The effect of kappa distributed electrons on the dust ion acoustic solitary wave in a collisional dusty plasma

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Abstract. The propagation of dust ion acoustic solitary waves (DIASWs) in a collisional dusty plasma with kappa distributed electrons is investigated. Applying the reductive perturbation technique (RPT), the deformed Korteweg-de Vries (dKdV) equation is derived for the propagation of weakly nonlinear DIASWs. Effect of the spectral index κ on the DIASWs is presented through numerical simulations. The spectral index κ plays a significant role on the excitation of the DIASWs.

Keywords: Solitary wave; Dust ion acoustic wave; Collisional dusty plasma.

I. INTRODUCTION

Dusty or complex plasma is described by the presence of the distinctive nature of highly charged and massive dust grains, in addition to the usual electrons, ions and neutral particles. Charged dust grains are omnipresent in industrial, astrophysical and space plasma environments. It has been recognized, that dusty plasma could have wide ranging applications in a diversity of plasma environments, such as planetary rings, commentary tails, interplanetary medium, interstellar clouds, and the lower parts of the Earth's ionosphere [1]-[2] and this has precipitated a rapid growth in studies of plasma. Nonlinear waves in dusty plasma have been an active area of research for past few decades[3]-[4].It is important to note that the presence of dust component in a plasma introduces new Eigen modes, such as, dust acoustic mode, dust ion acoustic mode, dust lattice mode, dust drift mode, etc. Dust grains, being negatively charged are considered to be immobile due to their heavy mass [5]. The phase velocity of the wave, which is called the dust ion acoustic wave (DIA) increases due to the decreased density of the electrons [6]-[7].DIA waves have been studied theoretically and experimentally. Luo et al. [8] studied of shock formation in dusty plasmas experimentally with collisionless plasma limit condition. Due to the heavier mass of the dust compared to ions, dust dynamics introduce low-frequency dust-modes below the ion-cyclotron frequency in dusty plasmas. In this low frequency limit, collision of the charged particles with the neutrals may play important roles in the properties of dust modes. Ghosh et al. [9] studied the effect of damping on nonlinear ion acoustic wave due to ion-dust collisions. Losseva et al. [10] studied the evolution of weakly dissipative hybrid dust-ion-acoustic soliton in inhomogeneous collisional plasma. Roychowdhury et al. [11] discussed the dust charge fluctuation effects on the phase velocity of dust acoustic wave. Popel et al. [12] showed that speed of the wave obtained by the experiment is larger than that of theoretical results. Losseva et al. [13] investigated arbitrary amplitude dust ion acoustic soliton like perturbation in dusty plasmas. Kruskal [14] investigated that the KdV equation is the unique asymptotically correct model for the gravity wave in an inviscid fluid. Many theoretical and experimental investigations on the dust ion acoustic solitary waves (DIAWs) have been made by a large number of researchers [15]-[18]. Ghorui et al. [19] studied the head on collision of DIAWs in magnetized dusty plasma. Recently, Ghosh et al. [20] studied the dynamic structures of ion acoustic waves in un-magnetized plasma with non-Maxwellian electrons and positrons applying the bifurcation theory of planar dynamical systems through direct approach. El-Hanbaly et al. [21] investigated the linear and nonlinear dust acoustic waves in dusty plasmas considering extremely massive, micron-sized negative dust grains. The authors showed the effects of non-thermal coefficient, ions temperature, and ions number density on the amplitude and width of soliton in dusty plasma. Saha and Chatterjee [22] investigated dynamical behavior of dust ion acoustic waves in super-thermal plasmas with q-nonextensive electrons using the bifurcation theory of planar dynamical systems through direct approach. Bacha et al. [23] studied dust ion-acoustic solitary waves in dusty plasmas with non-extensive electrons. Chattejee et al. [24] investigated the head on collision between two dust ion acoustic (DIA) solitons in dusty plasmas. Tribeche and Zerguini [25] studied the current-driven dust ion-acoustic instability in collisional dusty plasma with charge fluctuations. Recently, Maitra et al. [26] studied the dust ion acoustic solitary waves in collisional dusty plasma with dust grains having Gaussian distribution.

