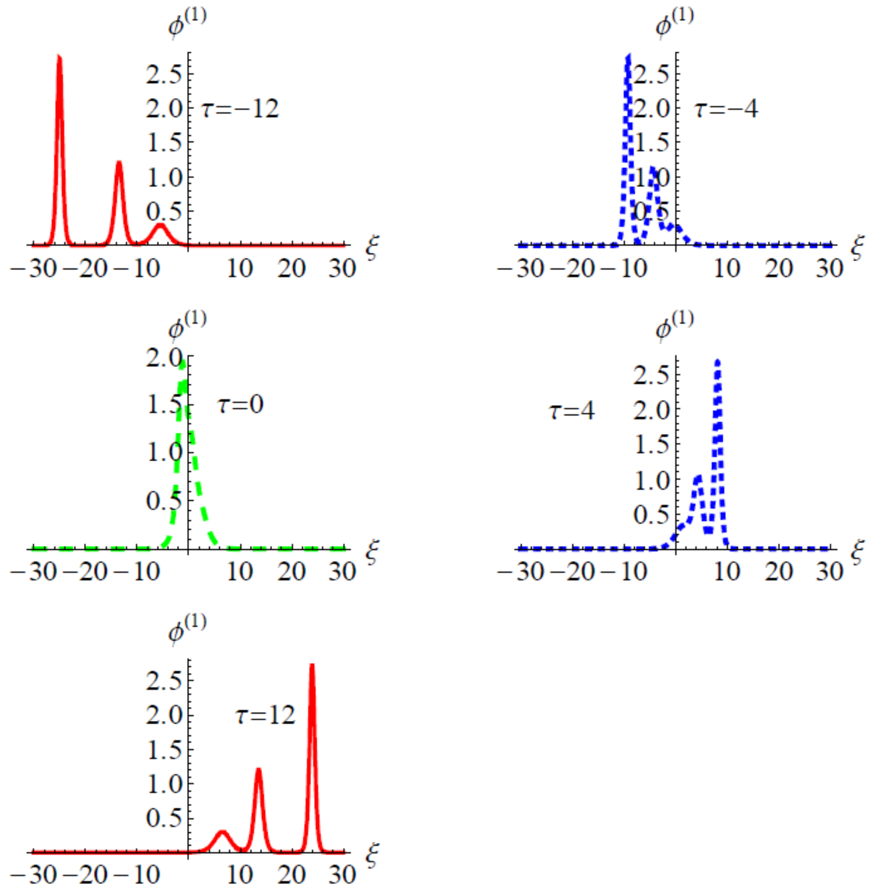


**FIGURE 4.** Variation of the combined compressive two-soliton profiles for different values of $\tau$ and other parameters are same as Fig 1.

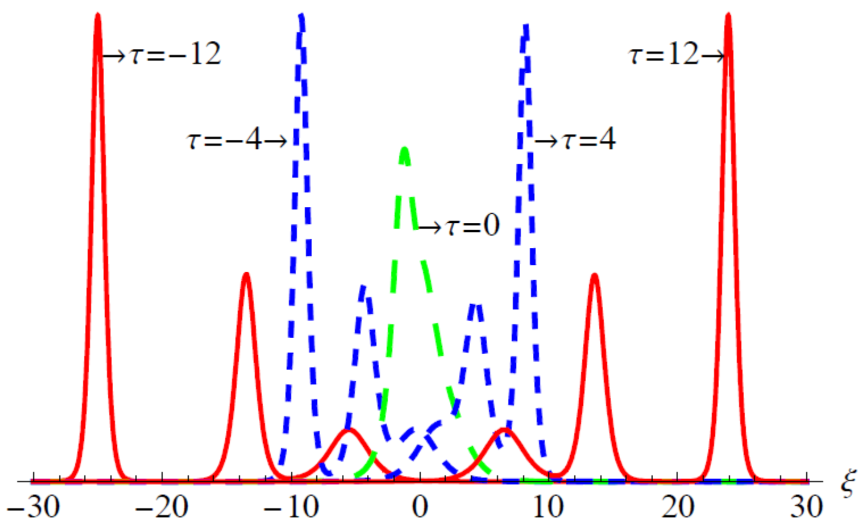


**FIGURE 5.** Variation of the combined compressive two-soliton profiles for different values of $\tau$ and other parameters are fixed as Fig 1.

## Conclusions

This manuscript reported an interesting study of nonlinear waves in complex dusty plasmas which may have potential applications in structure formation and wave phenomena in astrophysics plasmas. Theoretical analysis of nonlinear waves such as those in magnetised dusty plasmas are often challenging specially when strong nonlinearity is present. However RPT are widely used and is proven to be useful. In this paper, we carried out the 2SS of DIASWs with kappa distributed electron for KP equation. For this purpose we assumed a complex plasma model and plotted a number of figures. We applied RPT to obtain KP equation and using HBM which is a powerful mathematical tool to obtain 2SSof KP equation of DIASWs. At last, the numerical simulations obtained here may be useful to investigate the nature of nonlinear characteristics of magnetised dusty plasma with k distributed electrons as well as the two solitonic structures in magnetosphere of the earth, where super thermal electrons are present.

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