Most of these studies, Maxwellian velocity distribution of the plasma species is assumed, but rigorous studied on astrophysical system and space plasmas clearly indicate that these particles have velocity distributions which deviate from Maxwellian behavior [27]-[30] due to non-equilibrium stationary state. Recently, a significant interest has been paid on the study of nonlinear waves in non-Maxwellian plasma [31]-[34]. The kappa or the generalized Lorentzian distribution is used as a useful tool to study the kinetic modeling of waves and instabilities in space plasmas. In many cases, the plasma is either taken to be un-magnetized or in case of magnetized plasma. The wave propagation is assumed to be parallel to the ambient magnetic field [35]. The κ distribution obeys an inverse power law at high velocities and for all velocities Maxwellian distribution behaves as a special case of Kappa distribution. Kappa distributions are highly favored in any kind space plasma modeling, where a reasonable physical background is not apparent. The family of kappa distributions [26] is obtained from the positive definite part $12 \leq \kappa \leq \infty$, corresponding to $-1 \leq q \leq 1$of the general statistical formalism. If we take kappa distribution as $f_0(v) = \left[\frac{1+v^2}{\frac{\kappa}{\theta^2}}\right]^{-(\kappa+1)}$, where $v = v_x^2 + v_y^2 + v_z^2$, θ is the effective thermal speed $v_{th} = \left(\frac{2K_BT}{m}\right)^{\frac{1}{2}}$ given by$\theta^2 = \left[\left(\kappa - \frac{3}{2}\right)/\kappa\right] v_{th}^2$ and κ is a spectral index which measures the slope of the energy spectrum of super thermal particles forming the tail of velocity distribution [36]-[37]. This velocity distribution tends to Maxwellian at very large value of κ, while at low values of κ, they act as a hard energy spectrum with strong non-Maxwellian tail having power law form at high velocities. Typical values of κ for space plasma lie between 2 and 6. Samanta et al. [38] studied the bifurcations of nonlinear ion acoustic traveling waves in the frame of a Zakharov-Kuznetsov equation in magnetized plasma with a kappa distributed electron. In this work, we study DIASWs in collisional dusty plasma with kappa distributed electrons.

The organization of the present work is as follows: we introduce the basic equations in section II. In section III, the deformed KdV equation is derived and solution of the dKdV is obtained. The section IV is kept for results and discussions. Section V presents the conclusions of the work.

II. BASIC EQUATIONS

We consider a model of an un-magnetized collisional dusty plasma consisting N species of stationary dust grains with densities n_{dj} and charges given $Q_{dj}(= eZ_{dj,}$ where Z_{dj} is the number of charge residing on j^{th} dust grain) for $j = 1,2,3 \cdots N$. The basic equations for the ions are given by

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{1}$$
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -3n\frac{\partial n}{\partial x} - \upsilon_{id}u - \frac{\partial \varphi}{\partial x}, \tag{2}$$
$$\frac{\partial^2 \varphi}{\partial x^2} = \delta_1 + \delta_2 n_e - n. \tag{3}$$

The number density of electrons n_e is given by $n_e = \left(1 - \frac{\varphi}{\kappa - \frac{3}{2}}\right)^{-\left(\kappa + \frac{1}{2}\right)}$. The charge neutrality condition at equilibrium leads $n_{i0} = \sum_{j=1}^{N} Z_{dj0} n_{dj0} + n_{e0}$, which gives $\delta_1 + \delta_2 = 1$. The different physical quantities are normalized as follows: the ion density n, velocity u, and electronic potential φ are normalized by $n_{0,}$ C_i and $\frac{KT_i}{e}$ respectively, where C_i is the ion acoustic speed $(KT_i/m)^{1/2}$. The space coordinate x and time t are normalized by Debye length $\lambda_D = (KT_i/4\pi n_0 e^2)^{1/2}$ and ω_p^{-1} respectively, where ion frequency $\omega_p = (4\pi n_0 e^2/m)^{1/2}$, $\delta_1 = \frac{\sum_{j=1}^{N} Z_{dj0} n_{dj0}}{n_0}$ and $\delta_2 = \frac{n_{e0}}{n_0}$. T_i is the temperatures of ions. Here n_0 and n_{e0} are the densities of ions and electrons at equilibrium.

III. NONLINEAR ANALYSIS

The reductive perturbation technique is used to study small but finite amplitude dust ion acoustic waves. The stretched coordinates are taken as

$$\xi = \varepsilon^{\frac{1}{2}}(x - Mt), \tag{4}$$
$$\tau = \varepsilon^{\frac{3}{2}}\, t, \tag{5}$$

where M is the phase velocity of the wave and ε is a small parameter measuring the weakness of the dispersion. The dependent variables are expanded in the following manner:

$$n = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \varepsilon^3 n_3 \cdots, \quad (6)$$
$$u = 0 + \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 \cdots, \quad (7)$$
$$\varphi = 0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \varepsilon^3 \varphi_3 \cdots, \quad (8)$$
$$\upsilon_{id} \approx \varepsilon^{\frac{1}{2}} \upsilon_{id0.} \quad (9)$$

We have taken the dust ion collisional frequency as leads $\upsilon_{id0} = \sum a n_{dj0} r_{dj}^2$, a being a constant. Here, the ion density has nonzero equilibrium value. The boundary condition as $\xi \to \pm\infty$, $n \to 1$, $\varphi \to 0$ and $u \to 0$. Substituting equations (4)-(9) in equations (1)-(3) and equating coefficients of same power of ε from both sides, we obtain relations between the perturbed quantities. Considering the lowest order in ε, we obtain

$$n_1 = (1 - \delta_1) A \varphi_{1,} \quad (10)$$
$$u_1 = M(1 - \delta_1) A \varphi_{1,} \quad (11)$$
$$A = \frac{\kappa + \frac{1}{2}}{\kappa - \frac{3}{2}}. \quad (12)$$

The dispersion relation of the system is given by

$$M^2 = 3 + \frac{1}{(1 - \delta_1) A}. \quad (13)$$

The next order in ε yields

$$\frac{\partial \varphi_1}{\partial \tau} + A_1 \varphi_1 \frac{\partial \varphi_1}{\partial \xi} + B_1 \frac{\partial^3 \varphi_1}{\partial \xi^3} + C_1 \varphi_1 = 0, \quad (14)$$

where

$$A_1 = \frac{3A^3(1-\delta_1)^2(M^2+1) - B}{2A^2 M^2 (1-\delta_1)}, \quad (15)$$
$$B = \frac{\left(\kappa + \frac{1}{2}\right)\left(\kappa + \frac{3}{2}\right)}{\left(\kappa - \frac{3}{2}\right)^2}, \quad (16)$$
$$B_1 = \frac{1}{2A^2 M^2 (1-\delta_1)^2}, \quad (17)$$
$$C_1 = \frac{\upsilon_{id0}}{2M}. \quad (18)$$

Now it is observed that in absence of C_1 i.e., when $C_1 = 0$ then Eq.(14) becomes the KdV equation with the solitary wave solution with respect to a frame moving with phase speed λ is given by,

$$\varphi = \varphi_m \, sech^2 \left(\frac{\xi - \lambda \tau}{\omega}\right), \quad (19)$$

where φ is written in place of φ_1, with $\varphi_m = \left(\frac{3\lambda}{A_1}\right)$ is the amplitude and $\omega = 2\sqrt{\frac{B_1}{\lambda}}$ is the width of the solution. A damped KdV equation is considered to be a KdV equation with a linear driving force.

Now to determine the effect of collision on the solution given by Eq. (19), we consider here momentum conservation law. In presence of collision, this corresponds to

$$\frac{dI}{d\tau} = -2C_1 I, \tag{20}$$

$$I = \int_{-\infty}^{\infty} \varphi^2 \, d\xi. \tag{21}$$

Then the slow time dependence form of the solution of Eq.(14) is given by

$$\varphi(\tau) = \varphi_m \, sech^2\left(\frac{\xi - \lambda(\tau)\tau}{\omega(\tau)}\right). \tag{22}$$

After integrating Eq.(20) with respect to τ from initial time τ_0 to τ and using Eqs.(19)-(22), we obtain the time (τ) dependent soliton amplitude, width, and velocity given by:

$$\varphi_m(\tau) = \frac{3\lambda}{A_1} e^{-\frac{4}{3}C_1(\tau-\tau_0)}, \tag{23}$$

$$\omega(\tau) = 2\sqrt{\frac{B_1}{\lambda} e^{\frac{4}{3}C_1(\tau-\tau_0)}}, \tag{24}$$

$$\lambda(\tau) = e^{-\frac{4}{3}C_1(\tau-\tau_0)}. \tag{25}$$

IV. RESULT AND DISCUSSIONS

The reduction perturbation technique is employed to derive the deformed Korteweg-de Vries (dKdV) equation and we study the effect of κ on small amplitude dust ion acoustic solitary waves in collision dusty plasmas. The variation of amplitude and width of the solitary wave solution for the deformed Korteweg-de Vries (dKdV) equation is studied. It is important to note that the solitary wave is compressive $A_1 > 0$ and rarefactive if $A_1 < 0$.

In figure 1, variation of amplitude of solitary wave profile with κ is plotted for $\upsilon_{id0} = 0.6$, $\delta_1 = 0.2$, $\lambda = 1$, $\tau = 5$ and $\tau_0 = 0$. It is clear from figure 1 that amplitude increases as κ increases. In figure 2, variation of amplitude of the solitary wave with κ is plotted for $\upsilon_{id0} = 0.6$, $\delta_1 = 0.2$, $\lambda = 4$, $\tau = 5$ and $\tau_0 = 0$. It is clear from figure 2 that amplitude also increases with κ.

The behavior of dust ion acoustic solitary wave structure is shown for different values κ with $\upsilon_{id0} = 0.6$, $\delta_1 = 0.4$, $\lambda = 1.1$, $\tau = 5$ and $\tau_0 = 0$ in figure 3. It has been observed through numerical simulations that when $\kappa > 1.5$ (approximately), then the deformed Korteweg-de Vries (dKdV) equation has compressive solitary wave solution, where as the deformed Korteweg-de Vries (dKdV) equation has rarefactive solitary wave solution for $\kappa < 1.5$ (approximately).

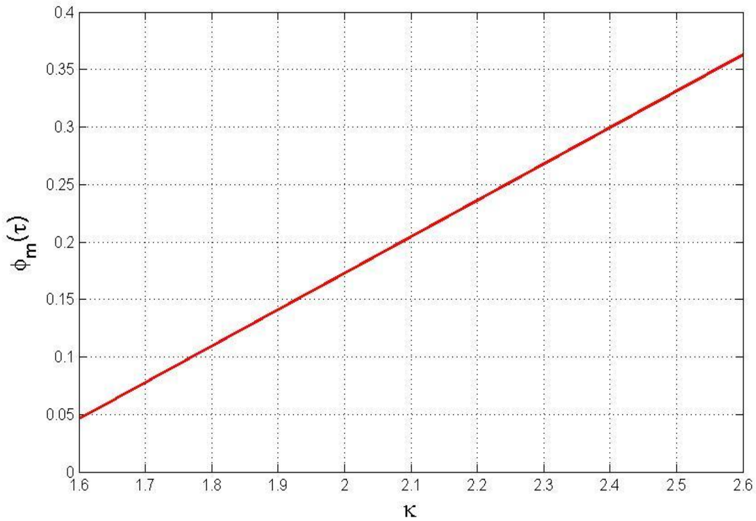

FIGURE 1. $\varphi_m(\tau)$ vs κ is plotted for $\upsilon_{id0} = 0.6$, $\delta_1 = 0.2$, $\lambda = 1$, $\tau = 5$ and $\tau_0 = 0$.

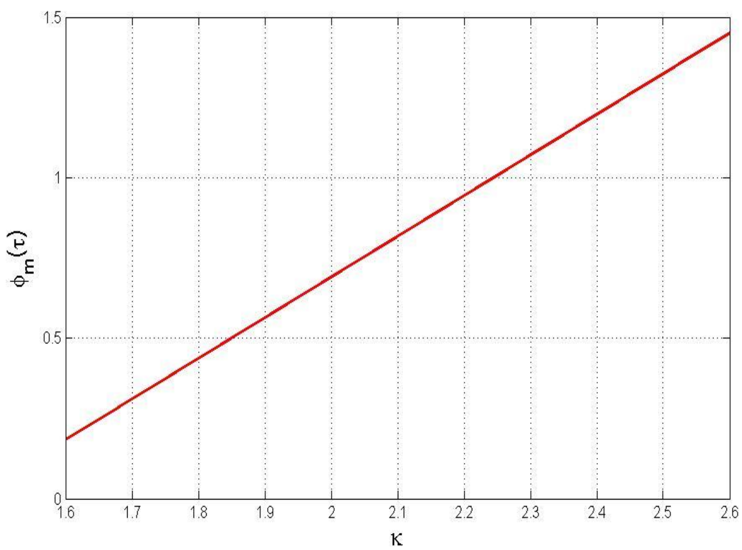

FIGURE 2. $\varphi_m(\tau)$ vs κ is plotted for $\upsilon_{id0} = 0.6$, $\delta_1 = 0.2$, $\lambda = 4$, $\tau = 5$ and $\tau_0 = 0$.

Furthermore, amplitude and width of the compressive (rarefactive) solitary wave increase with increase (decrease) in spectral index κ from its critical value. In other words, the dust ion acoustic solitary wave in collisional dusty plasma is noticed to flourish when electrons evolve far away from their Maxwell-Boltzmann equilibrium. In figure 4, the sudden change of the nature of DIA solitary wave from negative to positive is shown. Recently, Maitra and Banerjee [26] showed that increase in δ_1 diminish the dust ion acoustic solitary wave profile in collisional dusty plasmas with dust grains having Gaussian distribution.

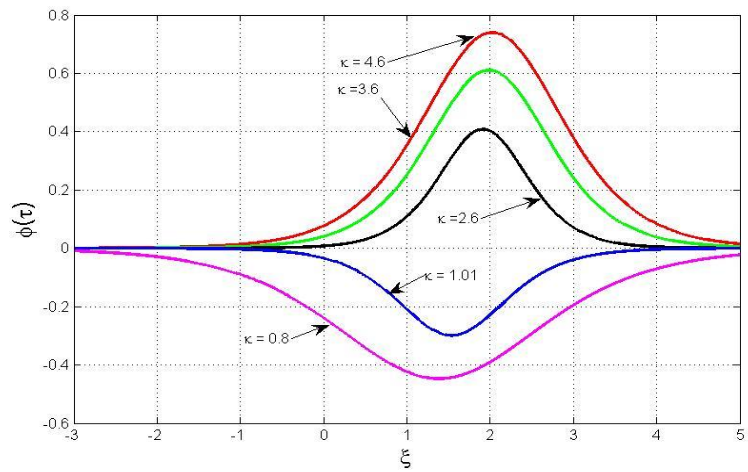

FIGURE 3. Plot of soliton solution $\varphi(\tau)$ for different values of κ with $v_{id0} = 0.6, \delta_1 = 0.4, \lambda = 1.1, \tau = 5$ and $\tau_0 = 0$.

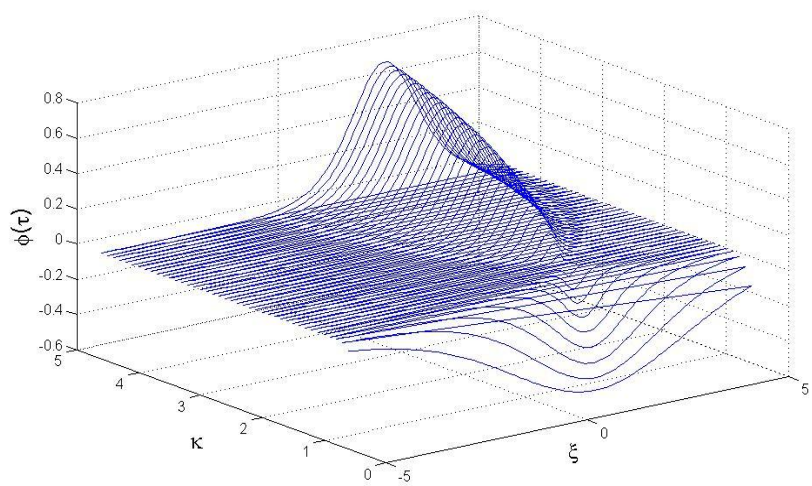

FIGURE 4. Plot of soliton profile against ξ and κ with other parameters same as FIGURE 3.

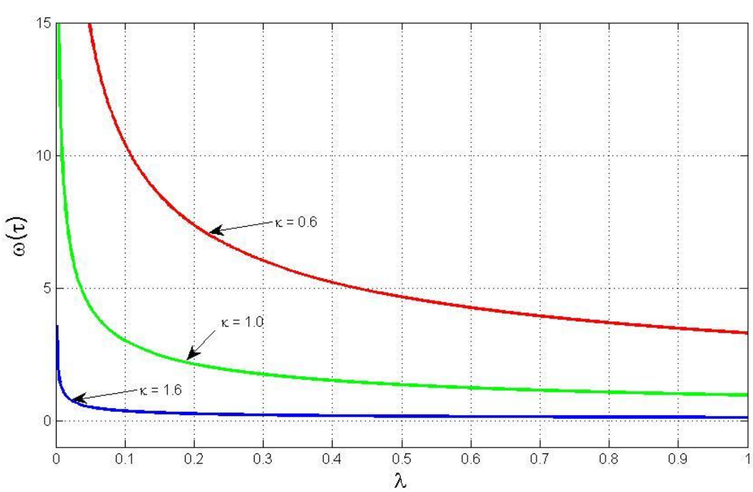

FIGURE 5. $\omega(\tau)$ vs. λ is plotted for different values of κ with other parameters are same as FIGURE 3.

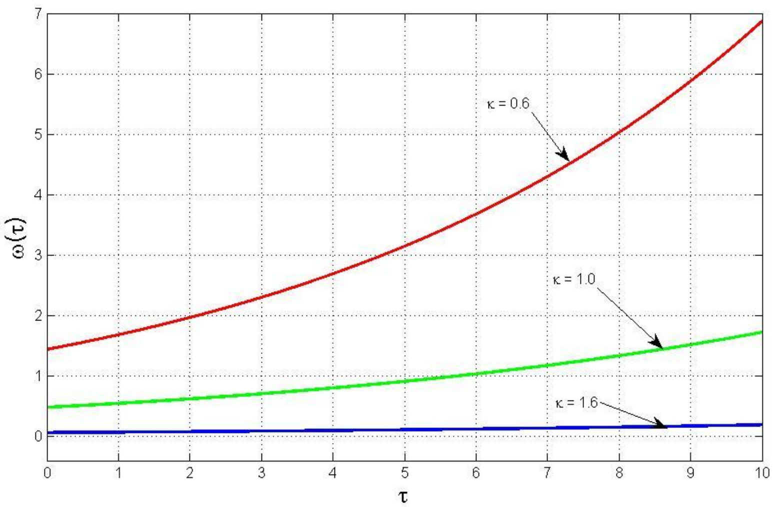

FIGURE 6. $\omega(\tau)$ vs. τ is plotted for different values of κ with other parameters are same as in FIGURE 3.

The dependence of width with λ is plotted in figure 5 for different values of κ, which shows that the width decreases as λ increases. The dependence of width with τ is plotted in figure 6 for different values of κ, which shows that the width increases with τ and also dust distribution affects DIA solitary waves. Luo et al. [8] performed an experimental investigation of the effect of negatively charged dust on ion acoustic shock formation in a Q machine. They observed ion acoustic compressional pulses to steepen as they traveled through a dusty plasma when the percentage of the negative charge in the plasma on the dust grains was greater than or equal to 75%. Thus, the results of the work (Luo et al. 1999) are in agreement with our results.

V. CONCLUSIONS

The characteristics of small but finite amplitude DIA solitary waves have been investigated in collisional dusty plasma with kappa distributed electrons. The deformed KdV equation has been derived using the reductive perturbation technique. From numerical simulations, it has been found that the plasma system has compressive solitary wave potential profile when the spectral index κ assumes values which are greater than the critical value $\kappa = 1.5$ (approximately) and the plasma system has rarefactive solitary wave potential profile when the spectral index κ assumes values which are less than the critical value. Moreover, the width of the solitary wave depends on velocity of the solitary wave profile and time.

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